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New Matrix Representation AndIsomorphism Identification Among The Planar Kinematic Chains By Skeleton Matrix

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Abstract:ASkelton [SM] matrixisused to identify the isomorphism among the kinematic chains (KCs). In this method the given KC's are represented in the Skelton [S] matrix. Thedeterminantof [SM] matrix is considered as an invariant of a kinematic chain which may used to detect isomorphism. With the help of these invariant/identification code the isomorphism among the kinematic chains are identified. No counterexample has been found. The proposed method is efficient and accurate and only one [SM]matrix for a given kinematic chain is sufficient to identify isomorphism. This method is examined for one degree of freedom (1-DOF), 6, 8, 10 links planar kinematic chains and 12 links 2-DOF planar kinematic chains.
Keywords:KC, [SM],

1. Introduction:

More and more Complex Mechanism has been developing since 19 centuries for finding new mechanism,as complexity of mechanism increases the chances of isomorphic chains increases so far this reason a precise and systematic approach are requiredduring the initial phase of conceptual design. To create new mechanism, Number synthesis(Structural Analysis) and Type Synthesis are applied. The basic foundation of Structural Analysis is closed and open kinematic chains. One of the key aspect of Number synthesis is to generate kinematic chains with required number of links and degree of freedom and all possible mechanisms derived from required kinematic chain so that new and useful mechanism can be found and also the designer has the liberty to take the decision for selecting best or optimum mechanism depending upon the requirement.

In the course of development of kinematic chains with required number of link and degree of freedom, duplication or isomorphism may be possible. So for the identification of duplication or isomorphism, the researchers have proposed several methods in the recent past.

In the past years, a major task encountered in the kinematic number synthesis is the elimination of isomorphism chains. Various method, such as are- Visual methods [1] were only suitable for kinematic chains with a small number of links. Characteristic polynomial method [2] which requires lengthy calculations and later counter examples were also reported [3]. Rao and Raju presented a method of loops for multi degree of freedom chains [4].Characteristic polynomial of structural matrix was proposed by Yan and Hwang [5]; but the method is uneconomical because of more computational time. Mruthyunjaya proposed a method of binary coding for structural synthesis of kinematic chains [6]. Agarwal and Rao proposed Variable permanent function to identify multi loop kinematic chains [7]. Method of Canonical coding of kinematic chains is presented by Ambekar and Agarwal a [8] but it is computationally uneconomical for large kinematic chain. Hamming number technique [9] is reliable and efficient, but when the primary Hamming string fails, it requires the computation of the secondary Hamming string which is time consuming. Shin and Krishna Murthy presents some rules for relabeling its vertices canonically for a given kinematic chain [10]. However, where a higher number of symmetry group elements in the kinematic chain are present. It becomes computationally inefficient The degree code [11] generated by the contracted link adjacency matrix of a chain was also proposed for testing the isomorphism. Yadav and Pratap present a method of link distance for the detection of isomorphism [12]. A method based on artificial neural network theory by Kong et al. was presented [13]. Quist and Soni give a loop method for kinematic chains [14]. A new method based on eigenvalues and eigenvectors of adjacency matrices of chains was also proposed [15]. The reliability of the existing spectral techniques for isomorphism detection was challenged by Sunkari, R.P., and Schmidt [16]. Huafeng Ding and Zhen Huang [17] shows that the characteristic polynomial and eigen value approach fail and proposed a method based on the perimeter topological graph and give rules for relabeling its vertices canonically. Hasan and Khan [18] presented a method based on degrees of freedom of kinematic pairs. All the above methods developed so far uses the graphs of the Kinematic chain and their adjacency matrices in one or the other way

The methods proposed so far are based on adjacency matrix [1] distance matrix [2,3] to determine the structurally distinct mechanisms of a kinematic chain; the flow matrix method [4], and the row sum of extended adjacency matrix methods [5,6] are used. Minimum code [7], characteristic polynomial of matrix [8], identification code [9], link path code [10], path matrices [11], Multivalued Neural Network approach [12], a mixed isomorphism approach [13], Hamming value [14], artificial neural network approach [15], theory of finite symmetry groups [16,17], the representation set of links by Vijayananda [18], Interactive Weighted Distance Approach [19], are used to characterize the kinematic chains. Most of these methods either have lack of uniqueness or very time consuming. The flow matrix method [4] is a lengthy process to identify the isomorphism , as n flow matrices are required to be developed. The row-sum of extended adjacency matrix method [5] distinguishes only 69 distinct mechanisms derived from the family of 8-link, 1-DOF kinematic chains instead of the 72 reported by other researchers. The characteristic polynomial of matrix [7] approach is a very lengthy process in which the invariants of each matrix are tabulated and compared to theidentification of distinct mechanism. Hence, there is a need to develop a computationally optimized method to detect isomorphism in kinematic chains.

In the proposed method, the kinematic chains are represented by the skeleton [SM] matrix which has the information about the type of the polygonal links existing in a kinematic chain and their connectivity to each other. The structural invariants are derived from [SM] matrix using software Matlab which is the determinant of [SM] matrix and called as |SMi|.

This unique invariant is treated as an identification or a characterization number of the kinematic chain. Therefore [SM] invariants are used to detect isomorphism among the kinematic chains. If |SMi|is same for two kinematic chains, they will be treated as isomorphic chains otherwise non isomorphic chains. No counterexample has been found in the detection of isomorphism in 6- link & 8- link, one dof kinematic chains. It is expected that proposed method will be able to detect isomorphism among the kinematic chains having number of links more than eight. There are distant invariants for all 6- link & 8- link, single dof kinematic chains shown in table – 1

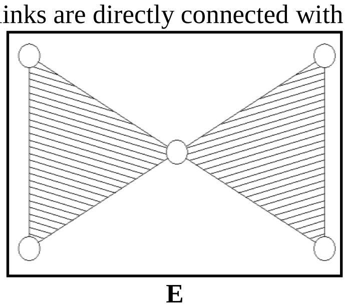
2. Architect of the proposed method:

2.1 Degree of the link d(li)

The degree of a link actually represents the type of the link, such as binary, ternary, quaternary links etc. Let the degree of i^{th} link in a kinematic chain be designated d(li) and d(li) = 2, for binary link, d(li) = 3, for ternary link, d(li) = 4, for quaternary link and d(li) = n, for n-nary link.

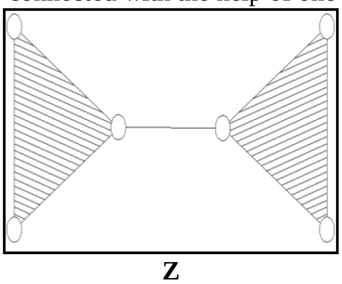
2.2 Type of connections

E-chain:When two polygonal links are directly connected with one joint ,



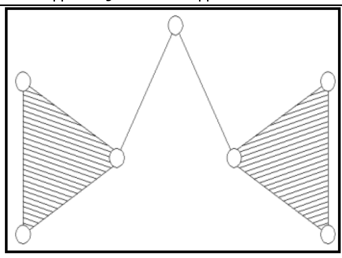
E

Z-Chain: When two polygonal links are connected with the help of one intermediate binary link



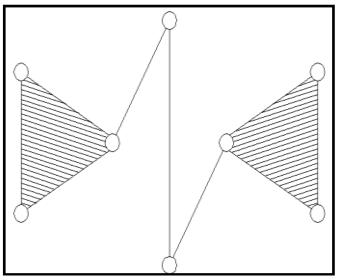
Z

V-Chain: When two polygonal links are connected with the help of two Intermediate binary link



V

D-Chain: When two polygonal links are connected with the help of threeIntermediate binary links.

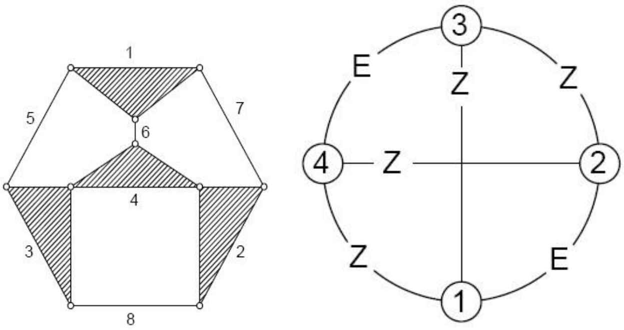


D

Fig. 1.

Skeleton Representation of the kinematic chain:

Skeleton diagram is the abstract topological representation of given kinematic chainsand was proposed by Frank[38]. A kinematic chain is the made up of polygonal links (links having degree more than 2) and binary strings, the polygonal links are connected to each other with the help of any combination of E-, Z-, D-, or V-chains. In the skeleton representation, all polygonal links are represented by the circles and these circles are connected to each other with the help of straight/curved lines showing the types of connections (E-, Z-, D-, or V-chains). The number of lines radiated from a circle represents the degree of that polygonal link. For example, Figure-2 (a) and (b) represents a 8 link, 10 joint, 1 F Kinematic chain (a) and its Skeleton (a'). In this KC there are 4 ternary links and 4 binary links. It contains 2 E-chain (in between polygonal link 1-3



(a) **(a')**

Fig. 2.

Fig. 2.8 link, 10 joint & 1-DOF Kinematic chain (a) and its Skeleton(a')

2.2 Skeleton matrix of the kinematic chains [SM]

The researcher has represented the kinematic chain by (0,1) adjacency matrix, shortest path distance matrix, joint-joint matrix and etc. The size of the matrix is nxn where the n = number of links existing in a kinematic chain. If the number of link large, the size of matrix is also very large & determination of some structural invariant is the problematic and time consuming. In the proposed work a new skeleton matrix develops, the size of which is reduced to the number of polygonal links in a kinematic chain. For example; in 8-link kinematic chain, if there are four (4) ternary link and 4 binary links then only a 4*4 size of the matrix is more than enough to represent a kinematic chain instead of 8*8 size of matrix. Therefore a skeleton matrix of size (nxn) is defined as

$$[\boldsymbol{SM}] = \{\boldsymbol{s_{ij}}\}_{\mathbf{nxn}}$$

$$[\boldsymbol{SM}] = \begin{cases} \{\boldsymbol{S_{ij}}\} \\ \boldsymbol{d(li)} \\ \mathbf{0} \end{cases}$$

Where $\{\boldsymbol{S_{ij}}\}$=Summation of squared value of type of chain between i_{th} and j_{th} Polygonal links those are connected, i.e. E=1, Z=2, D=3 and V=4

$\{\boldsymbol{s_{ij}}\}$ = $\boldsymbol{d(li)}$**,** If i=j squared value of degree of polygonal links

$\{\boldsymbol{s_{ij}}\}$= **0,** If i_{th} polygonal link is not directly connected to j_{th} link

Suppose two kinematic chains a and b with n polygonal links, their skeleton matrices can be written as S_1 and S_2 respectively as follows

$$\boldsymbol{SM1} = \begin{bmatrix} \boldsymbol{s_{11}} & \cdots & \boldsymbol{s_{1n}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{s_{n1}} & \cdots & \boldsymbol{s_{n\times n}} \end{bmatrix}_{\boldsymbol{n\times n}}$$

$$\boldsymbol{SM2} = \begin{bmatrix} \boldsymbol{s_{11}} & \cdots & \boldsymbol{s_{1n}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{s_{n1}} & \cdots & \boldsymbol{s_{n\times n}} \end{bmatrix}_{\boldsymbol{n\times n}}$$

Skeleton matricesSM1and SM2 shows polygonal link relationship among each other in skeleton diagram

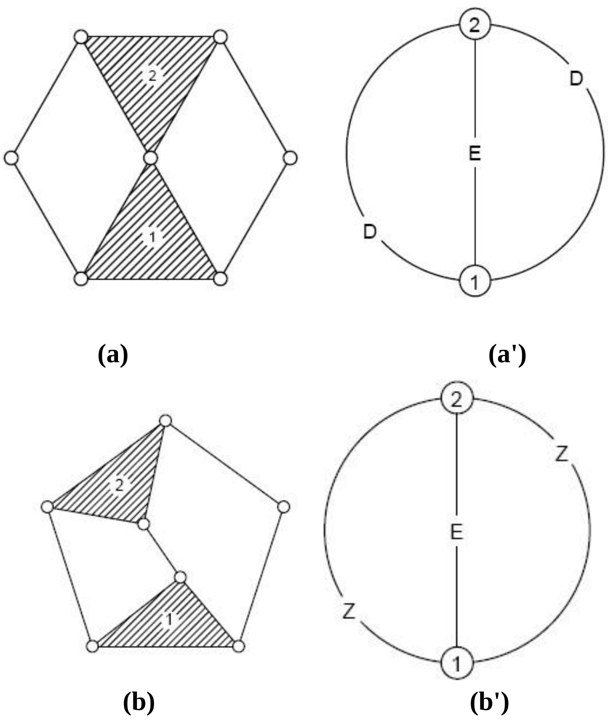


Fig. 3.

Skeleton matrix of the kinematic chains [S] Fig. 3. (a') and (b')

$$SM1 = \begin{bmatrix} 9 & 17 \\ 17 & 9 \end{bmatrix}$$

$$\text{SM2} = \begin{bmatrix} 9 & 19 \\ 19 & 9 \end{bmatrix}$$

Fig. 3. 6 links, 7 joint & 1-F Kinematic chain (a) and (b) and their Skeleton (a') and (b')

Determine the Determinant of Skeleton [SM] matrices of each kinematic chain

3. Procedure of identifying isomorphism

Step1: Convert kinematic chain into Skeleton

Step2: Development of Skeleton matrix [SM].

Step3: Determinant of [SM]matrixare usedas an invariant of a kinematic chain which may be used to detect isomorphism

Illustrative Example -1

1. This example consist of three 12 link KCs(a), KCs(b) & KCs© In figure. They have the same characteristic polynomial and Eigenvalues. The characteristic polynomial and adjacency matrix fail to detect isomorphism for these chains. These kinematic chains are tested for isomorphism by the proposed method.

Figure 1a

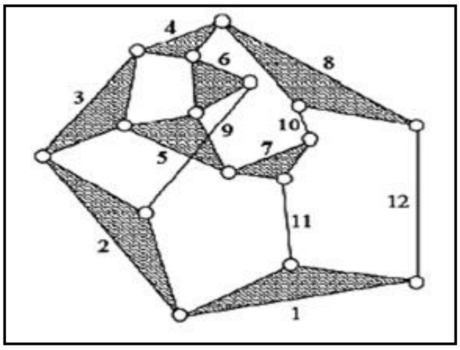


Figure 1b

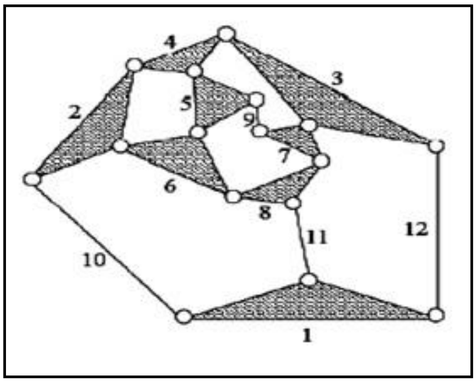
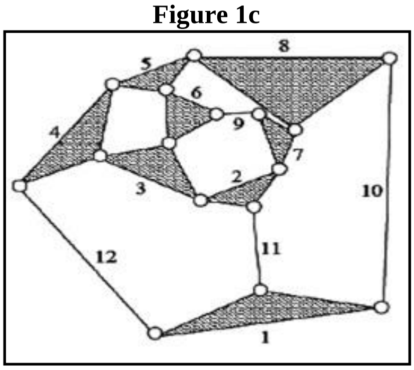
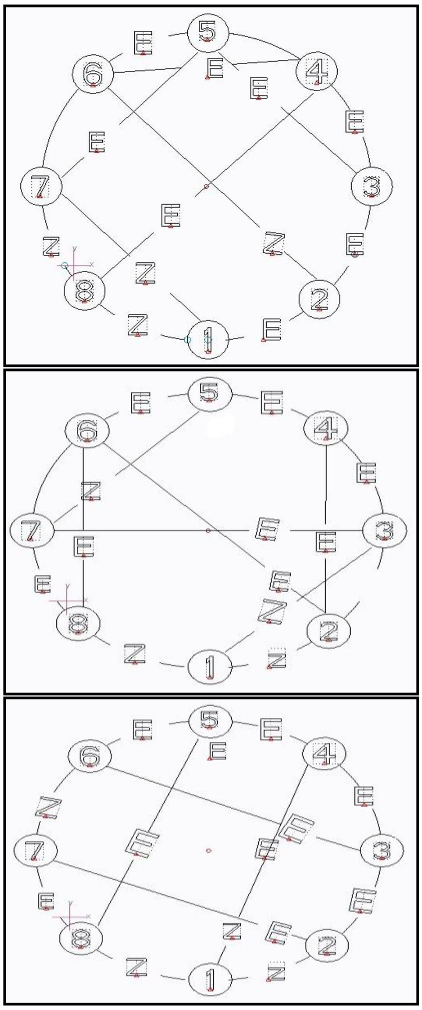


Figure 1c



Step-1



Step-2

Skeleton matrix of the kinematic chains [SM] Fig. 1(a)

$$SM := \begin{bmatrix} 9 & 1 & 0 & 0 & 0 & 0 & 4 & 4 \\ 1 & 9 & 1 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 9 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 9 & 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 & 9 & 1 & 1 & 0 \\ 0 & 4 & 0 & 1 & 1 & 9 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 & 9 & 4 \\ 4 & 0 & 0 & 4 & 0 & 0 & 4 & 9 \end{bmatrix}$$

Skeleton matrix of the kinematic chains [SM] Fig. 1(b)

$$[S] = \{d_{ij}\}_{nxn}$$

$$SMB := \begin{bmatrix} 9 & 4 & 4 & 0 & 0 & 0 & 0 & 4 \\ 4 & 9 & 0 & 1 & 0 & 1 & 0 & 0 \\ 4 & 0 & 9 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 9 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 9 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 & 1 & 9 & 0 & 1 \\ 0 & 0 & 1 & 0 & 4 & 0 & 9 & 1 \\ 4 & 0 & 0 & 0 & 0 & 1 & 1 & 9 \end{bmatrix}$$

Skeleton matrix of the kinematic chains [SM] Fig. 1(c)

$$SMC := \begin{bmatrix} 9 & 4 & 0 & 4 & 0 & 0 & 0 & 4 \\ 4 & 9 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 9 & 1 & 0 & 1 & 0 & 0 \\ 4 & 0 & 1 & 9 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 9 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 9 & 4 & 0 \\ 0 & 1 & 0 & 0 & 0 & 4 & 9 & 1 \\ 4 & 0 & 0 & 0 & 1 & 0 & 1 & 9 \end{bmatrix}$$

Step-5
Determinant of [SM]matrix of fig 1(a)

$$|SM_a| = 1.298 \times 10^7$$

Determinant of [SM]matrix of fig 1(b)

$$|SM_a| = 1.137 \times 10^7$$

Determinant of [SM]matrix of fig 1(c)

$$|SM_a| = 1.137 \times 10^7$$

Our method reports that both the KC shown in Fig.1 (a) and Fig.1 (b) are non-isomorphic as the values of the determinant of skeleton matrix [SM] of Fig. 1 (a) and (b) are different for both the KC.But kinematic chains of Fig.. (b) and © are isomorphic Note that by using another method summation polynomial [22], the same conclusion is obtained.

4. Results

The present work is proposed for the identification code for the given simple jointed kinematic chain. The methodology is applied on 3F, 10 link, 12 joint simple jointed kinematic chains
For storing and retrieving the structural information in the computer, the KC's are synthesized with the help of Skeleton matrix: [SM]. Applying the Skeleton matrix: [SM] in the C-programming and running the program in the MATLAB software to obtain desired results.

The only one structural invariant derived from Skeleton matrix: [SM] by using the MATLAB software. This structural invariant is same for identical or structurally equivalent chains and different for distinct chains. These invariants are used as the identification number of simple jointed kinematic chains and to detect isomorphism in the multiple jointed kinematic chains. If these invariants are the same the two simple jointed kinematic chains are isomorphic otherwise not.

5. Conclusions

The kinematic structural synthesis is the systematic development of kinematic chains and mechanisms derived from the kinematic chains. The kinematic chains can be represented by their Skelton which is an abstract representation of kinematic chains. During the course of development of kinematic chains duplication is possible. To avoid this duplication, an isomorphic test is required. For this purpose, numbers of methods are proposed in recent years. But those methods have either the lack of uniqueness or sometimes fail in the detection of isomorphism among the kinematic chains. Therefore the scope of further research is needed to detect isomorphism. In the proposed method, the kinematic chains are represented by the weighted squared shortest path distance [WSSPD] matrix which has the information of the type of the links existing in a kinematic chain and their connectivity to each other. The structural invariants are derived from [SM] matrix using software Matlab which is the determinant of [SM] matrix and called as |SM|. This unique invariants is treated as an identification or characterization number of the kinematic chain. Therefore |SM| invariants are used to detect isomorphism among the kinematic chains. If |SM| is same for two kinematic chains, they will be treated as isomorphic chains otherwise non isomorphic chains. No counterexample has been found in the detection of isomorphism in 6- link & 8- link, one dof kinematic chains. It is expected that proposed method will be able to detect isomorphism among the kinematic chains having number of links more than eight

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