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A New Exponentiated Two Parameter Distribution with Biometric Applications

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Abstract: In this paper we have introduced a new version of two parameter Akash distribution known as Exponentiated two parameter Akash distribution. We have also obtained its various estimation procedures namely, order Statistics, reliability analysis and its moments. We have also discussed its maximum likelihood estimation method for estimating its parameters. Finally the superiority and usefulness of the model is analyzed by applying the real lifetime data set.

Keywords: Exponentiated distribution, two parameter Akash distribution, Order statistics, Reliability analysis, Maximum likelihood Estimation.

1. Introduction

For understanding the nature of lifetime data generated from many fields of real life we have to analyze the data by fitting appropriate probability model to that data. A lot of probability models have been fitted by researchers over the years to lifetime data. Numbers of parameters in a probability model fitted to any kind of data play a significant role in providing the better fit to the data. More are the parameters in the model more will be the amount of variation captured by model from the data. Also for obtaining more and more flexibility in applying probability models to the data a model with more parameters is preferred than model with less number of parameters. Exponentiation technique is one of the techniques which we can use to add extra parameters to the classical or existing models. Researchers have shown that exponentiated technique finds greater applicability in the real life in many situations as exponentiated models fit better to real life data as compared to other existing models. Nadarajah (2011) studied exponentiated exponential distribution with properties and applications in real life. Gupta & Kundu (1999) introduced generalized exponential distributions. Dey, Kumar, Ramos & Louzada (2017) formulated Exponentiated Chen distribution and obtained its vital properties. Ashour & Eltehiwy (2015) introduced Exponentiated Power Lindley distribution and applied it to real life. Haq (2016) studied transmuted exponentiated inverse Rayleigh distribution and obtained its various properties. A new theory of distributions was introduced by Gupta et al. (1998), who discussed a new family of distributions namely the exponentiated exponential distribution. The family has two parameters scale and shape, which are similar to the weibull and gamma family. Later Gupta and Kundu (2001) studied some properties of the distribution. They observed that many properties of the new family are similar to those of the weibull or gamma family. Pal *et al.* (2006) studied the exponentiated weibull family as an extension of weibull distribution. Rodrigues *et al.* (2017) studied the exponentiated generalized Lindley distribution. Hassan et al. (2017) discussed the exponentiated Lomax geometric distribution with its properties and applications. Nasiru *et al.* (2018) obtained exponentiated generalized power series family of distributions. Rather and Subramanian (2019) obtained the exponentiated Ishita distribution with properties and applications. Maradesa Adeleke (2019) discussed exponentiated exponential Lomax distribution and its Properties. Nasir *et al* (2018) obtained the exponentiated Burr XII power series distribution with properties and its applications. Rather and Subramanian (2018) discussed the exponentiated Mukherjee-Islam distribution. Recently, Rather and Subramanian (2020) discussed the exponentiated Garima distribution with applications in engineering sciences.

Two parameter Akash distribution is a recently introduced two parametric lifetime model proposed by Shanker and Shukla (2017). The potentiality and usefulness of the proposed distribution in modeling lifetime data was greater as compared to other two parametric distributions. The two parameter Akash distribution is a special case of one parameter Akash distribution. The different mathematical and statistical properties of the proposed distribution has been derived and discussed such as order statistics, moments and associated measures hazard and mean residual life function, stochastic ordering, Bonferroni and Lorenz curves, Renyi entropy and stress strength reliability. The parameters of the proposed distribution are estimated by employing the method of maximum likelihood estimation and the method of moments. Finally the goodness of fit of the proposed two parameter Akash distribution has been described by analyzing the real life data set and the fit has been found quite satisfactory over exponential, Akash and lognormal distributions.

2. Exponentiated Two Parameter Akash Distribution

The probability density function of two parameter Akash distribution is given by

$$g(x)=\frac{\theta^3}{\alpha\theta^2+2}\left(\alpha+x^2\right)e^{-\theta x};\; x>0,\theta>0,\alpha>0 \qquad (1)$$

and the cumulative distribution function of two parameter Akash distribution is given by

$$G(x)=1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\; ;x>0,\theta>0,\alpha>0 \qquad (2)$$

A random variable X is said to have an exponentiated distribution, if its cumulative distribution function is given by

$$F_\alpha(x) = \left(G(x)\right)^\alpha \;\; ; x\in R^{'},\; \alpha>0 \qquad (3)$$

Then X is said to have an exponentiated distribution.

The probability density function of X is given by

$$f_\alpha(x)=\alpha\left(G(x)\right)^{\alpha-1}g(x) \qquad (4)$$

on Substituting (2) in (3), we will get the cumulative distribution function of Exponentiated two parameter Akash distribution

$$F_\alpha(x)=\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^\alpha \;\; ;\; x>0,\theta>0,\alpha>0 \qquad (5)$$

and the probability density function of Exponentiated two parameter Akash distribution can be obtained as

$$f_\alpha(x)=\frac{\alpha\theta^3\left(\alpha+x^2\right)e^{-\theta x}}{\alpha\theta^2+2}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1} \;\; ;x>0,\theta>0,\alpha>0 \qquad (6)$$

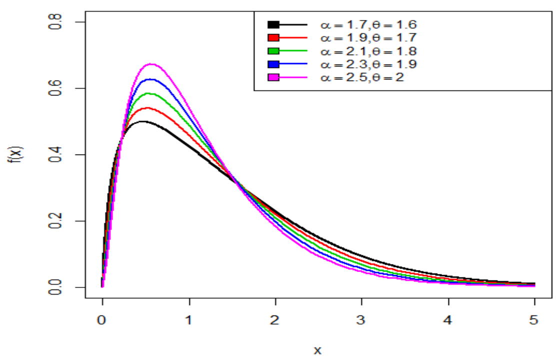

Fig.1 Pdf plot of Exponentiated Two Parameter Akash distribution

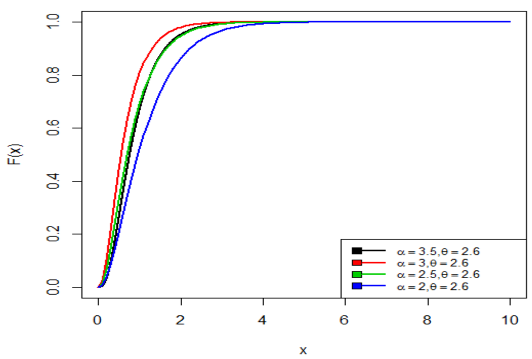

Fig.2 Cdf plot of Exponentiated Two Parameter Akash distribution

3. Reliability Analysis

In this section, we will discuss the survival function, hazard function and Reverse hazard rate function of the Exponentiated two parameter Akash distribution.

The survival function of Exponentiated two parameter Akash distribution is given by

$$S(x) = 1 - \left(1 - \left(1 + \frac{\theta x(\theta x + 2)}{\alpha\theta^2 + 2}\right)e^{-\theta x}\right)^{\alpha}$$

The hazard function is also known as hazard rate, instantaneous failure rate or force of mortality and is given by

$$h(x) = \left(\frac{\dfrac{\alpha\theta^3\left(\alpha + x^2\right)e^{-\theta x}}{\alpha\theta^2 + 2}\left(1 - \left(1 + \dfrac{\theta x(\theta x + 2)}{\alpha\theta^2 + 2}\right)e^{-\theta x}\right)^{\alpha - 1}}{1 - \left(1 - \left(1 + \dfrac{\theta x(\theta x + 2)}{\alpha\theta^2 + 2}\right)e^{-\theta x}\right)^{\alpha}} \right)$$

The reverse hazard rate is given by

$$h_r(x) = \frac{\alpha\theta^3\left(\alpha + x^2\right)e^{-\theta x}}{\theta x(\theta x + 2)e^{-\theta x}}$$

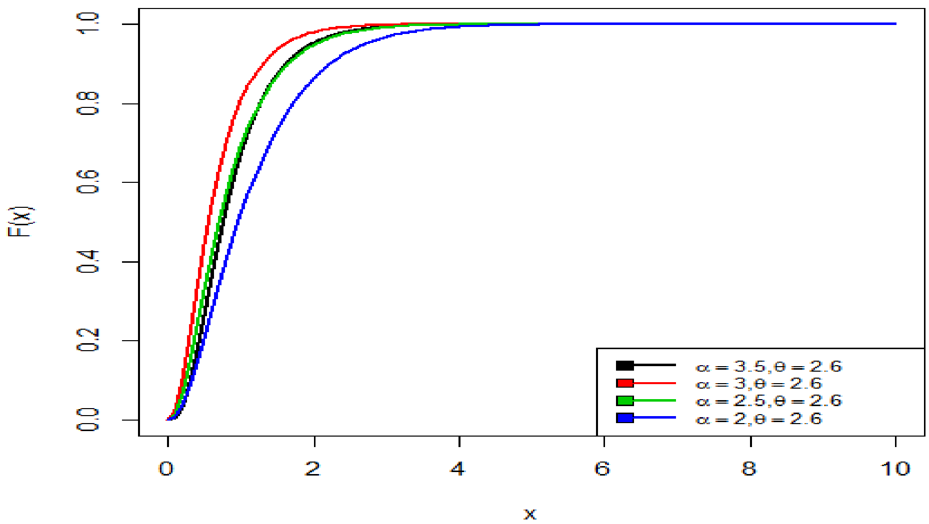

Fig.3 Survival plot of Exponentiated Two Parameter Akash distribution

4. Statistical Properties

In this section, we will discuss the different structural properties of the proposed Exponentiated two parameter Akash distribution.

4.1 Moments

Suppose X is a random variable following exponentiated two parameter Akash distribution with parameters α and θ, then the r^{th} order moment $E(X^r)$ for a given probability distribution is given by

$$E(X^r)=\mu_r{}'=\int_0^\infty x^r f_\alpha(x)dx$$

$$E(X^r)=\int_0^\infty x^r \frac{\alpha\theta^3\left(\alpha+x^2\right)e^{-\theta x}}{\alpha\theta^2+2}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1}dx$$

$$E(X^r)=\frac{\alpha\theta^3}{\alpha\theta^2+2}\int_0^\infty x^r\left(\alpha+x^2\right)e^{-\theta x}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1}dx \qquad (7)$$

Using Binomial expansion of

$$\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1}=\sum_{i=0}^\infty\binom{\alpha-1}{i}\left\{\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right\}^i(-1)^i$$

Equation (7) will become

$$E(X^r)=\frac{\alpha\theta^3}{\alpha\theta^2+2}\sum_{i=0}^\infty(-1)^i\binom{\alpha-1}{i}\int_0^\infty x^r\left(\alpha+x^2\right)e^{-\theta x(1+i)}\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)^i dx \qquad (8)$$

Again using Binomial expansion of

$$\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)^i=\sum_{k=0}^\infty\binom{i}{k}\left(\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)^k$$ equation (8) becomes

$$E(X^r)=\frac{\alpha\theta^3}{\alpha\theta^2+2}\sum_{i=0}^\infty\sum_{k=0}^\infty(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\left(\frac{\theta^2x^2+2\theta x}{\alpha\theta^2+2}\right)^k\int_o^\infty x^r\left(\alpha+x^2\right)e^{-\theta x(1+i)}dx$$

After simplification, we obtain

$$E(X^r)=\alpha\theta^3\sum_{i=0}^\infty\sum_{k=0}^\infty(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\frac{\left(\theta^2+2\theta\right)^k}{(\alpha\theta^2+2)^{k+1}}\left(\frac{\alpha\theta(1+i)^2\Gamma(r+3k+1)+\Gamma(r+3k+3)}{\theta(1+i)^{r+3k+3}}\right) \qquad (9)$$

Since equation (9) is a convergent series for all r ≥ 0, therefore all the moments exist.

Therefore

$$E(X)=\alpha\theta^3\sum_{i=0}^\infty\sum_{k=0}^\infty(-1)^i\binom{\alpha-1}{i}\binom{i}{k}\frac{\left(\theta^2+2\theta\right)^k}{(\alpha\theta^2+2)^{k+1}}\left(\frac{\alpha\theta(1+i)^2\Gamma(3k+2)+\Gamma(3k+4)}{\theta(1+i)^{3k+4}}\right)$$

$$E(X^2) = \alpha\theta^3 \sum_{i=0}^{\infty}\sum_{k=0}^{\infty} (-1)^i \binom{\alpha-1}{i}\binom{i}{k} \frac{\left(\theta^2+2\theta\right)^k}{(\alpha\theta^2+2)^{k+1}} \left(\frac{\alpha\theta(1+i)^2\,\Gamma(3k+3)+\Gamma(3k+5)}{\theta(1+i)^{3k+5}} \right)$$

Therefore the Variance of X can be obtained as

$$V(X) = E(X^2) - (E(X))^2$$

4.2 Harmonic mean

The Harmonic mean for the proposed Exponentiated two parameter Akash distribution can be obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^{\infty} \frac{1}{x} f_{\alpha}(x)dx$$

$$H.M = \int_0^{\infty} \frac{1}{x} \frac{\alpha\theta^3\left(\alpha+x^2\right)e^{-\theta x}}{\alpha\theta^2+2} \left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1} dx$$

$$H.M = \frac{\alpha\theta^3}{\alpha\theta^2+2} \int_0^{\infty} \frac{1}{x}\left(\alpha+x^2\right)e^{-\theta x} \left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1} dx \tag{10}$$

Using Binomial expansion in equation (10), we get

$$H.M = \frac{\alpha\theta^3}{\alpha\theta^2+2} \sum_{i=0}^{\infty} (-1)^i \binom{\alpha-1}{i} \int_0^{\infty} \frac{1}{x}\left(\alpha+x^2\right)e^{-\theta x(1+i)} \left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)^i dx \tag{11}$$

On using Binomial expansion in equation (11), we obtain

$$H.M = \frac{\alpha\theta^3}{\alpha\theta^2+2} \sum_{i=0}^{\infty}\sum_{k=0}^{\infty} (-1)^i \binom{\alpha-1}{i}\binom{i}{k} \left(\frac{\theta^2x^2+2\theta x}{\alpha\theta^2+2}\right)^k \int_0^{\infty} \frac{1}{x}\left(\alpha+x^2\right)e^{-\theta x(1+i)} dx \tag{12}$$

After the simplification of equation (12), we obtain

$$H.M = \alpha\theta^3 \sum_{i=0}^{\infty}\sum_{k=0}^{\infty} (-1)^i \binom{\alpha-1}{i}\binom{i}{k} \frac{\left(\theta^2+2\theta\right)^k}{\left(\alpha\theta^2+2\right)^{k+1}} \left(\frac{\alpha\theta(1+i)^2\,\Gamma(3k+1)+\Gamma(3k+3)}{\theta(1+i)^{3k+3}} \right)$$

4.3 Moment Generating Function and Characteristics Function

Let X have an exponentiated two parameter Akash distribution, then the moment generating function of X is obtained as

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_\alpha(x)dx$$

Using Taylor's series, we get

$$M_X(t) = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \ldots\right) f_\alpha(x)dx$$

$$= \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_\alpha(x)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!} \mu_j{}'$$

$$M_X(t) = \alpha\theta^3 \sum_{i=0}^\infty \sum_{j=0}^\infty \sum_{k=0}^\infty (-1)^i \binom{\alpha-1}{i}\binom{i}{k} \frac{t^j}{j!} \frac{\left(\theta^2 + 2\theta\right)^k}{(\alpha\theta^2 + 2)^{k+1}} \left(\frac{\alpha\theta(1+i)^2\Gamma(j+3k+1) + \Gamma(j+3k+3)}{\theta(1+i)^{j+3k+3}}\right)$$

Similarly, the characteristic function of Exponentiated two parameter Akash distribution is given by

$$\varphi_X(t) = \alpha\theta^3 \sum_{i=0}^\infty \sum_{j=0}^\infty \sum_{k=0}^\infty (-1)^i \binom{\alpha-1}{i}\binom{i}{k} \frac{it^j}{j!} \frac{\left(\theta^2 + 2\theta\right)^k}{(\alpha\theta^2 + 2)^{k+1}} \left(\frac{\alpha\theta(1+i)^2\Gamma(j+3k+1) + \Gamma(j+3k+3)}{\theta(1+i)^{j+3k+3}}\right)$$

5. Order Statistics

Order statistics has wide field in reliability and life testing. There is also an extensive role of order statistics in several aspects of statistical inference. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics of a random sample $X_1, X_2, \ldots X_n$ drawn from the continuous population with probability density function $f_x(x)$ and cumulative distribution function $F_x(x)$, then the pdf of r^{th} order statistics $X_{(r)}$ can be written as

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1 - F_X(x)\right)^{n-r} \quad (13)$$

Substitute the values of equation (5) and (6) in equation (13), we will obtain the pdf of r[th] order statistics $X_{(r)}$ for exponentiated two parameter Akash distribution and is given by

$$f_{x(r)}(x)=\frac{n!}{(r-1)!(n-r)!}\frac{\alpha\theta^3(\alpha+x^2)e^{-\theta x}}{\alpha\theta^2+2}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha(r-1)}$$

$$\times\left(1-\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha}\right)^{n-r}$$

The probability density function of higher order statistics $X_{(n)}$ can be obtained as

$$f_{x(n)}(x)=n\frac{\alpha\theta^3(\alpha+x^2)e^{-\theta x}}{\alpha\theta^2+2}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha(n-1)}$$

Similarly, the pdf of first order statistics $X_{(1)}$ can be obtained as

$$f_{x(1)}(x)=n\frac{\alpha\theta^3(\alpha+x^2)e^{-\theta x}}{\alpha\theta^2+2}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1}\left(1-\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha}\right)^{n-1}$$

6. Maximum Likelihood Estimation

In this section, we will discuss the maximum likelihood estimators of the parameters of exponentiated two parameter Akash distribution. Let $X_1, X_2, \ldots X_n$ be the random sample of size n from the Exponentiated two parameter Akash distribution, then the likelihood function can be written as

$$L(\alpha,\theta)=\frac{\left(\alpha\theta^3\right)^n}{(\alpha\theta^2+2)^n}\prod_{i=1}^{n}\left((\alpha+x^2)e^{-\theta x}\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right)^{\alpha-1}\right)$$

The log likelihood function is given by

$$\log L(\alpha,\theta)=n\log\alpha+3n\log\theta-n\log(\alpha\theta^2+2)+\sum_{i=1}^{n}\log\left(\alpha+x_i^{\,2}\right)-\theta\sum_{i=1}^{n}x_i$$

$$+(\alpha-1)\sum_{i=1}^{n}\log\left(1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x}\right) \qquad (14)$$

The maximum likelihood estimates of α, θ which maximizes (14), must satisfy the normal equations given by

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} - \frac{n\theta^2}{\alpha\theta^2 + 2} + \sum_{i=1}^{n}\left(\frac{1}{(\alpha + x^2)}\right) + \sum_{i=1}^{n} \log\left(1 - \left(1 + \frac{\theta x(\theta x + 2)}{\alpha\theta^2 + 2}\right)e^{-\theta x}\right) = 0$$

$$\frac{\partial \log L}{\partial \theta} = \frac{3n}{\theta} - n\left(\frac{2\alpha\theta}{\alpha\theta^2 + 2}\right) - \sum_{i=1}^{n} x_i + (\alpha - 1)\psi\left(1 - \left(1 + \frac{\theta x(\theta x + 2)}{\alpha\theta^2 + 2}\right)e^{-\theta x}\right) = 0$$

Where ψ (.) is the digamma function.

At this point it is important to mention that the analytical solution of the above system of non-linear equations is unknown. Algebraically it is very difficult to solve the complicated form of likelihood system of nonlinear equations. Therefore we use R and wolfram mathematics for estimating the required parameters of the proposed distribution.

7. Data Analysis

In this section, we use a real-life data set in exponentiated two parameter Akash distribution and the model has been compared with two parameter Akash distribution.

The following real life data set represents the survival times of 121 patients suffering from breast cancer reported by Lee (1992) and the data set is executed below in table 1

Table 1: Data regarding 121 patients suffering from breast cancer reported by Lee (1992)

0.3	0.3	4.0	5.0	5.6	6.2	6.3	6.6	6.8	7.4
7.5	8.4	8.4	10.3	11.0	11.8	12.2	12.3	13.5	14.4
14.4	14.8	15.5	15.7	16.2	16.3	16.5	16.8	17.2	17.3
17.5	17.9	19.8	20.4	20.9	21.0	21.0	21.1	23.0	23.4
23.6	24.0	24.0	27.9	28.2	29.1	30.0	31.0	31.0	32.0
35.0	35.0	37.0	37.0	37.0	38.0	38.0	38.0	39.0	39.0
40.0	40.0	40.0	41.0	41.0	41.0	42.0	43.0	43.0	43.0
44.0	45.0	45.0	46.0	46.0	47.0	48.0	49.0	51.0	51.0
51.0	52.0	54.0	55.0	56.0	57.0	58.0	59.0	60.0	60.0
60.0	61.0	62.0	65.0	65.0	67.0	67.0	68.0	69.0	78.0

80.0	83.0	88.0	89.0	90.0	93.0	96.0	103.0	105.0	109.0
109.0	111.0	115.0	117.0	125.0	126.0	127.0	129.0	129.0	139.0
154.0									

In order to compare the exponentiated two parameter Akash distribution with two parameter Akash distribution, we consider the Criterions like *BIC* (Bayesian information criterion), *AIC* (Akaike information criterion), *AICC* (Corrected Akaike information criterion) and -2logL. The better distribution corresponds to lesser values of *AIC*, *BIC, AICC* and *-2logL*. For calculating *AIC, BIC, AICC* and *-2logL* can be evaluated by using the formulas as follows.

$$AIC = 2k - 2\log L \qquad AICC = AIC + \frac{2k(k+1)}{n-k-1} \quad \text{and} \quad BIC = k\log n - 2\log L$$

Where *k* is the number of parameters in the statistical model, *n* is the sample size and -2log*L* is the maximized value of the log-likelihood function under the considered model.

Table 2: Performance of the fitted distribution

Distribution	MLE	S.E	-2logL	AIC	BIC	AICC
Exponentiated Two Parameter Akash	$\hat{\alpha} = 0.49572675$ $\hat{\theta} = 0.04572050$	$\hat{\alpha} = 0.06159590$ $\hat{\theta} = 0.00435844$	1157.502	1161.502	1167.094	1161.6036
Two Parameter Akash	$\hat{\alpha} = 3.023468$ $\hat{\theta} = 5.217657$	$\hat{\alpha} = 1.676065$ $\hat{\theta} = 4.526115$	1160.47	1164.47	1170.062	1164.5716

From table 2, it can be easily seen that the exponentiated two parameter Akash distribution have the lesser *AIC*, *BIC*, *AICC* and -2lo*gL* values as compared to the two parameter Akash distribution. Hence we can conclude that the exponentiated two parameter Akash distribution leads to a better fit than the two parameter Akash distribution.

8. Conclusion

In the present study, we have introduced a new generalization of two parameter Akash distribution called as exponentiated two parameter Akash distribution. The subject distribution is generated by using the exponentiated technique and the parameters have been obtained by using the maximum likelihood estimator. Some statistical properties along with reliability measures are discussed. The new distribution with its applications in real life-time data has been demonstrated. Finally, the result of real lifetime data set have been compared over two parameter Akash distribution and it has been found that the exponentiated two parameter Akash distribution provides a better fit than the two parameter Akash distribution.

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