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Structural Properties of Middle Graphs of Some Special Types of Trees

K. Vimala[1], M.Maniarasi[2], N. Abarna[3] and G.Sandhiya[4]
[1]*Assistant professor of Mathematics, STET Women's College, Mannargudi.*
[2,3,4] *Department of Mathematics, STET Women's College, Mannargudi.*

Abstract: In this paper, we focus on maximal binary tree, stand graph and path graph and then we obtain specific formula for the size of the middle graphs of these three trees.
Keywords: Maximal Binary Tree, Stand Graph, Path Graph, Middle Graph.

1. Introduction:

In this paper we discuss about only finite, simple and undirected graphs. For basic graph terminology, we refer [1,2 and 3]. Let G(V,E) be a simple graph with V={v_1,v_2,…….,v_n} and E={e_1,e_2,…….,e_n}. The middle graph M(G) [4,5 and 6], of a graph G is the graph whose vertex set is V(G)UE(G) in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

In this paper, we shown that, the size of M(B_n) is $\frac{(7n-11)}{2}$, the size of M(S_n) is (3n-2) and the size of M(P_n) is (3n-4).

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2. Preliminaries:

A linear graph (or)a graph G(V,E), consists of a set of objects V={v_1,v_2,…….,} are called vertices and another set E={e_1,e_2,……. } whose elements are called edges. Such that each e_k, is identified with an unordered pair (v_i,v_j) of vertices, these vertices are called the end vertices of 'e_k'. A graph G has neither self loops nor parallel edges is called a **simple graph**. A graph with finite number of vertices and edges is called a **finite graph**. Otherwise it is an **infinite graph**. A vertex v_i is an end vertex of some edge e_j ,v_i and e_j are said to be **incident** with each other.

The number of edges incident on a vertex v_i with self loops counted twice, is called **the degree**, d (v_i) of vertex v_i.A vertex of degree one is called **a pendent vertex** or **an end vertex**. The vertex of degree zero is called **an isolated vertex**.

Results:

The sum of the degree of the vertices of a graph is equal to twice the number of its edges. **(ie.,)** $\sum_{i=1}^{n} d(v_i)$ **= 2m,** where m is number of edges of the graph.

Theorem2.1: [7]
If G be a cycle with n vertices and n edges then its middle graph has 2n vertices and 3n edges.

Proof:

Let G be a cycle with n vertices and n edges. By definition of middle graph, whose vertex set is V(G)∪E(G) (i.e.,) the number of vertices of M(G) is n+n=2n. In a middle graph, the two vertices are adjacent if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

In any simple graph, every edge is incident with two vertices.

In particular, in a cycle every edge is adjacent to two edges. Therefore, in a middle graph of C_n, every edge create a new vertex, and the degree a newly formed vertex is four. The sum of the degree of the M(G) is equal to the sum of the degree of the vertices of G plus the sum of the newly formed vertices in M(G). (i.e.,) twice the number of vertices of G plus four times the number of vertices of G is equal to six times the number of vertices of G .But we know that, The summation of the degree of the vertices of G is equal to twice the number of edges of G. Here, twice the number of edges of M(G) is equal to six times the number of vertices of G.

Hence the number of edges of M(G) is equal to thrice the number of vertices of G Hence if a cycle with n vertices and n edges then its middle graph has 2n vertices and 3n edges.

3. The Main Results:

Definition 3.1:

A Binary Tree is defined as a tree in which there is exactly one vertex of degree two, and each of the remaining vertices is of degree one or three.

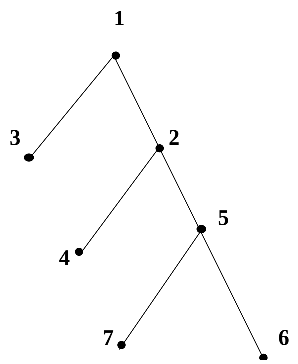
To construct a maximal binary tree for a given n such that the farthest vertex is as far as possible from the root, we must have exactly two vertices at each level, except at the 0 level.

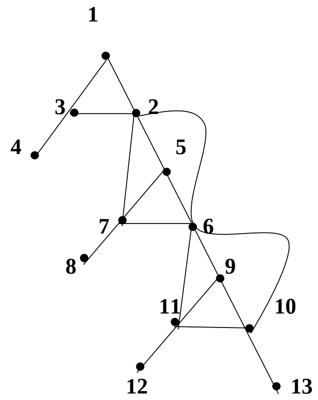
Therefore, $\max l_{max} = \frac{n-1}{2}$.

Definition3.2:

The middle graph M(G) of a graph G is the graph whose vertex set is V(G)UE(G) in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Example 3.1:





Maximal Binary Tree (B_7) **Middle GraphM(B_7)**

Theorem 3.1:

If B_n be a maximal binary tree with 'n' vertices and '(n-1) edges, then the middle graph of the maximal binary tree M (B_n), has (2n−1) vertices and $\frac{(7n-11)}{2}$ edges.

Proof:

Let B_n be a maximal binary tree with 'n' vertices and 'm' edges. Clearly, 'm' is equal to (n−1). Let $M(B_n)$ is a middle graph of B_n, whose vertex set is $V(B_n) \cup E(B_n)$ and in which two vertices are adjacent if and only if either they are adjacent edges of B_n or one is a vertex of B_n and the other is an edge of incident with it. Clearly, the number of vertices of $M(B_n)$ is equal to (m+n). It is denoted by p. Already, we know that ,the degree sum of the graph is equal to twice the number of edges. Here, the graph $M(B_n)$has (n+1)/2 pendent vertices, one vertex has degree two, (n-1)/2 vertices are degree three, (n-1)/2 vertices are degree four, one has degree five and (n-5)/2 vertices are degree six. By simple calculation, we get that, the degree sum of $M(B_n)$ is equal to 7n-11, which is equal to twice the number of edges. Then the number of edges of $M(B_n)$ is equal to (7n-11)/2 (Here 'n' always odd).Thus we get the p is equal to 2n-1 and q is equal to(7n-11)/2.Hence the size of $M(B_n)$ is (7n-11)/2.

TABLE 3.1 Number of vertices and Number of edges of Maximal Binary Tree and its Middle Graph

S.No	Maximal Binary Tree		Middle Graph	
	Number of vertices n	Number of edges m	Number of vertices p	Number of edges q
1	3	2	5	5
2	5	4	9	12

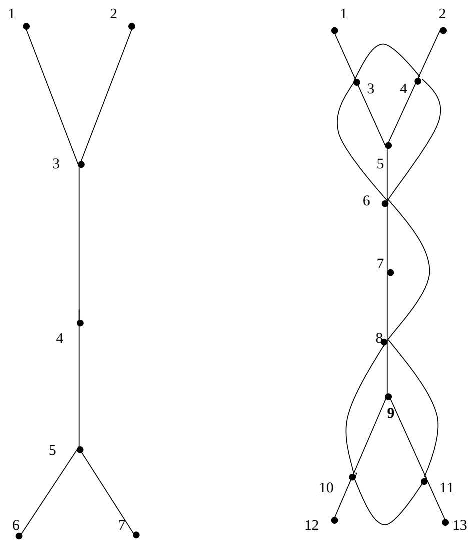
3	7	6	13	19
4	9	8	17	26
5	11	10	21	33
6	13	12	25	40
7	15	14	29	47
8	17	16	33	54
9	19	18	37	61
10	21	20	41	68
. . . .	. . . .	. . . .	. . . .	. . . .

Hence we conclude that, in Maximal Binary Tree,
Number of vertices in M(B_n) = 2n-1
Number of edges in M(B_n) = $\frac{7n-11}{2}$
Maximum degree Δ(M(B_n)) = 6
Minimum degree δ(M(B_n)) = 1

Definition 3.3:

Let G be a simple graph with n vertices and 2 vertices has a degree 3, all other vertices if any has degree 2. Then the graph is called **Stand graph.** A stand graph with n ≥ 6 vertices is denoted as $\mathbf{S_n}$.

Example 3.2:



Stand graph (S_7)Middle graph M (S_7)

Theorem3.2

If S_n be a stand graph with 'n' vertices $(n \geq 7)$, then the middle graph ofthe stand graph $M(S_n)$ has $(2n-1)$ vertices and $(3n-2)$ edges.

Proof:

Let S_n be a stand graph with 'n' vertices $(n \geq 7)$. Let $M(S_n)$ be the middle graph of S_n, whose vertex set is $V(S_n) \cup E(S_n)$ and in which two vertices are adjacent iff either they are adjacent edges of G of one is a vertex of G and the other edge incident with it. Clearly, the number of vertices of $M(S_n)$ is equal to $(m + n)$ i.e.,$(n-1)+n,$ $(ie)(2n-1)$. Here, the number of vertices of $M(S_n)$ denoted by p, is equal to $(2n-1)$. Already, we know that, sum of the degree of the graph is equal to twice the number of edges. Here, the graph $M(S_n)$ has four pendent vertices , $(n-6)$ vertices of degree two , two vertices of degree three, $(n-3)$ vertices of degree four and two vertices of five so by simple calculations, we get that, the degree sum of $M(S_n)$ is equal to $(6n-4)$ i.e., $2(3n-2)$, which is equal to twice the number of edges. Then the number of edges equal to $(3n-2)$.We get that, middle graph of the stand graph has$(2n-1)$ vertices and $(3n-2)$ edges. Hence the size of M(S_n) is (3n-2).

TABLE 3.2: Number of vertices and Number of edges of Stand Graph and its Middle Graph

S.NO	Stand Graph		Middle Graph	
	Number of vertices n	Number Of edges m	Number of vertices p	Number Of edges q
1	6	5	11	16
2	7	6	13	19
3	8	7	15	22
4	9	8	17	25
5	10	9	19	28
6	11	10	21	31
7	12	11	23	34
8	13	12	25	37
9	14	13	27	40
10	15	14	29	43
. . . .	. . . .	. . . .	. . . .	. . . .

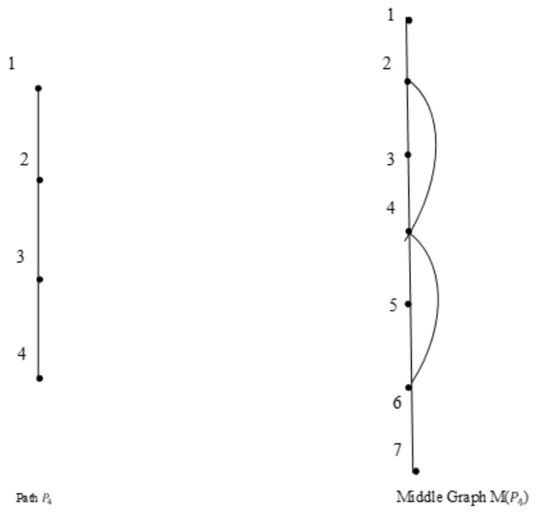
Hence we conclude that, in stand graph,

Number of vertices in M(S_n) = 2n-1
Number of edges in M(S_n) = 3n-2
Maximum degree Δ(M(S_n)) = 5
Minimum degree δ(M(S_n)) = 1

Definition3.4:

The **Path** is a trail with all vertices are distinct. It is denoted by P_n , where *n* is the number of vertices. The path P_n has (n-1) edges.

Example 3.3:



Theorem 3.3:

If P_n be a path with 'n' vertices and 'n-1' edges, then the middle graph of path M(P_n) has (2n-1) vertices and (3n-4) edges.

Proof:

Let P_n be a path with 'n' vertices and 'm' edges. Clearly, 'm' is equal to (n-1). Let M(P_n) is a middle graph of P_n, whose vertex set is V(P_n) ∪ E(P_n) and in which two vertices are adjacent iff either they are adjacent edges of P_n or one is a vertex of P_n and the other is an edge of incident with it. Clearly, the number of vertices of M(P_n) is equal to (m + n). It is denoted by p. Already, the degree sum of the graph is equal to twice the number of edges, Here , the graph M(P_n) has two pendent vertices, (n-2) vertices has degree two, 2 vertices has degree three and (n-3) vertices has degree four, By simple calculation, we get that, the degree sum of M(P_n) is equal to 2(3n-4) which is equal to twice the number of edges, then the number of edges M(P_n) is equal to (3n-4). Thus we get the P is equal to (2n-1) and q is equal to (3n-4). Hence the size of M(P_n) is (3n-4).

TABLE 3.3: Number of vertices and Number of edges of path graph and its middle graph

S.NO	**Path**		**Middle Graph**	
	Number of Vertices (n)	**Number of Edges (m)**	**Number of Vertices (n)**	**Number of Edges (m)**
1	3	2	5	5
2	4	3	7	8
3	5	4	9	11
4	6	5	11	14
5	7	6	13	17
6	8	7	15	20
7	9	8	17	23
8	10	9	19	26
9	11	10	21	29
10	12	11	23	32
. . . .	. . . .	. . . .	. . . .	. . . .

Hence we conclude that, in path graph,

Number of vertices in M(P_n) = 2n-1
Number of edges in M(P_n) = 3n-4
Maximum degree Δ(M(P_n)) = 4
Minimum degree δ(M(P_n)) = 1

Conclusion:

We have showed that, ifB_n be a Maximal Binary Tree with 'n' vertices and '(n-1)' edges. Here we find out, the middle graph of an maximal binary tree M(B_n) has (2n-1) vertices and $\frac{7n-11}{2}$ edges, if S_n be a Stand Graph with 'n' vertices $(n \geq 7)$, then the middle graph of the stand graph $M(S_n)$ has $(2n-1)$ vertices and $(3n-2)$ edges and If P_n be a path with 'n' vertices and 'n-1' edges, then the middle graph of path M(P_n) has (2n-1) vertices and (3n-4) edges. Hence the size of M(B_n) is $\frac{7n-11}{2}$, the size of M(S_n) is $(3n-2)$, andthe size of M(P_n) is (3n-4).

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