International Journal of Latest Engineering and Management Research (IJLEMR)
ISSN: 2455-4847
www.ijlemr.com || Volume 03 - Issue 02 || February 2018 || PP. 01-04

Application of Gert Analysis in Management

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Abstract: Graphical Evaluation and Review Technique, commonly known as **GERT**, is a network analysis technique used in project management that allows probabilistic treatment of both network logic and estimation of activity duration. The technique described in 1966 by Dr. Alan B. Pritskerof Purdue University and WW Happ was reviewed. Compared to other techniques, GERT is only rarely used in complex systems. Nevertheless, the GERT approach addresses the majority of the limitations associated with PERT/CPM technique. GERT allows loops between tasks. The fundamental drawback associated with the GERT technique is the complex programme (Monte Carlo simulation) required to model the GERT system. This paper results GERT to analyze a hypothetical R&D project; improving project planning performance, determining project labor, equipment, resource needs, and also helps the project managers to modify project strategies and set contract prices.

Introduction

A very simple rule for finding the least-cost testing sequence has been proposed by Boothroyd.1 The tests were non-destructive and such that, at each stage, only those items found to be acceptable were subjected to the next test. However, the items found to be rejectable will be generally reworked and submitted to the retest. The items which fail the retest are scrapped, while the items which pass the retest are sent to the next test. An Optimal rule for determination of testing sequence is given which minimizes the total mean cost including not only testing cost but also reworking and scrapping cost.

Features of Gert Network

Some of the features in-cluded in GERT networking are:
1) Probabilistic branching (stochastic networks),
2) Network looping (feedback loops),
3) Network modification during execution (learning capability),
4) Multiple sink nodes (multiple process outcomes),
5) Multiple node realizations (repeat events),
6) Specified activity releases (activity completions to realize a node may be less than, equal to, or greater than number of activities terminating at a node),
7) Multiple probability distributions (nine different probability distributions which may be associated with activity parameters),
8) Multiple types of node input (four types of logic may be related to node realization),
9) Multiple types of time statistics collection (five different types of time statistics may be collected at net-work nodes),
10) Statistics on the probability of node realization (probability of realizing various outcomes),
11) Cost statistics (may be collected corresponding to any one of the five types of time statistics),
12) Queuing networks (networks of queuing systems may be analyzed), and
13) Limited resources (networks involving up to three different types of limited resources, associated with activities, may be analyzed).

The potential range of applications of GERT goes far beyond the applications of such familiar networking models as PERT and CPM.

Brief Review of Gert

GERT was initiated by Pritsker and Happ (1966), Pritsker and Whitehouse (1966) and Whitehouse and Pritsker (1969) as a procedure for the analysis of stochastic networks having the following features:
(1) Each network consists of logical nodes (or events) and directed branches (or activities). (2) A branch has a probability that the activity associated with it is performed.
(3) Other parameters describe the activities represented by the branches. In this paper, however, reference made to a sample size parameter only.

The sample size n associated with a branch is characterised by the moment generating function (*mgf*) of the form $M_n(\theta) = \sum_n \exp(n\theta) f(n)$, where $f(n)$denotes the density function of n and θ is any real variable. The probability φthat the branch is realised is multiplied by the *mgf* to yield the *W*-function suchthat
$W(\theta) = \varphi M_n(\theta)$
The *W*-function is used to obtain the information on the relationship whichexists between the nodes.
NOTATIONS

T_i = Series of test
Ct = Cost per test
q_t = Average acceptance rates
T_h = Rejectable test
Cp = Rework cost
$T_i^{[r]}$ = Series of retest.
$C_{ti}^{[r]}$ = Re test cost
$q_i^{[r]}$ = Average acceptance rates in retest
C_s = Scrapped cost

Gert Analysis of Testing System

Suppose there are a series of tests T_i (i = 1, 2... n) costing Ct. per item tested, with average acceptance rates q_t. If an item is found to be rejectable at test T_h the item is reworked costing Cp. and submitted to the retest $T_i^{[r]}$ whose cost is $C_{ti}^{[r]}$ and average acceptance rate $q_i^{[r]}$The item which fails the retest is scrapped costing C_s(const.). If an item passes either of the test T_i or the retest T;$^{[r]}$ after reworking, it is sent to the next test. Assume that the C's and q's are unaffected by the order in which the tests are executed, and so mutually independent.

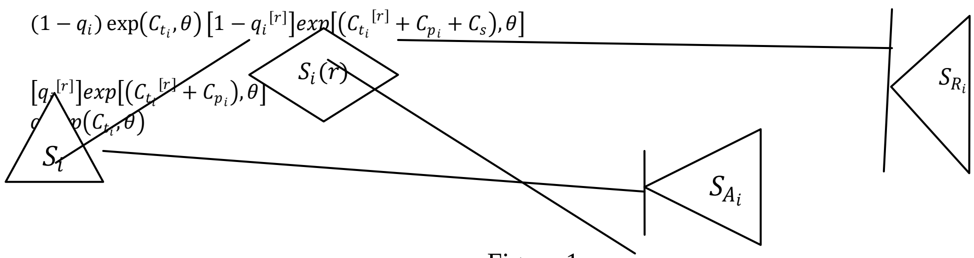

Figure-1.

The possible states in the above testing system can be defined as follows:
S_i = initial state of the test T_i
$S_i^{[r]}$ = state in which the reworking and the retest $T_i^{[r]}$ of the item rejected at the test T_i are required,
S_{Ai} = state in which the item is found to be acceptable at the test T_i or the retest $T_i^{[r]}$ (in this case the item may be sent to the next test), and.
S_{Ri}. = state in which the item is found to be rejectable at the retest $T_i^{[r]}$ (in this instance the item may be scrapped).

The GERT network representation of states of the test T_i(i = 1,2,..., n) can be constructed as shown in Figure 1. For example, to derive the W-function on the branch (S_i, S_{Ai}), first remember the probability that the item is found to be acceptable is given by
$\emptyset_1 = q_i$ (1)

The m.g.f. of the cost parameter C_t or the cost of testing (in this case the cost of reworking, retest, and scrapping is not necessary), is given by
$M_{C_1}(\theta) = \exp(C_{t_i}, \theta)$(2) because
$C_1 = C_{t_i}(const.)$(3)

The probability that the branch is realized is multiplied by the moment generating functions to yield a W-function. From equations (1) and (2),

$$W_1(\theta) = \emptyset_1 M_{C_1}(\theta) = q_i \exp(C_{t_i}, \theta) \quad (4)$$

The W-function on the other branches may be obtainable in the similar way. This function is used to obtain information on a relationship which exists between the nodes.

The application of Mason's rule(1953) on the representation in Figure 1 gives

$$W_{A_i}(\theta) = q_i exp(C_{t_i}, \theta) + (1 - q_i)q_i^{[r]} exp[(C_{t_i} + C_{t_i}^{[r]} + C_{p_i}), \theta] \quad (5)$$

and

$$W_{R_i}(\theta) = (1 - q_i)[1 - q_i^{[r]}] exp[(C_{t_i} + C_{t_i}^{[r]} + C_{p_i} + C_s), \theta] \quad (6)$$

where $W_{A_i}(\theta)$ and $W_{R_i}(\theta)$ are respectively the equivalent W-functions from the initial node s_i to the terminal nodes S_{A_i} and S_{R_i}. From the definition of W-function

$$P_{A_i} = W_{A_i}(\theta)\big]_{\theta=0} = q_i + (1 - q_i)q_i^{[r]} \quad (7)$$

and

$$P_{R_i} = W_{R_i}(\theta)\big]_{\theta=0} = (1 - q_i)(1 - q_i^{[r]}) \quad (8)$$

are obtained, where P_{A_i} stands for the probability that the item passes the test and is sent to the next test, and P_{R_i}. gives the probability that the item is found to be rejectable arid is scrapped. The m.g.f. of C_{A_i} Which will be costed from the initial node s_i to the terminal node S_{A_i} is given by

$$M_{A_i}(\theta) = W_{A_i}(\theta)/P_{A_i} \quad (9)$$

Similarly, the m.g.f. of C_{R_i} which will be costed from the initial node s_i to the terminal node S_{R_i} is written as

$$M_{R_i}(\theta) = W_{R_i}(\theta)/P_{R_i} \quad (10)$$

From equations (5)-(10) the mean $E[C(t_i)]$ and variance $V[C(t_i)]$ of the total cost peritem at test T_i is

$$E[C(t_i)] = P_{A_i} \frac{d}{d\theta} M_{A_i}(\theta)\Big]_{\theta=0} + P_{R_i} \frac{d}{d\theta} M_{R_i}(\theta)\Big]_{\theta=0}$$

$$= q_i C_{t_i} + (1 - q_i)q_i^{[r]}(C_{t_i} + C_{t_i}^{[r]} + C_{p_i}) + (1 - q_i)[1 - q_i^{[r]}](C_{t_i} + C_{t_i}^{[r]} + C_{p_i} + C_s)$$

And

$$V[C(t_i)] = P_{A_i}\left[\frac{d^2}{d^2\theta} M_{A_i}(\theta)\Big]_{\theta=0} - \left\{\frac{d}{d\theta} M_{A_i}(\theta)\Big]_{\theta=0}\right\}^2\right] + P_{R_i}\left[\frac{d^2}{d^2\theta} M_{R_i}(\theta)\Big]_{\theta=0} - \left\{\frac{d}{d\theta} M_{R_i}(\theta)\Big]_{\theta=0}\right\}^2\right]$$

$$= (C_{t_i}^{[r]} + C_{p_i})^2 q_i q_i^{[r]}(1 - q_i)/\{q_i + q_i^{[r]}(1 - q_i)\}$$

While the above mean and variance can be also derived from first principles using the probability function for the cost, the analysis by GERT has the advantage of clear visual representation of test systems and reduction of calculation of the statistics which gives useful information in the determination of the least-cost testing sequence

Gert as a Tool for Control

Many management planning tools incorporate control features. PERT/CPM in particular is rather effective in this respect: it determines amount of work remaining, compliance of completed work with the specified plans, and probability of completing the remainder of the project at the specified time based on experience to that point. In planning, GERT is substantially more sophisticated than PERT; however, GERT's probabilistic branching attribute makes it more difficult to use as a control technique. As implementation of the project begins, information as to the realism of estimates in the GERT net-work begins to accumulate, and the situation can then be re-evaluated and new information used to replace the probabilities or the time parameters. The impact of changes in certain identifiable, critical branching or activities can be assessed by a series of simulations in which these points are changed and the effect of the changes on completion time noted. If the expected probability of taking a branch is .50, but a probability of .60 causes a substantial decrease in completion time or cost, then emphasis can be placed on raising that expected probability and ensuring that it does not decline.

Conclusion:

GERT allows for probabilistic treatment of both network logic and activity duration estimates. GERT is mainly used on project activities that are only performed in part, as well as those activities that may be performed more than once. Since this activity will be performed more than once, GERT enable us to calculate the entire duration of the activity.The optimal testing sequence which minimizes the mean of the total cost including testing, reworking and scrapping cost, and makes the variance of the total cost smaller.

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