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Multiobjective Optimization for Tuning a PID Controller

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Abstract: The tuning of PI and PID controllers is typically performed by classical techniques such as Ziegler-Nichols and Cohen-Coon, these techniques require interventions in the plants and do not provide suggestions of preference for the responses of the systems. This paper proposes a tuning for a controller applied to a thermal system using a Multiobjective Genetic Algorithm, NSGA-II, and selecting the best tuning through a Multicriteria Decision Making method, the Weighted Sum Aggregated Product Assessment. The results were compared with classic techniques and differential evolution, where one can observe the effectiveness of the proposed method.
Keywords: PID, Tuning, Multiobjective Genetic Algorithm, NSGA-II, Multicriteria Decision Making.

1. INTRODUCTION

With the increasing of competitiveness in the industrial sector the search for optimal systems has grown, causing the researches and development of optimal methods for tuning controllers to become a differential in the minimizing of cost of production.

The Proportional-Integral-Derivative (PID) controller and its variants make up about 90% of the control loops in industry (Aström and Hägglund, 2001). Despite the importance of PID controllers, a significant percentage operates in manual and about 50% of the control loops operating in automatic show large variances (Oviedo et al. 2006).

Classical methods, Zeigler-Nichols (ZN) and Cohen-Coon (CC) for tuning controllers are traditionally applied. These classical methods are reasonable for systems that have a single output and are stable, but the adjustment process will be impossible when dealing with multiple outputs and unstable plants, and do not present the possibility of exploring the best possible results for systems.

In the last decades a large number of stochastic techniques have been applied in PID controllers tuning, as can be seen in: (Souza et al, 2011.) (Barbosa et al., 2010), (Rani et al., 2012) and (Popov et al., 2005). The researches indicate the Genetic Algorithms (GAs) as a powerful tool for this application.

The GAs are stochastic search methods that mathematically imitate the natural mechanisms of evolution of species, including processes of "genetic evolution" of populations, survival and adaptation of individuals (Barbosa et al., 2010). Due to the wide applicability of the GAs, they are the most studied techniques of evolutionary computation.

The NSGA-II, presented by Deb et al. (2002), can be applied to tuning PID controller due to its versatility and ability to approach the Pareto Front even in non-linear and nonconvex problems. After approaching the Pareto Front, the best individual will be selected using the multicriteria decision method Weighted Aggregated Sum Product Assessment (WASPAS).

The objective of this study is to compare the results obtained when tuning a PID controller, which was applied to a heat exchanger, using the classical methods and Differential Evolution obtained in Souza et al. (2011) to the results of NSGA-II algorithm

2. GENETIC ALGORITHMS

GAs are based on the evolutionary theory of Charles Darwin, where the most adapted individuals tend to survive and reproduce generating adapted individuals. This method was introduced by Holland (1975) and, according to Linden (2008) has been successfully applied in several optimization problems.

To understand the GAs is necessary to expose some definitions described by Takarashi, R. (2007) and cited by Takarashi, F. (2015):

- Individual: It is a possible solution to the optimization problem;
- Population: It is a set of solutions (individuals) formed in every interaction of the algorithm;
- Generation: It is an interaction where the population is subject to operators and a new population is formed;
- Fitness: It is the measure of the quality of the solutions obtained from the objective and constraint functions;
- Genetic Operators: These are the basic rules of the GAs to generate new populations. The main genetic operators are:

- Selection: According to the fitness function choose the individuals who will suffer crossover.
- Crossover: According to a probability, the algorithm combines information from two or more individuals (parents) to form a new generation (children);
- Mutation: Modifies randomly an individual to form another.

In the multi-objective optimization problems, we are working with different objectives, often conflicting. In these types of problems, solutions that are no worse than any other solution are desired, these solutions are called non-dominated. In these solutions, the improvement of a goal would be directly linked to the worsening of another. Based on that, instead of a single optimal solution, we will find a set of non-dominated solutions, this set is called Pareto-Optimal set. The image of the Pareto-Optimal set in the objective space is called Pareto-Optimal front (Takahashi, 2015).

2.1 O NSGA-II

The NSGA-II uses an elitist approach (permanence of the fittest individuals for future generations) that provides a higher speed in convergence towards Pareto front (Deb et al., 2002), the classification of individuals is linked to dominance relation, "Fast Non-dominated Sorting Approach", where individuals are classified into different Pareto fronts according to the dominance criteria. During the classification may appear individuals who do not have dominance among themselves, then the NSGA-II algorithm suggests a second classification based on density solutions, where individuals not dominated and with less density are more apt. This classification is known as Crowding Distance and allows a better distribution of solutions on the Pareto front.

Another criterion adopted when selecting individuals is the degree of feasibility, this criterion seeks to favor individuals who violate fewer constraints, or those violations with smaller magnitude.

The basic structure of NSGA-II can be analyzed by using the Figure 1 and the flow presented below:

1. Configuration of optimization problem parameters (Population Size, Maximum Number of Generations, Crossover Rate, Mutation Rate and Optimization Variables Bounds);
2. Creation of the Initial Population (Using the random method);
3. Creation of a population of children through Selection, Crossover and Mutation;
4. Creation of a temporary population through the junction of the children and the previous population;
5. Sorting individuals through Fast Non-dominated Sorting Approach and degree of feasibility (Creation of Pareto Fronties);
6. Sorting of individuals of Pareto Frontiers through Crowding Distance;
7. Extraction of the next population considering the population size set in step 1;
8. If the maximum generations number is not reached, go back to step 3.

2.1.1 The initial population

In NSGA-II the initial population is generated from points generated randomly throughout the space of variables (Deb et al., 2002), this method is efficient, since it covers the feasible region for the solutions (Renner, 2003). The size of the initial population is important to the convergence of GA, being necessary to select a large number of individuals to ensure a good exploration of the search space, but is important to note that an increase in population size causes increase in computational effort (Takahashi, 2007).

For this study populations with 40 and 50 individuals were tested and the populations with 50 individuals obtained better results and were adopted.

2.1.2 Selection

The selection stage seeks to select individuals of the population to be used as the next generation parents, one can make an analogy to the theory of natural solution of species. The literature suggests various algorithms for selection, such as: Roulette wheel selection, Random Selection, Selection Rank, among others.

The NSGA-II uses the method of the binary tournament Selection, this method consists of randomly select two individuals in the population organized in order of rank and Crowding distance, and compare them according to Crowded Comparison Operator, the fittest individual is then saved for the crossover stage. This stage is repeated until it has randomly selected a number of individuals equal to the population size (Deb et al., 2002).

2.1.3 Crossover

The crossover stage of the NSGA-II algorithm uses the Simulated Binary Crossover (SBX) (Deb et al., 2002) and in this study was used as the distribution index n_C=20. The distribution index defines the probability distribution of the children regarding to the distance from its parents, the greater the distribution ratio, the greater the proximity to the children with its parents.

2.1.4 Mutation

For the mutation stage is used Polynomial Mutation, this method consists in using a distribution index (n_m) which generates a disturbance in the target variable in the order of: (Ub – Lb)/n_m. Where Ub is the maximum limit of the variable and Lb is the minimum limit (Deb and Agrawal, 1999). In this study we used the distribution index n_m=100.

3. DECISION-MAKING METHODS

The decision methods assist in decision-making, these methods can be used in different situations and provide comparisons between conflicting criteria.

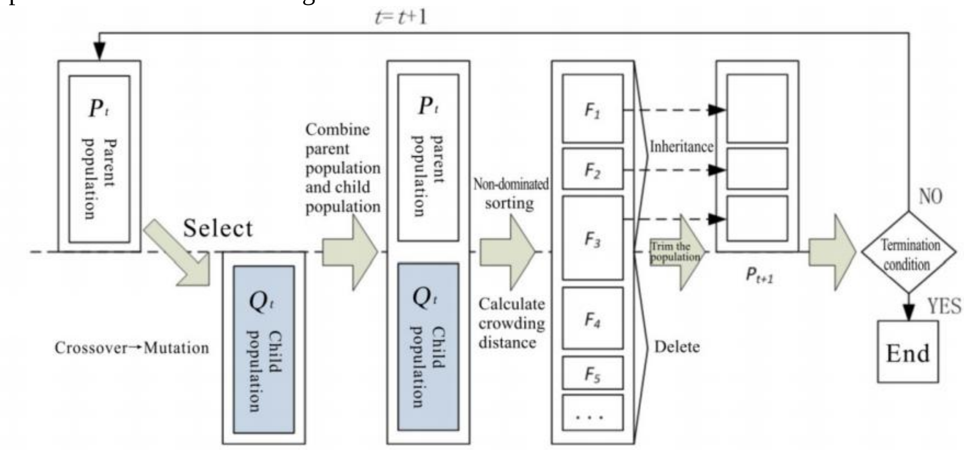

Figure 1. Process of NSGA-II (Jiang, 2014)

There are several decision methods that follow basically two schools: American and French school. Among various methods, we can mention: Analytic Hierarchy Process (AHP), Simple Multi-Attribute Rating Technique (SMART), Weighted Sum (WSM), Weighted Product (WPM), Weighted Aggregated Sum Product Assessment (WASPAS), Elimination and Choice Expressing Reality (ELECTRE), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) and so on.

In this paper we selected the WASPAS method due to its ability to work with multi-dimensional and convex problems (Zavadskas et al., 2012), which is the case of Pareto front presented in this work.

3.1 Weighted Aggregated Sum Product Assessment

The WASPAS method it is the union of the WSM and WPM methods, where the accuracy of WASPAS is superior to the methods used separately (Zavadskas, 2012).

To use the method is necessary the application of the WSM using the equation below:

$$Q_i^{(1)} = \sum_{j=1}^{n} \bar{x}_{ij} * w_j \qquad (1)$$

Where*n*is the number of decision criteria, w_j is the criteria weight and $\bar{x}_{ij}$ is the linear normalization of performance values of each alternative *i* when it evaluation in terms of the criteria *j*.In cases of minimizing the linear normalization, $\bar{x}_{ij}$, is performed through the following equation(Zavadskas et al., 2012):

$$\bar{x}_{ij} = \frac{min_i x_{ij}}{x_{ij}} \qquad (2)$$

According to WPM the total relative importance of alternative *i*, denote as $Q_i^{(2)}$, is defined by following (Triantaphyllou and Mann, 1989):

$$Q_i^{(2)} = \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} \qquad (3)$$

Finally, Zavadskas (2012) proposes the integration of the two previous methods for the construction of WASPAS seeking greater accuracy in decision-making.

$$Q_i = \lambda Q_i^{(1)} + (1-\lambda) Q_i^{(2)}, \lambda = 0 \ldots 1. \qquad (4)$$

4. THE CONTROL SYSTEM

The system to be controlled in this study consists of a heat exchanger shell-tube type in counter flow. This system seeks to heat a water flow from another hot water flow. The heat exchanger can be seen in Figure 2, where: $Q_{c,e}$ and $T_{c,e}$ respectively represent "the flow and the temperature" of the hot fluid input; $Q_{t,e}$ and $T_{t,e}$ represent "the flow and temperature" of cold fluid input; T_c and T_t are respectively the temperature of the fluid in the shell and the temperature of the fluid in the exchanger tubes.The control system seeks to maintain the output temperature of the cold fluid $T_{t,s}$ at 40°C by controlling the hot fluid flow $Q_{c,e}$ through a control valve, the system feedback is provided by a set thermocouple/temperature transmitter.

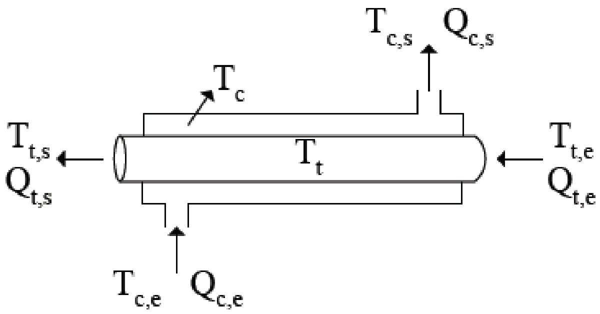

Figure 2. Heat exchanged (Garcia, 2005)

The mathematical modeling of the system is described by Garcia (2005), and equations for the model non-linearized of the heat exchanger are described below:

$$\frac{d(T_{t,s})}{dt} = \frac{\rho_{t,e}.Q_{t,e}.C_{p,a}(T_{t,e} - T_{t,s}) + U.A.\Delta T_{CT}}{\rho_t.V_t.C_{p,a}} \qquad (5)$$

$$\frac{d(T_{c,s})}{dt} = \frac{\rho_{c,e}.Q_{c,e}.C_{p,a}(T_{c,e} - T_{c,s}) - U.A.\Delta T_{CT}}{\rho_c.V_c.C_{p,a}} \qquad (6)$$

$$[T_{t,s}(0)\ T_{c,s}(0)] = [40°\text{C}\ 74{,}4°\text{C}] \qquad (7)$$

$$\Delta T_{CT} = \frac{\Delta T_{max,CT} - \Delta T_{min,CT}}{\ln(\frac{\Delta T_{max,CT}}{\Delta T_{min,CT}})} \qquad (8)$$

Where, $\rho_{t,e}$ is the specific mass of inlet hot water (equal to 998,21 kg/m³); ρ_t is the specific mass of the hot water in the pipes (equal to 995,65 kg/m³); $\rho_{c,e}$ is the specific mass of inlet cold water (equal to 965,31kg/m³); ρ_c is the specific mass of the cold water in the hull (equal to 995,65 kg/m³); V_t is the volume of fluid in the tubes (equal to $3{,}385.10^{-3}$ m³); V_c is the volume of fluid in the hull (equal to $4{,}557.10^{-3}$ m³); $C_{p,a}$ is the specific heat of water (equal to 4186,8 J/kg*K); UA is the overall coefficient of thermal transfer (equal to 961,35 W/K); $\Delta T_{c,t}$ is the logarithmic mean temperature difference.

The behavior of the temperature sensor (thermocouple) and transmitter can be represented by a 1st-order system with a dead time of 13,9s, where the temperature is calculated by:

$$\frac{d(T_t)}{dt} = \frac{K_t.(T_{m,mV} - 1{,}019) - T_t}{t_t} \qquad (9)$$

Where, K_t is the gain of the transmitter (equal to 10,2 mA/mV); t_t is the time constantof the set (equal to 2s); $T_{m,mV}$ is the temperature measured by thermocouple in mV (the conversion table for the thermocouple is found in Garcia (2005)). The initial condition is: $T_t(0) = 14{,}6\,mA$, and the temperature measured in mA is: $T_{t,mA} = \mathrm{T}_t + 4$.

The behavior of the I/P converter along with the control valve can be represented by a 1st-order system, where the valve stem position (X emp.u.) is calculated by:

$$\frac{d(X)}{dt} = \frac{K_{AT}.(v-3) - X}{t_{AT}} \qquad (10)$$

Where, K_{AT} is the static gain of the actuator (equal to 1/12 p.u./psi); t_{AT} is the time constant of the set (equal to 10s); v is the converter output signal in psi (equal to $v = 0{,}75[m(mA) - 4] + 3$, where m is the 4-20mA signal received by the I/P converter). The initial condition is set to$X(0) = 0{,}60869\ p.u.$

The flow behavior across the valve is:

$$Q_{c,e}\left(\frac{m^3}{S}\right) = K_v.C_v.R^{X-1}\sqrt{\frac{\Delta P_v(bar)}{\rho_{c,e}(\frac{kg}{m^3})}} \qquad (11)$$

Where, K_v is the adjustment factor (equal to $7{,}6.10^{-3}$); C_v is the flow coefficient (equal to 27 gpm/psi); R is the rangeability (equal to 30); $\Delta P_v$is the pressure drop across the valve (assumed constant and equal to 0.2 bar).

The structure of the controller used was the ideal PID. Its function can be seen in Equation 12.

$$G_c = K_c\left[1 + \frac{1}{T_i S} + T_d S\right] \qquad (12)$$

5. THE CONTROL SYSTEM

After obtaining the model of the system is necessary to multiobjective optimization problem formulation. First are determined the optimization variables and their limits, in this study the variables are Kc (Proportional Gain) and Ti (Integral Time) and its limits are: 0 < Kc < 100 e 0 < Ti < 100.

To this problem two design objectives are considered: (1) minimal ITSE (integral time squared error) and (2) minimal control (u) effort. By minimizing the first objective is looking for a betterdisturbance rejection and good reference tracking, minimizing the second reduces the quantity of hot water and thus the cost of the control (Popov et al., 2005). The mathematical description of the objectives is:

$$J_1 = \int_0^{Ts} te^2(t)dt \qquad (13)$$

$$J_2 = \int_0^{Ts} |u(t)|dt \qquad (14)$$

Where Ts = 1000s is the simulation time.

The use of ITSE criterion is based on the developed by Killingsworth and Krstic (2006) and cited by Arruda et al. (2008), where the ITSE criterion shows better results in closed loop than other criteria such as ITAE (Integral Time of Absolute value Error) and IAE (Integral of Absolute value Error), besides having greater selectivity to changes in process parameters when compared to ISE (Integral of Square Error)(Caon, 1999).The last part of the optimization problem formulation is the definition of system constraints. For this work the following restrictions have been defined: the maximum overshoot (Mp) less than 1% and the settling time (ts) for the criterion of 5% less than 150s, the representation of these criteria can be seen at Figure 3.

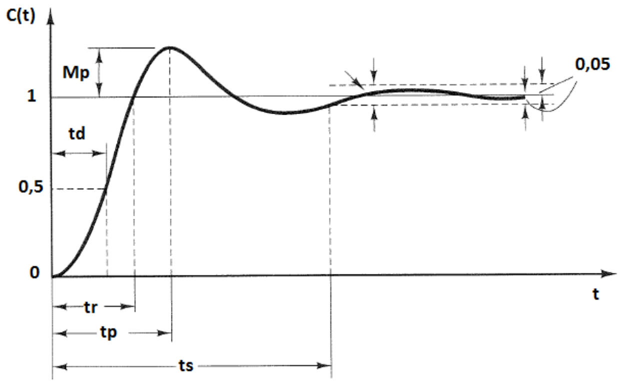

Figure 3. Maximum overshoot (Mp) and settling time (ts) (Ogata, 1998)

The model of the control loop was simulated in Simulink and consists of a black box to the GA, where are inserted the optimization variables and returned values of goals and step response. The restrictions are calculated in a function of MATLAB and are returned to the GA. In Figure 7 can be seen the system simulation.

6. RESULTS

The NSGA-II was performed several times using different parameters in order to obtain a better approximation ofPareto Front. Tests were made varying the distribution ratio parameters for the mutation 20, 50 and 100, another parameter tested was the maximum number of generations of 100, 200 and 250. The best results were obtained by a mutation distribution index equal to 100 and maximum number of generations equal to 250. The results for three simulations with the best parameters are presented in the following Figures 4, 5 and 6:

For a matter of better distribution was selected the approach ofPareto frontier shown inFigure 6. The WASPAS method using weights$w_1 = 0{,}6$, $w_2 = 0{,}4$ and $\lambda = 0{,}5$found the individual 2 of the population as the fittest. In the space of variables, the selected individual consists of:$Kc = 2{,}77$and$Ti = 20{,}50$.

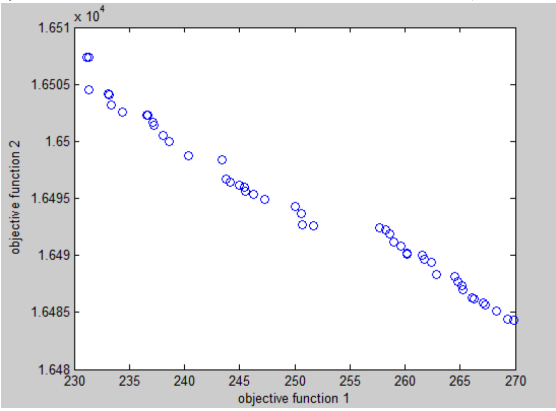

Figure 4. ApproachingPareto-Front 1

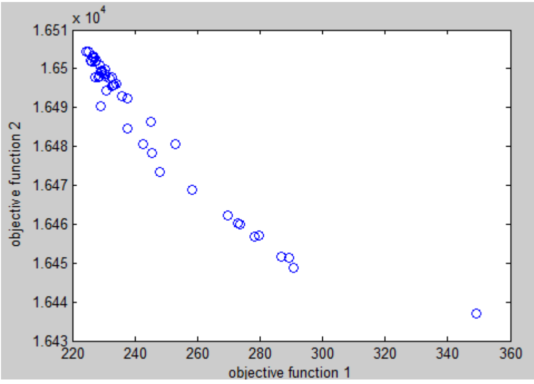

Figure 5. ApproachingPareto-Front 2

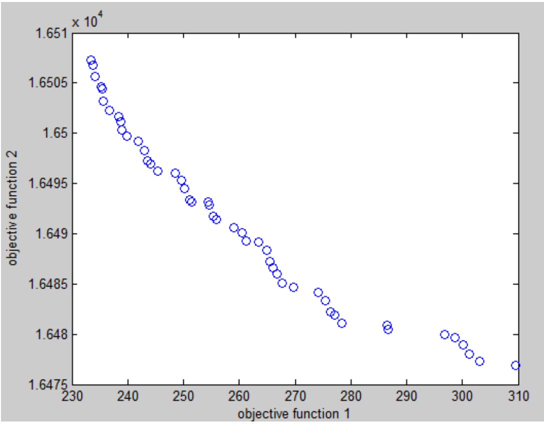

Figure 6. Approaching Pareto-Front 3

The Table 1presents the gainsfor a PI controller, obtained in the literature (Garcia, 2005), obtained in Souza et al. (2011) by the methods: Ziegler-Nichols, through sensitivity threshold (ZN-SL) and reaction curve (ZN-CR), Cohen-Coon (CC-CR) and Differential Evolution method (ED). It also shows the gains obtained in this study using the NSGA-II (NSGA-II). The Figure 8, shows the system response to a step set point, from 40 °C to 41 °C, for all tunings of PI controllers. One can observe the efficiency of the tuning through the NSGA-II method compared to the others.

Table 1. Controller Parameters

Method	**Parameters**	
	Kc	**Ti**
PI (Garcia, 2005)	0,40	2,50
PI (ZN-SL)	4,41	50,00
PI (ZN-CR)	2,52	44,95
PI (CC-CR)	2,72	16,63
PI (ED)	1,00	13,38
PI (NSGA-II)	2,77	20,50

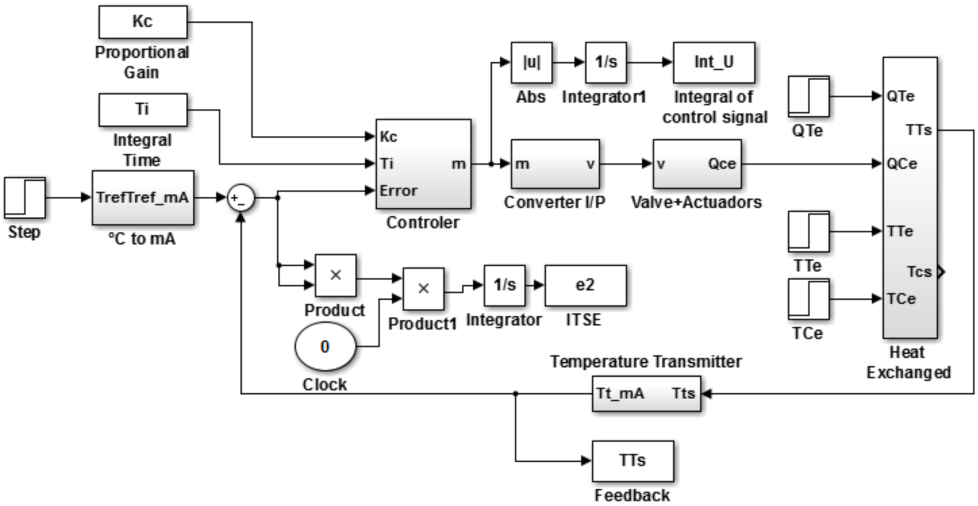

Figure 7. Simulation of System

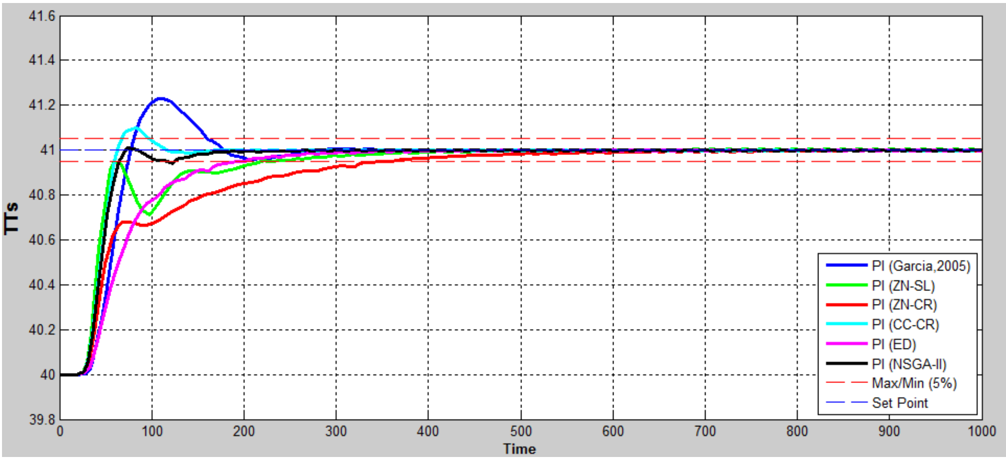

Figure 8. Response to a step set point, from 40 °C to 41 °C, for all tunings of PI controllers.

7. CONCLUSION

This paper provided the study of multiobjective genetic algorithms and decision making method applied to the tuning of PID controllers. The subject method, NSGA-II, was adequate and promising for the task, and presents very interesting results when compared to the classical methods and differential evolution.

As can be seen in the results is important to run the algorithm several times and select the adequate parameters.The GA can converge prematurely and does not provide a good solution, this occurs because it is a stochastic method or because non adequate parameters.

As a proposal for future studies, it is proposed the application of the methods presented in multi-variable and unstable systems.

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