

Meta-Informatics and Intuitionistic Fuzzy Logic

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Abstract: This paper concerns itself with the context of modern university education and the extent to which it is more focused on research than teaching, more given to training than education, more concerned with facts than information, let alone with knowledge for living as well as earning a living. In particular it outlines the salient features of intuitionistic fuzzy logic and its role in decision making under conditions of uncertainty. It raises questions about sorting the sources of information and the motives of informants.

Keywords: Fuzziness, decidability, tautology, information, critical faculty, expository writing.

Mathematics Subject Classification 2010: 00A99; 97A40

Introduction

Informatics deals with the storage and retention of data for future use. In most university courses the emphasis is on the important technical aspects of the storage and retention. The use is generally restricted to the use of an application. Here we want to consider the ‘meta’; that is, the use from the point of view of distinctions among data, fact, information, knowledge and wisdom. A similar approach will be taken with intuitionistic fuzzy logic (IFL). Thus, this paper is not about the IFL and Informatics *per se*, but *per accidens* in relation to the non-specialist educated graduate.

The reasons to justify these constraints on their ordinary expositions are twofold:

- IFL deserves to be better known and part of the repertoire of the 21st century educated engineering and computing scientists, and
- there is a danger for universities to focus on what is measurable in training technocrats and meeting the key performance indicators of research at the expense of the scholarship of teaching in the sense of Boyer [1].

Two situations in support of these points come to mind. One is the worry about performance in university league tables, whether focused on research measures or teaching satisfaction (which are inversely related in some countries). The other is a consequence of these league games with a lowering of the importance of good expository writing. Evidence for these can be found in mediocre literature reviews which skim along the chronological surface with descending to any real critical depth where unexpected links can lead to the creation of new knowledge from old. When one meets a really good literature review in a doctoral dissertation and praises it as worthy of publication in its own right, one gets a brief thank-you but is firmly told that it does not count as a research performance indicator. Yet, those of us who value the skills of good expository writing (real research awareness and readability, know what great value they add to the academy.

The points were getting at though were well expressed by Professor Diane Silvers Ravitch in a similar ceremony at Reed College in the USA when she said: “The person who knows ‘how’ will always have a job. The person who knows ‘why’ will always be the boss!”[2,3]. This gets to attitudes and the critical faculty, and even why digressions are not unimportant because they can shape attitudes, particularly our own self-confidence and the ability to challenge popular opinions. This paper is then more discursive and expository than technical, although it touches on areas now known as soft computing. It has two parts

- an outline of Intuitionistic Fuzzy Logic to sensitize the reader to its essential features as an extension of binary logic, and
- a case study of the actual complications which surround the flow of information

Intuitionistic Fuzzy Logic

IFL is a generalisation of the mathematical intuitionism of Brouwer [4] and the fuzzy sets of Zadeh [5]. Fuzzy logic essentially copes with the fact that not every member of a set necessarily shares the same degree of membership of that set; for instance, members of a given set of warm things vary in their degrees of warmth. Fuzzy logic is often seen as incompatible with Aristotelian logic, but this is not the case: fuzzy logic is a refinement of traditional binary logic [5]. Intuitionistic fuzzy logic goes further with the provision of formal non-membership of a fuzzy set [6]:

In classical terms, to each proposition p , we assign a truth value denoted by 1 (truth) or 0 (falsity). In IFL we assign a truth value, $\mu(p) \in [0,1]$, for the degree of truth, and a falsity value, $\nu(p) \in [0,1]$ [8] where the degrees satisfy

$$0 \leq \mu(p) + \nu(p) \leq 1$$

This assignment is provided by an evaluation function V , which is defined over a set of propositions S ,

$$V : S \rightarrow [0,1] \times [0,1]$$

such that

$$V(p) = \langle \mu(p), \nu(p) \rangle$$

is an ordered pair. If the values $V(p)$ and $V(q)$ of the propositions p and q are known, then V can be extended:

$$V(\neg p) = \langle \nu(p), \mu(p) \rangle$$

$$V(p \wedge q) = \langle \min(\mu(p), \mu(q)), \max(\nu(p), \nu(q)) \rangle,$$

$$V(p \vee q) = \langle \max(\mu(p), \mu(q)), \min(\nu(p), \nu(q)) \rangle,$$

$$V(p \supset q) = \langle \max(\nu(p), \mu(q)), \min(\mu(p), \nu(q)) \rangle;$$

and, for the propositions $p, q \in S$:

$$\neg V(p) = V(\neg p),$$

$$V(p) \cap V(q) = V(p \wedge q),$$

$$V(p) \cup V(q) = V(p \vee q),$$

$$V(p) \rightarrow V(q) = V(p \supset q).$$

Intuitionistic Fuzzy ishence an improvement and extension of the traditional fuzzy approach. “One of the main ideas behind the traditional fuzzy approach is that, since we cannot elicit the expert’s degrees of confidence in all possible propositional combinations of their original statements $S_i, i = 1, 2, \dots, n$, we:

- extract the degrees of confidence $d(S_i)$ in these statements, and then
- use negation, “and”-, and “or”-operations to estimate the expert’s degrees of belief in different propositional combinations.

To make these estimates more accurate, a natural idea is to extract, from the expert, not just his/her degrees of confidence in the original statements, but also degrees of confidence in some propositional combinations of these statements—at least the simplest ones” [9]. A *tautology* and an *intuitionistic fuzzy tautology* (IFT) are then defined respectively by

- “A is a tautology” if, and only if, $V(A) = \langle 1, 0 \rangle$;
- “A is an IFT” if, and only if, $V(A) = \langle a, b \rangle \rightarrow a \geq b$.

An advantage of the novel idea of non-membership occurs in applications, when used with Generalized Nets [10]; e.g., in medicine [11], education [12], ecology [13] and engineering [14] to name but four fields.

Fuzziness

“Fuzziness” itself is open to misinterpretation. It goes far beyond the notion of vagueness [15] because the simplest explanation which fits the facts has been the prevailing confirmation in science. Scientists are professionally skeptical: they want to see the evidence, but being human they can be prone to disregard facts which do not fit the available evidence, however vague, if their source is from authority less prestigious than the recognized authorities in their field.

Argument from authority within the “scientific establishment” [16] in science has historically hampered scientific progress which is usually marked by confirmation [17] or refutation [18], although in practice the working scientist operates within a framework which contains a collection of hypotheses where there can be disagreement between empirical data and individual conjectures without destroying the theory as a whole [19].

We have witnessed in the 2020 Covid-19 pandemic the practical problems for decision makers with multiple sources of information [20] in recommending action by medical specialists and citizens in minimizing the spread of the virus. “Decidability” (whether there is an effective procedure that decides whether the formula is a member of a given set) should be part of the core of many degrees from the liberal arts to the most highly technical specialty [21]. Not that we are suggesting that decisions should be left to artificial intelligence alone, but it does set out the relative merits of alternative actions among a choice of difficult choices [22]. Of course, the internal sense of intuition, based on experience and shrewd guessing plays a part in this, though Singer, a traditional philosopher, says: “... can we really know anything through intuition? ... mathematics as a system of tautologies, the basic elements of which are true by virtue of the meanings of the terms used” [23,24].

Bertrand Russell attempted a century ago to reduce mathematics to ‘tautologies’ (logical truths) but it proved impossible, because industrial and other research mathematicians simply do not work this way. For example, the standard presentation of the foundations of mathematics includes the “axiom of infinity” which states “there exists an infinite set”; you just have to take it (by intuition) or leave it. In no way is it a logical truth [25]. Research mathematicians work with intuitive insights rather than logic, though they may justify their conclusions with logic acceptable to their peers [26].

Mathematics is not an intellectual word game even if some scientists confuse efficient and formal causality [27]. Mathematical notation is more than a form of words: it is a tool of thought [28]. For instance, the relationship between subscripts and powers (as superscripts) in the umbral calculus reveals ideas latent in the original mathematical language [29,30].

Informatics and Teaching

At the then Sydney Teachers’ College sixty years ago, we learnt that our teaching had to inform the knowledge, skills and attitudes of our students. The word ‘attitudes’ has been replaced in the lexicon of the current Australian Qualifications Framework by the word ‘applications’, which, while not unimportant, is not the same.

Our graduates build on their knowledge as they progress in their professions. Some of their skills will become obsolete, but university education needs to shape their intellectual attitudes, particularly through a distinctive and cohesive core curriculum in the liberal arts, which is a foundation for life as well as for a living: for maintaining and developing their critical faculty and the currency of their knowledge and skills - for work well done – because work prepared well and completed to the last detail, no matter what the work is, is important for our nation, the person and their family – that is, for the common good.

Whatever the work might be, it will certainly involve the processing of information and the need to distinguish information and opinion, knowledge and information, wisdom and knowledge, even accuracy and precision, and the need to seek evidence and reasons. Thus the relatively new revolutionary Artificial Intelligence text generation system, called GPT2, which can write false but convincing news fiction [31] – dubbed “*deepfakes for text*” – which is so good that its potential for misuse is extremely high, and we are not yet prepared as a society for the legal and social ramifications of this type of technological breakthrough [32]: another reason for liberal education as well as technical training! The emerging power of this technology seems to be far more powerful than current computerization.

The founder of modern information theory was Claude Shannon when he applied Boolean algebra to electrical circuits at the Massachusetts Institute of Technology in 1937. However, the real foundation of the theory of information began exactly five hundred years previously with Lorenzo Valla, a Renaissance philologist, born in 1407. His interest in textual criticism was based on a distinction as relevant now as it was six centuries ago, a distinction between authentic and secondary, derived or forged information [33].

Valla’s major work established that a document supposedly donated by the Emperor Constantine to the papacy in the fourth century was actually a forgery, which had been used to justify Papal temporary power for more than a thousand years. Even so, Valla may have had an axe to grind as he particularly disliked the Pope at the time, Eugene IV (1431-1447), who is supposed to have bribed the Cardinals before his election. The Council of Basel subsequently deposed Eugene as a heretic in 1439 and the anti-Pope Felix V was elected [34].

Valla’s conflict of interest was that his employer was King Alfonso V of Naples who had his greedy eyes on the then considerable Papal territories thanks to the forged donation of Constantine. Thus, not only the *authenticity* of information, but also the motivation of its source, must be challenged [35]. This is increasingly difficult with the moral bankruptcy of modern social media where cowardly anonymity accompanies absence of editorial responsibility, though now, as in the fifteenth century, the money trail will usually tell you a lot! [36].

Concluding Comments

While the works of Orwell were sometimes dismissed as pessimistic, the current training of students has led to a crisis in university education as higher education seeks a purpose beyond ‘manpower’ and research rankings [37]. Many students and some staff at various universities do not want to engage in critical debate and discussion, and so “the devaluation of judgment and the positive endorsement of mandatory validation has had a formidable impact on academic education” [38]. As Einstein said: “It is easier to dismantle an atom than a prejudice!” [39]. Yet continuing to cultivate your critical faculty is an important part of real professional progress, irrespective of what position is achieved or how much wealth is accumulated.

There is an urgent need now for leaders in the profession to understand the human implications of the electronic surveillance by the technical *few* on the less technical *many*, particularly the effects on the unemployed and the under-employed [40]. While the works of Foucault and Bentham were also sometimes dismissed as pessimistic, Shoshana Zuboff has recently taken a serious and informed look at data harvesting and

surveillance from the perspective of social psychology [41]. There are state-of-the-art techniques in Deep Learning and Image Processing being developed to protect users so we can be optimistic, but the 2020 pandemic has illustrated the speed of rumours and mis-information in the absence of critical thinking about data,

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