

## Calculation of Life Insurance Premium using Markov Chain

Mohamat Fatekurohman<sup>1</sup>, I Made Tirta<sup>\*2</sup>, Putri Rahma Nugiezta<sup>3</sup>,

<sup>1,2,3</sup>(Department of Mathematics, Faculty Mathematics and Natural Science, University of Jember, Indonesia)

<sup>\*</sup>Correspondence Author

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**Abstract:** BNI Life Insurance is an insurance company that is growing rapidly with various insurance products offered to customers. One of them is a whole life insurance product, namely the Old Age Guarantee Program. Participants who take part in the insurance program are required to make premium payments according to what has been agreed. The Markov chain is a statistical method that can be used to analyze life insurance premium calculations. The aim of this research is to determine the premium value of Old Age Security Program participants using the Markov Chain method. Several other independent variables that are needed are the participant's survival probability value, the probability density value of the participant's risk of failure, the interest rate, and the sum insured proposed by the insurance participant. The results of this research are that the older the insurance participant gets, the smaller the premium value that must be paid by the insurance participant.

**Keywords:** insurance premium, survival probability value, Markov Chain

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### 1. Introduction

Insurance is currently an important thing that is much needed and in demand by most people in Indonesia. Insurance has the benefit of guaranteeing these workers if something unexpected happens in the future that requires them to stop working within a certain period of time or beyond, causing these workers to no longer have income. Workers who take part in the insurance program are required to make payments over a certain period periodically with a predetermined nominal amount of money, these payments are usually known as premiums. Insurance or coverage is a reciprocal agreement between the insurer and the insurance cover, where the insurer undertakes to compensate for losses, and/or pay an amount of money (compensation) determined at the time of closing the agreement, to the insurance cover or other person appointed, at the time the event occurs, while the insurance cover commits itself to paying the premium money [1]. Insurance consists of various types, including life insurance, education insurance, health insurance, property and personal goods insurance, investment insurance, and so on [2]. The program that guarantees the welfare of workers in financial terms, as mentioned earlier, is a type of life insurance program. Analysis of premium calculations often uses an economics approach. It is possible to carry out this analysis using a mathematical approach, especially in the field of statistics, one of which is using the survival function. The premium value is one of the factors that has a significant influence on participants' ability to pay insurance premiums [3]. One study that explains the influence of risk factors on the value of pensions and life annuity obligations, especially on unobserved risk factors, namely insurance participants who have a history of illness or hereditary disease and insurance participants who are under medical treatment (illness or accident) using several models, one of which is the Markov Chain [4]. Markov chains are an alternative method in statistics that can be used to analyze life insurance premium calculations. One of them is analyzing the single premium calculation and risk in multistate cases using homogeneous continuous time Markov Chains [5]. The Kolmogorov differential equation is a system used to obtain a probability distribution at time  $t + h$  by conditioning on conditions at future and previous times in analyzing the birth and death process using Markov Chains in continuous time [6]. In life insurance, the risk is only in one possible transition, namely the transition from state 1 (healthy) to state 2 (death), in other words, life insurance does not provide coverage if the insured experiences illness but will receive a certain amount of compensation if the insured dies [7].

In this article, we will examine further the influence of age, survival probability value, interest rate, and proposed sum insured on the premium value obtained by the insurance participant using the Markov Chain method.

### Whole Life Insurance

Life insurance is divided into several types, one of which is whole life insurance. Whole life insurance is insurance that if the policy holder starts from the approval of the insurance contract until he dies, the insurance money will be paid [8].

**Survival Function**

The survival function is denoted  $S(t)$ , by indicating the probability that an individual survives over time  $t$ , where  $t \geq 0$

**Hazard Function**

The hazard function is denoted  $h(t)$ , as showing the probability or chance of an individual's risk of failure at time  $T$ . defined:

$$h(t) = \frac{f(t)}{S(t)} \quad (1)$$

$$h(t) = \frac{d_j}{n_j} \quad (2)$$

Where, number of events risk value, so, based on equations (1) and (2) the value of probabilitas densety function  $f(t)$ .

**Transition**

Transition is a move from one state to another state. In the case of whole life insurance, there is a transition, namely from a capable state (state 0) to a failed or incapable state (state 1). An illustration of the two-state model in the case of whole life insurance can be described as follows:

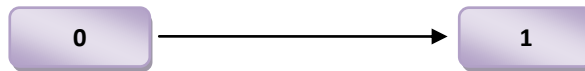


Figure1. Image of the two-state model

It is assumed that movement/transition can only occur from status 0 to status 1 and it is not possible to do the opposite, namely from status 1 to status 0. An illustrative explanation of Figure 1 is presented in Table 1.

Table1. Two-state model table

Status	0	1
0	$S(t)_{xt}^{00}$	$f(t)_{xt}^{01}$
1	0	1

From Table 1, a matrix can be formed whose order is the transition matrix of insurance participants in year categories at time  $t$  as follows:

$$P_{xt} = \begin{bmatrix} p_{xt}^{00} & q_{xt}^{01} \\ 0 & 1 \end{bmatrix}$$

Where,

$$p_{xt}^{00} = S(t)_{xt}^{00} = \text{the value of the insurance participant's survival chances}$$

$$q_{xt}^{01} = f(t)_{xt}^{01} = \text{probability density value of insurance participants}$$

**Net Single Premium**

The net single premium (of continuous life insurance can be determined by the following formula:

$$A_x = c_{ij_x} \int_0^t v^t \cdot p_{xt}^{00} \cdot q_{xt}^{01} dt \quad (3)$$

In actuarial science, the net single premium on whole life insurance is denoted by  $A_x$ , where

$$v^t \text{ (annual discount factor)} = \left[ \frac{1}{(1+i)} \right]^t$$

$p_{xt}^{00}$  = insurance participant survival probability value (from status 0 to status 0)

$q_{xt}^{01}$  = probability density value of risk of failure for age insurance participants who experience the transition from status 0 to status 1

$c_{ij_x}$  = the proposed sum assured

## 2. Methods

The data used in this research is secondary data from the last few periods. This data is data from participants in the old age insurance program for BNI Life Insurance whole life insurance. The steps taken are as follows.

- a. Study of literature.
- b. Data retrieval
- c. Analyze the variables that will be used
- d. Determining the value of the insurance participant's survival chances
- e. Determine the density value of the insurance participant's risk of failure
- f. Determine the transition opportunity for insurance participants from a capable state to failure
- g. Determine the annual interest rate
- h. Determining the single premium with the Markov.

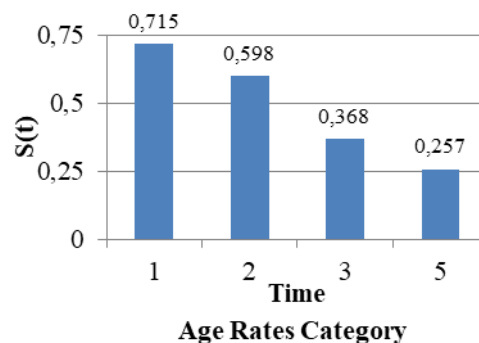
## 3. Results and Discussions

From the analysis of the insurance participant age variable which is divided into four categories, Table 2 is obtained.

Table 2. Analysis of participant age category variables

Age Category	Number of Customers	Status	
		Failed	Able
1	10	5	5
2	8	5	3
3	25	12	13
4	9	4	5

Table 2 explains the number of customers who have failed status (unable to pay premiums according to the payment period) and the status of being able to pay premiums according to the payment period. After analyzing the age variable, the survival probability value for each age category was searched using the R-Studio program and obtained Graph 1.



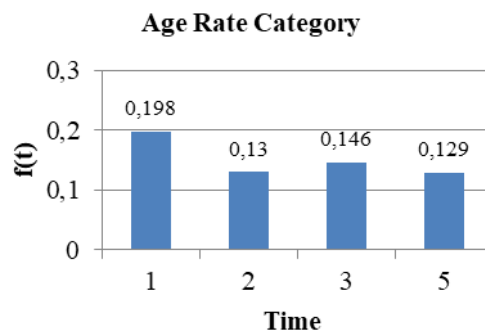
Graph 1. Graph of the average survival probability value for insurance participant

From Graph 1, it can be seen that there is a decrease in the value of survival opportunities relative to time, the longer the time, the smaller the chance of the insurance participant's ability to pay premiums according to the predetermined payment period. Based on equation 2, the value of the probability density function  $f(t)$  can be determined if the survival probability and hazard values are known. The opportunity density value in question is the opportunity for insurance participants in each age category to fail or be unable to make payments according to the predetermined time period, so that Table 3 is obtained.

Table 3. Probability density function of insurance participants

Time (t) \ Usia (x)	1	2	3	5
1	0,160	0,098	0,128	0,129
2	0,234	0,150	–	–
3	0,202	0,051	0,164	–
4	–	0,222	0,148	–

Then from the results of Table 3, the average value is found and presented in Graph 2.



From the results of Graph 2, it can be seen that the graph of the probability of failure density value versus time experiences ups and downs, this is because the value of the probability of failure is very dependent on the hazard value. While, the hazard value depends on the number of events in each age category. If the number of events increases, the hazard value will become greater, which will cause the chance of failure  $f(t)$  to also become greater.

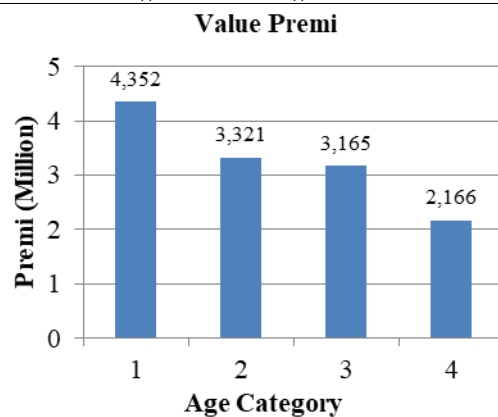
After calculating the probability of survival value and also knowing the probability density value for each participant category, a transition matrix can be order  $2 \times 2$  which  $p_{xt}^{00}$  has an order which is denoted as the probability value of insurance participants in the age category  $x$  who are in good health who will continue to survive in making premium payments within the specified time period. has been established. While  $q_{xt}^{01}$ , it is denoted as the probability value of an insurance participant in the age category  $x$  who is in good health who will fail to make premium payments within the specified time period. The survival value  $p_{xt}^{00}$  and probability density value of failure  $q_{xt}^{01}$  for insurance participants are presented in Table 4.

Table 4. Table of survival probability values and failure probability density values

Usia (x)	Time (t)	$p_{xt}^{00}$	$q_{xt}^{01}$
1	1	0,800	0,160
1	2	0,686	0,098
1	3	0,514	0,128
1	5	0,257	0,129
2	1	0,625	0,234
2	2	0,375	0,150
3	1	0,720	0,202
3	2	0,665	0,051
3	3	0,369	0,164
4	2	0,667	0,222
4	3	0,222	0,148

The premium calculation in this research uses the latest reference interest rate from Bank Indonesia, namely the BI 7-Day Repo Rate. Based on the BI 7-Day Repo Rate, the latest constant interest rate used as of November 21 2019 is 5.00%.

Based on equation (3), where  $c$  is the sum assured,  $t$  is time,  $v^t$  is the annual discount factor  $\alpha$  with the interest rate used,  $p_{xt}^{00}$  is the probability of survival value, and  $q_{xt}^{01}$  is the probability density value of participants. So, the results of calculating the net single premium for the old age insurance program are based on the factors presented in Appendix Table 5. From Table 5, it is then presented in the form of Graph 3.



Graph 3. Graph of the average premium value of insurance participants for each age category.

Graph 3 shows that there is a decrease in the premium value obtained by insurance participants as they get older. This is because as the age (older) of the insurance participant increases, the ability to pay premiums also decreases so that the value of the premium that must be paid also adjusts, namely becoming smaller.

#### 4. Conclusion

The value of the survival probability and the probability density value of the insurance participant's risk of failure greatly influence the premium calculation using the continuous Markov chain model. The greater the value of the survival chances and the density of opportunities for insurance participants, the greater the premium value that must be paid by the participant.

#### Authors' Contributions

The first author mainly contributed on Idea and desain of the research, the second author contributed on preparing type of statistics data analyses, reviewing english publication. The third authors contributed mainly on collectingg data, preparing the draft of manuscript. All authors involved in finishing the manuscript

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