

## Bölcsföldi-Birkás-Bíró monotone prime numbers

József Bölcshöldi<sup>1</sup> György Birkás<sup>2</sup> Tamás Bíró<sup>3</sup>

<sup>1</sup> (Eötvös Loránd University Budapest and Perczel Mór Secondary Grammar School Siófok, Hungary)

<sup>2</sup> (Baross Gábor Secondary Technical School Siófok, Hungary)

<sup>3</sup>(DRV Siófok, Hungary)

**Abstract:** After defining, monotone prime numbers will be presented: the monotone increasing primes from 23 to 5555555777, the monotone decreasing primes from 733 to 777777777533. How many monotone prime numbers are there in the interval  $(10^{p-1}, 10^p)$ , where  $p$  is a prime number? On the one hand, it has been counted by computer among the prime numbers with up to 43-digits. On the other hand, the function (1) gives the approximate number of monotone prime numbers in the interval  $(10^{p-1}, 10^p)$ . Near-proof reasoning has emerged from the conformity of Mills' prime numbers with monotone prime numbers. The sets of monotone prime numbers are probably infinite.

### 1. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ( $F_0=0, F_1=1, F_n=F_{n-1}+F_{n-2}$ ), Gauss-primes (in the form  $4n+3$ ), Leyland-primes (in the form  $x^y+y^x$ , where  $1 \leq x \leq y$ ), Pell-primes ( $P_0=0, P_1=1, P_n=2P_{n-1}+P_{n-2}$ ), Bölcshöldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found 2 further sets of special prime numbers within the set of prime numbers. It are the sets of monotone prime numbers.

### 2. Monotone increasing prime numbers [3], [9], [10], [11], [12].

Definition: a positive integer number is a monotone increasing prime number, if a/ the positive integer number is prime, b/the digits of number are monotone increasing primes, c/ the number of digits is prime, d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are monotone increasing prime numbers (Fig.1, Fig.2).

Monoton increasing prime number  $p$  has the following sum form:

$$p = \sum_{j=0}^{k(p)} e_j(p) \cdot 10^j \quad \text{where } e_j(p) \in \{2, 3, 5, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3, 7\} \text{ and } \sum_{j=0}^{k(p)} e_j(p) \text{ is prime.}$$

**The monotone increasing prime numbers are as follows (the last digit can only be 3 or 7):**

23,  
 223, 227, 337, 557, 577,  
 33377,  
 2222333, 2233337, 2235557, 3337777, 3355777, 5555777,  
 2222222223, 2222222357, 2222333333, 2222333577,  
 22223335777, 22223357777, 2233335777, 2235555777,  
 2255577777, 3333333377, 3355555557, 5555555777, ... etc

2

$M(p)$  is the factual frequency of monotone increasing prime numbers in the interval  $(10^{p-1}, 10^p)$ .  
 $M(2)=1, M(3)=5, M(5)=1, M(7)=6, M(11)=12, M(13)=19, M(17)=19, M(19)=32, M(23)=26, M(29)=42,$   
 $M(31)=66, M(37)=80, M(41)=91, M(43)=105, \dots$  etc.  
 $S(p)$  function gives the number of monotone increasing prime numbers in the interval  $(10^{p-1}, 10^p)$ . We think that the power funktion is  **$S(p)=2,44p$** , where  $p$  is a prime  
 (1)

The factual number of monotone increasing primes and the number of monotone increasing primes calculated according to function (1) is as follows:

Number of digits p	The factual number of monotone increasing primes in the interval $(10^{p-1}, 10^p)$ M(p)	The number of monotone increasing primes according to functions (1) $S(p)=2,44p$	M(p)/S(p)
2	1	4,88	0,20
3	5	4,07	1,23
5	1	12,20	0,08
7	6	17,08	0,35
11	12	26,84	0,45
13	19	31,72	0,60
17	19	41,48	0,46
19	32	46,36	0,69
23	26	56,12	0,46
29	42	70,76	0,59
31	66	75,64	0,87
37	80	102,49	0,78
41	91	100,04	0,91
43	105	104,92	1,00

Fig.1

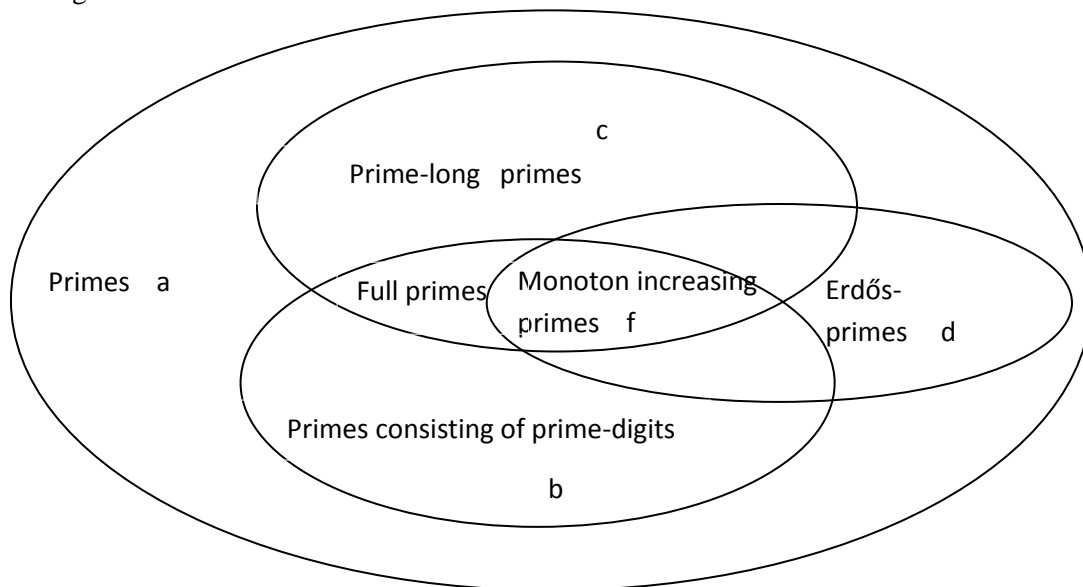
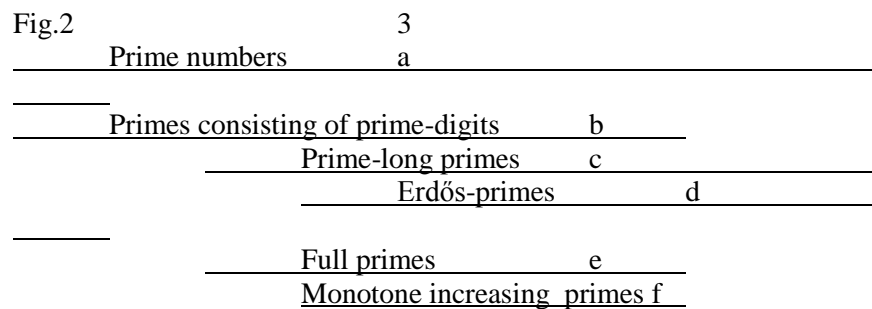


Fig.2



**3. Monotone decreasing prime numbers [3], [9], [10], [11], [12].**

Definition: a positive integer number is a monotone decreasing prime number, if a/ the positive integer number is prime, b/ the digits of number are monotone decreasing primes, c/ the number of digits is prime, d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are monotone decreasing prime numbers (Fig.3).

Monotone decreasing prime number p has the following sum form:

$$p = \sum_{j=0}^{k(p)} e_j(p) \cdot 10^j \quad \text{where } e_j(p) \in \{2, 3, 5, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3\} \text{ and } \sum_{j=0}^{k(p)} e_j(p) \text{ is prime.}$$

The monotone decreasing prime numbers are as follows (the last digit can only be 3):

733, 773,  
 55333, 75533, 77773,  
 7553333, 7555333, 7775533, 777753,  
 5555333333, 7777775553, 7777777533,  
 777533333333, 777777555333, 77777777533, ... etc.

M(p) is the factual frequency of monotone decreasing prime numbers in the interval (10<sup>p-1</sup>, 10<sup>p</sup>).  
 M(2)=0, M(3)=2, M(5)=3, M(7)=4, M(11)=3, M(13)=3, M(17)=10, M(19)=5, M(23)=8, M(29)=11, M(31)=10,  
 M(37)=10, M(41)=10, M(43)=9, M(47)=11, M(53)=11,  
 M(59)=19, M(61)=13, M(67)=15, M(71)=27, M(73)=24, ... etc.

Q(p) function gives the number of monotone decreasing prime numbers in the interval (10<sup>p-1</sup>, 10<sup>p</sup>). We think that the function is linear:

$$Q(p) = 0,325p + 1, \quad \text{where } p \text{ is a prime.} \tag{1}$$

4

The factual number of monotone decreasing primes and the number of monotone decreasing primes calculated according to function (1) are as follows:

Number of digits p	The factual number of monotone decreasing primes in the interval (10 <sup>p-1</sup> , 10 <sup>p</sup> ) M(p)	The number of monotone decreasing primes according to function (1) Q(p)	M(p)/Q(p)
2	0	1,65	0
3	2	1,98	1,01
5	3	2,63	1,14
7	4	3,28	1,22
11	3	4,58	0,66
13	3	5,23	0,58
17	10	6,53	1,53
19	5	7,18	0,70
23	8	8,48	0,94
29	11	10,43	1,05
31	10	11,08	0,90
37	10	13,03	0,77
41	10	14,33	0,70
43	9	14,98	0,60
47	11	16,28	0,68
53	11	18,23	0,60
59	19	20,18	0,94
61	13	20,83	0,62
67	15	22,78	0,66
71	27	24,08	1,12
73	24	24,73	0,97

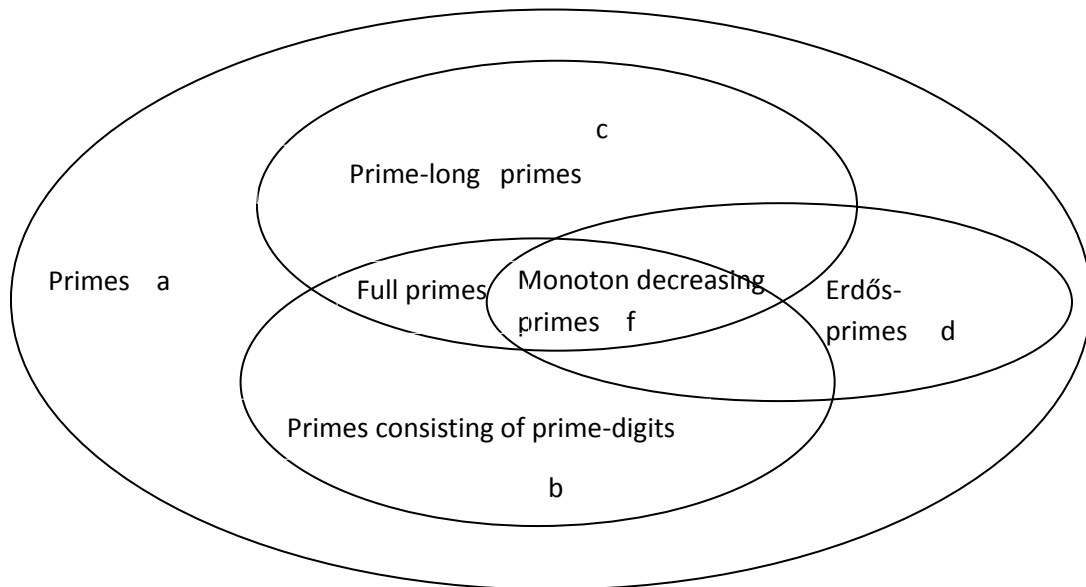
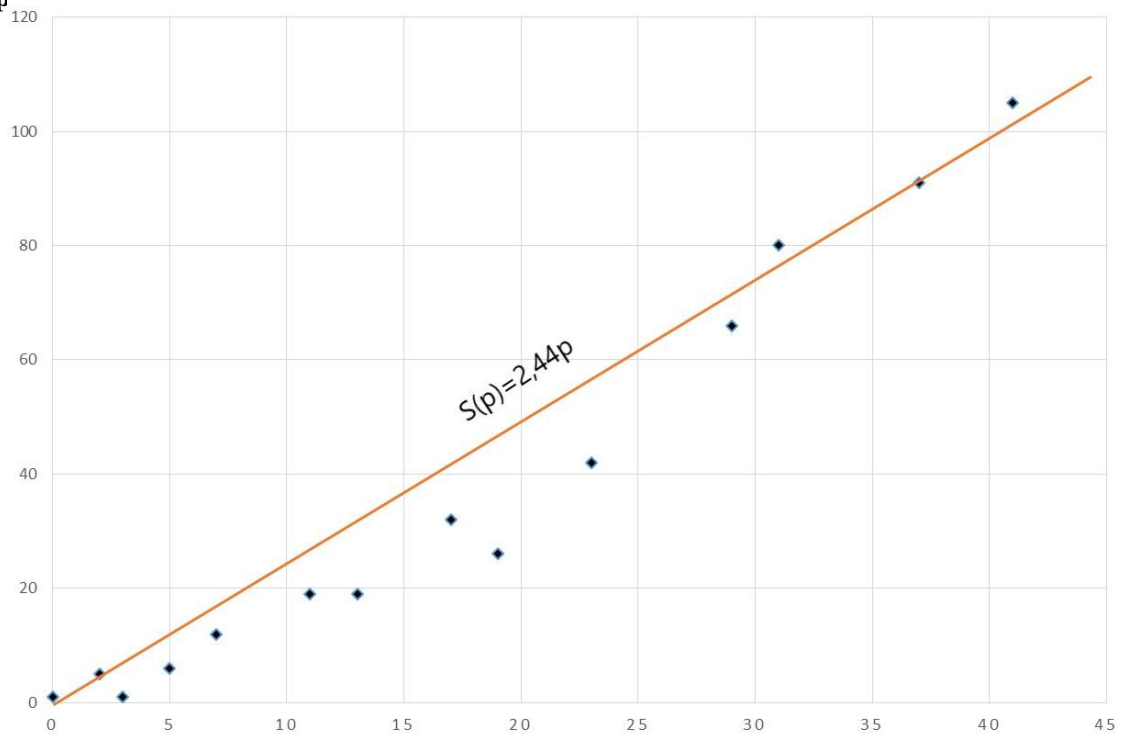


Fig.3

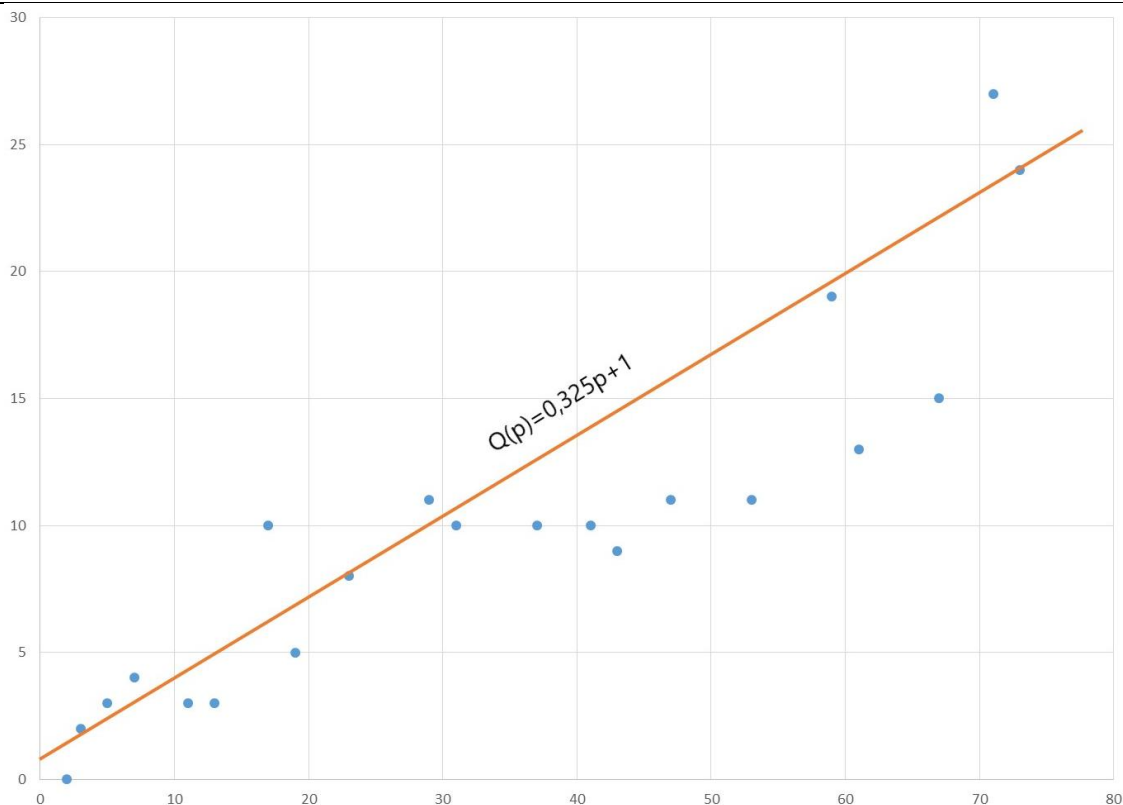
**Approximate function of monotone increasing prime numbers**

$M(p)$  and  $S(p)=2.44p$



**Approximate function of monotone decreasing prime numbers**

$M(p)$  and  $Q(p)=0.325p+1$



6

#### 4. Number of the elements for example of the set of monotone increasing prime numbers [3], [9], [10], [11], [12].

Let's take the set of Mills' prime numbers!

Definition: The number  $m = [M \text{ ad } 3^n]$  is a prime number, where  $M = 1,306377883863080690468614492602$  is the Mills' constant, and  $n = 1, 2, 3, \dots$  is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following:  $m = 2, 11, 1361, 2521008887, \dots$

The connection  $n \rightarrow m$  is the following:  $1 \rightarrow 2, 2 \rightarrow 11, 3 \rightarrow 1361, 4 \rightarrow 2521008887, \dots$ . The Mills' prime number  $m = [M \text{ ad } 3^n]$  corresponds with the interval  $(10^{m-1}, 10^m)$  and vice versa. For instance:  $2 \rightarrow (10, 10^2)$ ,  $11 \rightarrow (10^{10}, 10^{11})$ ,  $1361 \rightarrow (10^{1360}, 10^{1361})$ , etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals  $(10^{m-1}, 10^m)$  that contain at least one Mills' prime number is infinite. The number of monotone growing primes in the interval  $(10^{m-1}, 10^m)$  is  $S(m) = 2,44m$ . The number of monotone increasing prime numbers is probably infinite:  $\lim_{p \rightarrow \infty} M(p) = \infty$  is probably, where  $p$  is prime, if  $p \rightarrow \infty$ .

#### 5. Conclusion

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

**Acknowledgements:** The authors would like to thank you for publishing this article.

#### References:

- [1]. <http://ac.inf.elte.hu> → VOLUMES → VOLUME 44 (2015) → VOLLPRIMZAHLENMENGE → FULL TEXT
- [2]. <http://primes.utm.edu/largest.html>
- [3]. <http://mathworld.wolfram.com/SmarandacheSequences.html>
- [4]. Dubner, H.: "Fw: (Prime Numbers) Rekord Primes All Prime digits" Februar 17. 2002
- [5]. <http://listserv.nodak.edu/scripts/wa.exe?A2=ind0202&L=nbrthry&P=1697>

- [6]. Harman, Glyn: Counting Primes whose Sum of Digits is Prime.
- [7]. Journal of Integer Sequences (2012. , Vol. 15, 12.2.2.)
- [8]. ANNALES Universitatis Scientiarum Budapestiensis de Rolando Eötvös Nominatae Sectio Computatorica, 2015, pp 221-226
- [9]. International Journal of Mathematics and Statistics Invention, February 2017:  
<http://www.ijmsi.org/Papers/Volume.5.Issue.2/B05020407.pdf>
- [10]. International Organisation of Scientific Research, April 2017
- [11]. Bölcsföldi Birkás prime numbers:
- [12]. [http://www.iosrjournals.org/iosr-jm/pages/v13\(2\)Version-4.html](http://www.iosrjournals.org/iosr-jm/pages/v13(2)Version-4.html) or
- [13]. <http://dx.doi.org> or [www.doi.org](http://www.doi.org) Article DOI is: 10.9790/5728-1302043841
- [14]. Ács-Bölcsföldi-Birkás prime numbers 2018: <http://irjes.com/volume7issue6.html>

**József Bölcsföldi**

Perczel Mór Secondary Grammar School  
H-8600 Siófok  
Március 15 park 1  
8624 Balatonszárszó, Rákóczi utca 53.  
Hungary

**György Birkás**

Baross Gábor Secondary Technical School  
H-8600 Siófok  
Kardvirágköz 7/a  
Hungary

**Tamás Bíró**

DRV Siófok  
H-8636 Balatonszemes  
Bem utca 12.  
Hungary