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# Comparison of Performance Effects of Mixed Matrix for FastICA Algorithm and JADE Algorithm

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**Abstract:** On the basis of expounding the theory principle and algorithm steps of FastICA algorithm and JADE algorithm, blind source separation processing experiments for image were used to do comparative analysis on the performance of the mixing matrix effects to the FastICA algorithm and JADE algorithm. It is found that the FastICA algorithm can only get the blind source separation results when the mixed matrix is not singular, however JADE algorithm is not very sensitive to singularity of mixed matrix.
**Keywords**: blind source separation, FastICA algorithm, JADE algorithm, mixed matrix, the singularity

## I. Introduction

Blind source separation (BSS) technology is an effective method of modern signal processing, which can recover various original signals that cannot be directly observed from several observed mixed signals [1]. Since the 1990s, BSS has become a hot topic in signal processing field. In 1991, after Jutten and Herault [2], blind source separation began to be studied internationally. In 1994, Comon formally proposed the concept of ICA and gave its mathematical model [3]. In 1995, Bell and Sejnowski [4] applied information theory criteria combined with neural network to obtain an ICA separation algorithm, which successfully solved the cocktail party problem and realized the separation of the voices of ten people, which caused a great sensation in the signal processing academia and greatly promoted the development and deepening of ICA research. Cardoso and Laheld [5] summarized previous studies in 1996. Hyvarinen and Oja [6] proposed a classical ICA algorithm called fixed point algorithm (FastICA) in 1997. Amari [7] made a detailed analysis of ICA's natural gradient algorithm in 1998. After 2000, the research on ICA focused more on discussing practical problems. Shi et al.[8] proposed a new ICA fast fixed-point algorithm in 2004. Park et al.[9] proposed a modified information maximization algorithm for blind source separation in 2006. Via et al.[10] studied the ICA problem of quaternion of second order statistics in 2011. In this paper, at first, the theory principle and algorithm steps of FastICA algorithm and JADE algorithm were introduced. And then, for comparative analysis on the performance of the mixing matrix effects to the blind source separation processing, experiments of image processing by FastICA algorithm and JADE algorithm were carried out. At last, the conclusion were gotten by us.

## II. Principle of Two Kinds of Algorithms

Blind source separation [11-12] refers to the process of separating each source signal from the observation signal when the observer only knows the overlapped observation signal and cannot accurately obtain the theoretical model and source signal of the observation signal. The meaning of "blind" here refers to "undetectable source signal" and "unknown characteristics of mixed system".

**1.1 Fastica Algorithm**

Hyvarinen and Oja[6] proposed a fixed-point algorithm--FastICA algorithm by using entropy optimization method. The advantage of this algorithm is that there is no problem of choosing learning factor, and the convergence speed is fast. In addition, the algorithm does not limit the Kurtosis of source signal, that is, super-gaussian or sub-gaussian of source signal. The objective function of the algorithm is:

$$\mathbf{J_G}(\mathbf{Y}) = \left[\mathbf{E}\{\mathbf{G}(\mathbf{Y})\} - \mathbf{E}\{\mathbf{G}(\mathbf{Y_{Gauss}})\}\right]^2 \quad (1)$$

Where, $G$ is a nonquadratic nonlinear function. The learning rule of FastICA is to find a direction that maximizes the non-Gaussianity of $\mathbf{Y} = \mathbf{w}^{\mathbf{T}}\mathbf{x}$. Under the constraint condition of $E\{(\mathbf{w}^T\mathbf{x})^2\} = \|\mathbf{w}\|^2 = 1$, according to the Lagrange condition, the optimal solution of $E\{\mathrm{G}(\mathbf{w}^T\mathbf{x})\}$ can make the gradient of the Lagrangian function be equal to 0:

$$E\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\} + \beta\mathbf{w} = 0 \quad (2)$$

Where, $\beta$ is a constant. At the optimal value $\mathbf{w}^{\mathrm{opt}}$ of $\mathbf{w}$, we get $\beta = E\{(\mathbf{w}^{\mathrm{opt}})^T\mathbf{x}g((\mathbf{w}^{\mathrm{opt}})^T\mathbf{x})\}$. Now let's use the Newton method to solve the equation. Using $F$ denotes the function on the left side of equation (2), then its Jacobi matrix $J\{F(\mathbf{w})\}$ is:

$$J\{F(\mathbf{w})\} = E\{\mathbf{x}\mathbf{x}^T g'(\mathbf{w}^T\mathbf{x})\} - \beta\mathbf{I} \quad (3)$$

Since $\mathbf{x}$ is the signal after bleaching pretreatment, the approximate expression can be obtained: $E\{\mathbf{x}\mathbf{x}^T g'(\mathbf{w}^T\mathbf{x})\} \approx E\{\mathbf{x}\mathbf{x}^T\}E\{g'(\mathbf{w}^T\mathbf{x})\} = E\{g'(\mathbf{w}^T\mathbf{x})\}\mathbf{I}$. So the Jacobi matrix becomes a diagonal matrix, and it is easier to invert. Thus, the approximate Newtonian iterative formula can be obtained:

$$\begin{cases} \mathbf{w}^* = \mathbf{w} - \left[E\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\} - \beta\mathbf{w}\right] / \left[E\{\mathbf{x}g'(\mathbf{w}^T\mathbf{x})\} - \beta\right] \\ \mathbf{w} = \mathbf{w}^* / \|\mathbf{w}^*\| \end{cases} \quad (4)$$

Here $\mathbf{w}^*$ is the new value of $\mathbf{w}$, $\beta = E\{\mathbf{w}^T\mathbf{x}g(\mathbf{w}^T\mathbf{x})\}$, and normalization improves the stability of the solution. After simplification, the iterative formula of FastICA algorithm can be further obtained:

$$\begin{cases} \mathbf{w}^* = E\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\} - E\{\mathbf{x}g'(\mathbf{w}^T\mathbf{x})\}\mathbf{w} \\ \mathbf{w} = \mathbf{w}^* / \|\mathbf{w}^*\| \end{cases} \quad (5)$$

## 1.2 Jade Algorithm

JADE algorithm [13] is a representative blind signal separation algorithm proposed by Cardoso in 1993, which requires fourth-order cumulants of variables.

For the $d$-dimension complex random vector $\mathbf{v} = (v_1, \cdots, v_d)$ and the finite fourth-order cumulant, the fourth-order cumulants set is defined as the following:

$$\mathcal{C}_v \triangleq \{\text{Cum}(v_i, v_j, v_k, v_l) \mid 1 \le i, j, k, l \le d\} \quad (6)$$

Add noise in channel and environment to the model and consider the mixed model with noise:

$$\mathbf{x} = \mathbf{As} + \mathbf{n} \quad (7)$$

Where $\mathbf{n}$ is the noise. Let $\mathbf{z}$ is the spheroidized observation signal vector $\mathbf{z}=[z_1, z_2, \cdots, z_d]^T$, $\mathbf{M}$ is an arbitrary $d \times d$ matrix, and the four-dimension cumulant matrix $\mathbf{Q_z(M)}$ of $\mathbf{z}$ is defined as:

$$\begin{aligned}\mathbf{N}&=\mathbf{Q_z}(\mathbf{M}) \triangleq n_{ij} \\ &= \sum_{k=1}^{d}\sum_{l=1}^{d} \text{Cum}(z_i, z_j, z_k, z_l) m_{lk} \quad 1 \le i, j \le d\end{aligned} \quad (8)$$

Where $\text{Cum}(z_i, z_j, z_k, z_l)$ is the four-dimension cumulant of the $ith, jth, kth, lth$ component of the vector $\mathbf{z}$, $m_{kl}$ is the $kth, lth$ element of the matrix $M$.

The principle of JADE method is to compress the fourth-order cumulant matrix (or second-order correlation moment) of the mixed signal after whitening into a diagonal matrix through $U$ transformation, so as to solve the unitary matrix $U$. The specific process is as follows:

For any $n \times n$ matrix $M$, its fourth-order cumulant matrix is defined as:

$$\begin{aligned}&\mathbf{N} = \mathbf{Q_z}(\mathbf{M}) \\ &\Leftrightarrow n_{ij} = \sum_{k,l=1\sim n} \text{Cum}(z_i, z_j^*, z_k, z_l^*) m_{kl} \\ &(1 \le i, j \le n)\end{aligned} \quad (9)$$

Where, $\text{Cum}(x_i, x^*_j, x_k, x^*_l)$ is the ith-row and jth-column element of the $(k,l)-th$ fourth-order cumulative quantum matrix.

Firstly, the observation signal $\mathbf{x}$ is bleached by the bleaching transformation $\mathbf{Q}$, and the bleached signal is:

$$\begin{aligned}\mathbf{z} &= \mathbf{Qx} = \mathbf{Q}(\mathbf{As}+\mathbf{n}) \\ &= \mathbf{Q}(\tilde{\mathbf{x}}+\mathbf{n}) \\ &= \mathbf{Us}+\mathbf{Qn}\end{aligned} \quad (10)$$

Since the high-order cumulant of white noise is 0, the fourth-order cumulant of the above equation can be written as:

$$\begin{aligned}\mathbf{Q_z}(\mathbf{M}) &= \mathbf{QQ_{\tilde{x}}}(\mathbf{M})\mathbf{Q}^H \\ &= \mathbf{QAQ_s}(\mathbf{M})(\mathbf{QA})^H \\ &= \mathbf{UQ_s}(\mathbf{M})\mathbf{U}^H\end{aligned} \quad (11)$$

Since the source signal is linearly statistically independent, the fourth-order cumulant $\mathbf{Q}_z(\mathbf{M})$ of the source signal in the above equation is a diagonal matrix, and equation (11) can be written as:

$$\mathbf{Q_z}\left(\mathbf{M}\right)=\mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H \quad (12)$$

$$\mathbf{U}^H\mathbf{Q_z}\left(\mathbf{M}\right)\mathbf{U}=\boldsymbol{\Lambda} \quad (13)$$

It can be seen that on theory the determination of matrix $U$ can be accomplished by the joint diagonalization of the fourth-order cumulant of the albino signal, but it is difficult to get it directly in the practical operation. Therefore, in order to solve the unitary matrix $U$, Cardoso's JADE method converts the problem into $K$ fourth-order cumulative matrixes of the whitening signal, and the non-diagonal element be requested to get the smallest after unitary. The objective function is:

$$\begin{aligned} C(\mathbf{U}) &= \min\left\{\sum_k C(\mathbf{Q_z}\left(\mathbf{M}\right),\mathbf{U})\right\} \\ &= \min\left\{\sum_{k,i} off\left(u_i^*\mathbf{Q_z}\left(\mathbf{M}\right)u_i\right)\right\} \quad \left(k,i=1\sim n\right) \end{aligned} \quad (14)$$

## III. Image Blind Source Separation

Blind source separation of images [14] refers to image separation without prior knowledge of source images and their mixing modes. Image blind source separation as one of the key technology of image processing, has received wide attention and research in recent years, and has a broad application prospect, such as extracting target image from noise image, fingerprint image extracted from fingerprints, and various kinds of medical image blind source signal recovery and so on.

Mixed image can be divided into superposition mixed image and permutation mixed image according to different ways of mixing. Superposition mixed image means that several images are directly mixed according to some linear or nonlinear function, and permutation mixed image means that part content of one image is replaced by part content of another image. This paper discusses the former mixed mode of image.

## IV. Research of Mixed Matrix

When blind source separation is applied, we often choose a random matrix as the mixed matrix and conduct subsequent processing. However, the author finds that not all random matrices can be used as the mixed matrix, and the selection of the mixed matrix should follow certain rules. The following two images are compared using FastICA algorithm and JADE algorithm for blind source separation.

As shown in figure 1 below, (a) and (b) are two original grayscale images respectively, and (c) is their mixed images. According to the digital image processing method[15], they can be regarded as matrices. For the sake of simplicity, the hybrid matrix selected here is a two-dimensional matrix, and is set as $\mathbf{A}=\begin{pmatrix}\mathbf{a}_{11} & \mathbf{a}_{12}\\ \mathbf{a}_{21} & \mathbf{a}_{22}\end{pmatrix}$.(in the picture below $\mathbf{A}=\begin{pmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{1} & \mathbf{0}\end{pmatrix}$).


Fig.1 the original image (a,b) and mixed image (c)

The separation figure obtained after FastICA algorithm processing of (c) in Fig.1 is shown in Fig.2 below, where the left figure is the restored image of (a) in Fig.1 (hereinafter referred to as image 1), and the right figure is the restored image of (b) in Fig.1 (hereinafter referred to as image 2).

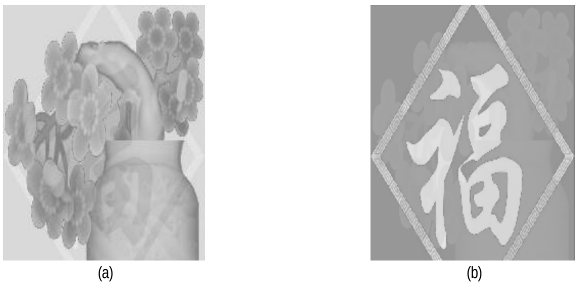


Fig.2 the image obtained after blind source separation (FastICA algorithm)

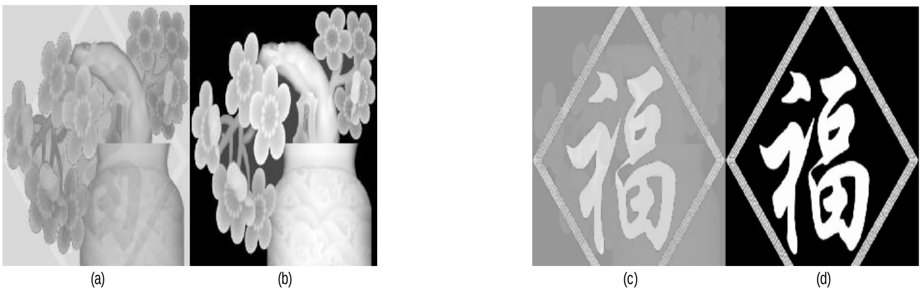


Fig.3 the image comparison before and after blind source separation
(a,c: before processing; b,d: after processing)

In order to analyze and compare the performance of the two blind source processing algorithms, peak signal-to-noise ratio (PSNR) and structure similarity method (SSIM) are adopted here [16].

The peak signal-to-noise ratio (PSNR) is a common method to measure image quality. Set the number of rows and columns of the image as $\mathbf{M}, \mathbf{N}$ respectively, and use $\mathbf{f}_{\mathbf{ij}}'$、$\mathbf{f}_{\mathbf{ij}}$ to represent the pixel value at the position $(\mathbf{i}, \mathbf{j})$ of the restored image and the original image, then the PSNR is defined as:

$$\mathbf{MSE}=\frac{1}{\mathbf{MN}}\sum_{\mathbf{i}=1}^{\mathbf{M}}\sum_{\mathbf{j}=1}^{\mathbf{N}}(\mathbf{f}_{\mathbf{ij}}^{'}-\mathbf{f}_{\mathbf{ij}})^2,\quad \mathbf{PSNR}=10\log(\frac{255^2}{\mathbf{MSE}}) \quad (15)$$

SSIM is a method to measure the similarity between recovered image and original image based on structural information. This method is consistent with the subjective perception of human eyes.

Set $\mathbf{x}$ represents the original image, and $\mathbf{y}$ represents the image information of the restored image at the same position. The measure of image brightness is represented by mean sum $\mu_{\mathbf{x}}$ and $\mu_{\mathbf{y}}$, the measure of contrast is represented by standard deviation sum $\delta_{\mathbf{x}}$ and $\delta_{\mathbf{y}}$, and the measure of structure similarity is represented by covariance $\delta_{\mathbf{xy}}$, so the formula of brightness, contrast and structure similarity is shown as follows:

$$\mathbf{l}(\mathbf{x},\mathbf{y})=\frac{2\mu_{\mathbf{x}}\mu_{\mathbf{y}}+\mathbf{C}_1}{\mu_{\mathbf{x}}^2+\mu_{\mathbf{y}}^2+\mathbf{C}_1} \quad (16)$$

$$\mathbf{c}(\mathbf{x},\mathbf{y})=\frac{2\delta_{\mathbf{x}}\delta_{\mathbf{y}}+\mathbf{C}_2}{\delta_{\mathbf{x}}^2+\delta_{\mathbf{y}}^2+\mathbf{C}_2} \quad (17)$$

$$\mathbf{s}(\mathbf{x},\mathbf{y})=\frac{\delta_{\mathbf{xy}}+\mathbf{C}_3}{\delta_{\mathbf{x}}\delta_{\mathbf{y}}+\mathbf{C}_3} \quad (18)$$

Where $\mathbf{C}_1,\mathbf{C}_2,\mathbf{C}_3$ is the constant added to prevent instability, usually a small positive number. Then SSIM is defined as:

$$\mathbf{SSIM}(\mathbf{x},\mathbf{y})=\left[\mathbf{l}(\mathbf{x},\mathbf{y})\right]^{\alpha}*\left[\mathbf{c}(\mathbf{x},\mathbf{y})\right]^{\beta}*\left[\mathbf{s}(\mathbf{x},\mathbf{y})\right]^{\gamma} \quad (19)$$

Among them, these three parameters $\alpha$, $\beta$, $\gamma$ are used to adjust the weight of brightness, contrast and structural information respectively.

By taking different values of the mixed matrix $\mathbf{A}$ successively, the performance comparison parameters of blind source separation by the two algorithms are shown in the following TABLE 1.

TABLE 1 Performance comparison of the two algorithms

| Mixed matrix $\mathbf{A}$ | Mothed | PSNR | PSNR | SSIM | SSIM |
|---|---|---|---|---|---|
| $\mathbf{A}=\begin{pmatrix}\mathbf{a}_{11} & \mathbf{a}_{12}\\ \mathbf{a}_{21} & \mathbf{a}_{22}\end{pmatrix},(\lvert\mathbf{A}\rvert\neq 0)$ | FastICA algorithm | 6.4239 | 5.5308 | 0.4424 | 0.2050 |
| | JADE algorithm | 10.8032 | 6.4991 | 0.6145 | 0.2306 |
| $\mathbf{A}=\begin{pmatrix}\mathbf{a}_{11} & \mathbf{a}_{12}\\ \mathbf{a}_{21} & \mathbf{a}_{22}\end{pmatrix},(\lvert\mathbf{A}\rvert=0)$ | FastICA algorithm | none | none | none | none |
| | JADE algorithm (i.e. $\mathbf{A}=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$) | 7.7394 | 5.5604 | 0.1766 | 0.1621 |

Through multiple experiments, it is found that when FastICA algorithm is used, no matter what values of elements $\mathbf{a}_{11}, \mathbf{a}_{12}, \mathbf{a}_{21}, \mathbf{a}_{22}$ in the mixed matrix $\mathbf{A}$ are selected, as long as the determinant of $\mathbf{A}$ is not equal to zero (i.e. $|\mathbf{A}| \neq 0$), the results of blind source separation are as follows: PSNR of image 1 (a in Fig.3) is 6.4239, and PSNR of image 2 (c in Fig.3) is 5.5308. The SSIM of image 1 is 0.4424, and the SSIM of image 2 is 0.2050.When JADE algorithm is used, regardless of the specific value of elements $\mathbf{a}_{11}, \mathbf{a}_{12}, \mathbf{a}_{21}, \mathbf{a}_{22}$ in the mixed matrix $\mathbf{A}$, as long as the determinant of $\mathbf{A}$ is not equal to zero (i.e. $|\mathbf{A}| \neq 0$), the result of blind source separation is: PSNR of image 1 is 10.8032 and PSNR of image 2 is 6.4991.The SSIM of image 1 is 0.6145, and the SSIM of image 2 is 0.2306. Therefore, it is concluded that as long as the mixed matrix $\mathbf{A}$ is not singular, the separation results obtained by the two algorithms are fixed regardless of the specific values of the elements of $\mathbf{A}$. Moreover, the separation effect of JADE algorithm is superior to that of FastICA algorithm.

At the same time, it is found that, regardless of the specific values of elements $\mathbf{a}_{11}, \mathbf{a}_{12}, \mathbf{a}_{21}, \mathbf{a}_{22}$ in the mixed matrix $\mathbf{A}$, as long as the determinant of $\mathbf{A}$ is equal to zero (i.e. $|\mathbf{A}|=0$), FastICA algorithm cannot obtain the separation result image, which means that FastICA algorithm does not converge at this time. When the determinant of $\mathbf{A}$ is equal to zero (i.e. $|\mathbf{A}|=0$), the JADE algorithm can be used to obtain the separated result image. After many experiments, it is found that the blind source separation results are different when $\mathbf{a}_{11}, \mathbf{a}_{12}, \mathbf{a}_{21}, \mathbf{a}_{22}$ are taken as different values. For example, when $\mathbf{a}_{11} = \mathbf{a}_{12} = \mathbf{a}_{21} = \mathbf{a}_{22} = 1$, the PSNR of image 1 is 7.7394, and the PSNR of image 2 is 5.5604. The SSIM of image 1 is 0.1766, and the SSIM of image 2 is 0.1621. At this time, JADE algorithm has a slightly worse separation effect than when $|\mathbf{A}| \neq 0$, but the general picture of the original image can still be seen. The following TABLE 2 lists the separation effect in the condition of $|\mathbf{A}|=0$ when element values in the first row of $\mathbf{A}$ are unchanged and element values in the second row are changed, indicating that JADE algorithm can be run at this time. Therefore, JADE algorithm is not particularly sensitive to the singularity of mixed matrix, that is to say, JADE algorithm has strong applicability.

TABLE 2 The separation result of JADE algorithm at $|\mathbf{A}|=0$

| Mixed matrix $\|\mathbf{A}\|=0$ | Image | PSNR | SSIM |
|---|---|---|---|
| $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ | Image 1 | 7.7891 | 0.1772 |
| | Image 2 | 5.5604 | 0.1621 |
| $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}$ | Image 1 | 7.6526 | 0.1868 |
| | Image 2 | 5.5604 | 0.1621 |
| $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}$ | Image 1 | 8.0862 | 0.2509 |
| | Image 2 | 5.5604 | 0.1621 |

## V. Conclusion

As important algorithms in blind source separation, FastICA algorithm and JADE algorithm are widely used. At first, this paper introduces the principle and derivation of the two algorithms in detail, and then from the angle of effect of the mixing matrix to the performance of the algorithm, through the experimental analysis of two image blind source separation, and the comparative analysis to blind source separation results by two kinds of evaluation methods of peak signal-to-noise ratio (PSNR) and structural similarity law (SSIM), it is concluded that these two kinds of algorithm have different affected performance of mixed matrix, namely for FastICA algorithm only under the condition of the mixed matrix with singular blind source separation results are obtained, while for the JADE algorithm it is not very sensitive to whether or not the mixed matrix is singular. In addition, this paper also obtains the analysis results that JADE algorithm performs better in blind source separation than FastICA algorithm in general.

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