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Air and refrigerant temperature optimal stabilization for the fridge unit with one thermostat valve

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Abstract: The article discusses air temperature control system in the refrigerating chamber with one thermostatic valve. The study of the system during the identification experiment showed that it can be considered as two – dimensional one since its initial coordinates form two – dimensional vector, the components of which are deviations of air temperature in the chamber from the set value and deviations of the refrigerant temperature at the evaporator outlet. It has been proven that it is possible to increase the efficiency of a refrigeration unit with one thermostatic valve through the use of an optimal system for stabilizing the air temperature in a refrigerating chamber. To build such a system, a new method has been created for synthesizing optimal multidimensional systems for stabilizing objects with delays in control signals. This method allows you to determine the matrix of discrete transfer functions of the regulator in the conditions of multidimensional stationary disturbances and measurement noise. When applying this method mathematical operations with transcendental functions do not occur.
Keywords: disturbance, optimum system, refrigeration unit, regulator, thermostatic valve

1. Introduction

The system of automatic control of air temperature in a refrigerating chamber with one thermostatic valve, as indicated in [1, 2], has two control paths: the first path is designed to stabilize the air temperature, and the second one - to produce cold. Observation of signals in the control paths of this system, carried out under operating conditions, showed that regulation occurs under conditions of uncontrolled disturbances and is accompanied by measurement noise [1, 2].

It is possible to increase the efficiency of a refrigeration unit with one thermostatic valve (TMV) by using an optimal system for stabilizing the air temperature in the refrigerating chamber. The creation of such a system is hampered by the presence of a delay in the control signals and the action of a two-dimensional vector of perturbations, the dynamics of which are significantly different from white noise. In such conditions, it is necessary to substantiate the choice of the synthesis method for the optimal stabilization system and to develop a method for its application.

2. Analysis of recent research and publications

The study of methods for the synthesis of systems for stabilizing dynamic objects with a delay [3-11] allows us to divide them into two classes. The first class consists of methods that are based on adding an additional element (compensator) to the structural diagram of the system, due to which an equivalent replacement of a control object occurs with a dynamic object without delay with using known management system development procedures. The main limitations in the application are associated with high requirements for the accuracy of determining the model of the control object, neglecting the effect of perturbations on the real object, the complexity of ensuring the robustness of the synthesized control system.

The second class includes methods aimed at using the positive effect of lateness on the properties of closed-loop control systems. These are methods that use equations of state to synthesize control systems for scalar objects. The necessary conditions for their use are: the ability to measure data on state variables (output coordinates) in an ideal way, complete controllability and observability of the control object, the possibility of affecting the system of random disturbances only in the form of white noise.

The second group combines methods for calculating optimal multidimensional stabilization systems for stable control objects in the space the conditions that are designed to work with regular program signals and noise, provided that all components of the state vector are perfectly measured.

The third group of methods is intended for the synthesis of multidimensional control systems in the frequency domain that are optimal by a quadratic criterion. The main disadvantage of these methods is associated with the complexity of calculating the residuals of transcendental functions that appear during the execution of the synthesis of the stabilization system of an object with a delay.

The fourth group of methods uses the *z*-transform apparatus to develop object management systems with lateness in control signals. Their use does not cause the emergence of transcendental functions in the development of an optimal stabilization system. However, existing methods allow us to develop an optimal system for stabilizing the motion of a stable control object in the absence of noise, setting a program signal and measuring initial coordinates.

Considering the results of a study of the dynamics of a supermarket refrigerating chamber with one thermostatic valve and a restriction on the use of known synthesis methods for optimal control systems for multidimensional objects with a delay, there is a need to develop a new synthesis method. Characteristic features of the method should be the synthesis in the frequency domain by the quadratic quality criterion and the absence of transcendental functions when performing calculations.

3. Purpose and task of the research

The aim of the research is to develop a method for synthesizing optimal stabilization systems for multidimensional objects with a delay in the frequency domain under conditions where stationary random noise-type vector processes act on the inputs of a linearized model of a multidimensional object, measuring the output signals of the control object is accompanied by stationary noise whose dynamics is different from white noise.

An analysis of the literature has shown that the system for controlling air and coolant temperatures in a refrigerating chamber with one thermostatic valve can be represented as shown in Fig. 1.

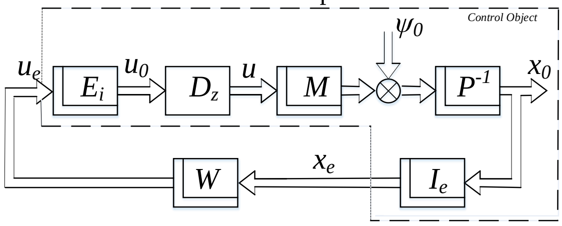

FIG.1. BLOCKDIAGRAM OF AIR AND REFRIGERANT TEMPERATURE CONTROL SYSTEM IN A REFRIGERATING CHAMBER WITH ONE TMV

The system consists of a continuous and discrete part and contains a control object and a W controller. The control object has two inputs: the first one has an m-dimensional vector of lattice control functions u_e, and the second input in the stabilization mode is acted upon by a generalized disturbance vector ψ_0 formed by additive mixtures of a two-dimensional disturbance vector and a vector of measurement noise and nonlinear distortion. The structure of the control object includes: a multidimensional extrapolator of zero order with a matrix of transfer functions $\underline{E_i}$

$$E_i = \left(1 - e^{-T_c s}\right)\frac{1}{s} E_m$$

where$s$ is a complex variable; E_m is the $m \times m$ unit matrix; D_z is the delay element with the transfer function matrix equal to

$$D_z = \begin{bmatrix} e^{-\lambda_1 s} & 0 & \cdots & 0 \\ 0 & e^{-\lambda_2 s} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-\lambda_m s} \end{bmatrix} \tag{1}$$

whereλ_i is the delay time of the i-th component of the vector u_0 of the control signals; pulse element I_e.

The synthesis task can be formulated as follows.

Suppose that the dynamics of the control object is characterized by a system of ordinary differential equations

$$P\,x_0 = M\;u + \psi_0 \tag{2}$$

where P is a given $n \times n$ matrix whose elements are operator polynomials from the differentiation operator; x_0 is the n-dimensional vector of the output signals of the control object; u is the m-dimensional vector of control signals; M is a given polynomial matrix of size $n \times m$, which determines the sensitivity of an object to a change in the control signal. The disturbance dynamics is characterized by the well-known spectral density matrix $S_{\psi0\psi0}$.

It is necessary to find the matrix of discrete transfer functions of the multidimensional regulator W by the known matrices P, M, $S_{\psi0\psi0}$, which connecting to the feedback circuit ensures stability of the closed control system and delivers the minimum of the sum of the weighted dispersions of the components of the vectors u_e and x_e, represented as the following functional

$$J = \left\langle x_e^{/} R x_e \right\rangle + \left\langle u_e^{/} C u_e \right\rangle \tag{3}$$

where "/" is the symbol of the "transpose" operation - the symbol of the expectation, *R* is a positive definite diagonal weight matrix; *C* is an inherently defined diagonal weight matrix.

4. Methodology of solving the problem

The solution of the problem occurs in two stages:

1. search for a discrete model of the dynamics of the control object with a delay;
2. synthesis of an optimal multidimensional stabilization system of a multidimensional object with a delay in control signals under random effects.

4.1 Determination of the discrete model of the dynamics of the control object with a delay

The purpose of the first stage is to move from a continuous control object to an equivalent discrete object (Fig. 2).

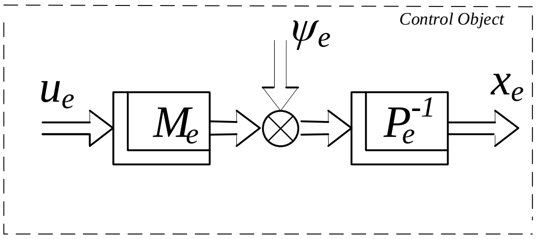

FIG. 2. EQUIVALENT DISCRETE OBJECT

The achievement of this goal is carried out as a result of applying the modified *z* - transformation. It is necessary to determine the rules for calculating the polynomial matrices M_e and P_e of a complex argument

$$z = e^{T_c s} \tag{4}$$

as well as the rules for finding the dynamic characteristics of the discrete perturbation vector ψ_e (matrix of discrete spectral densities $Z\psi\psi$).

As proved in the monograph [9] for a scalar control object, the equivalent discrete transfer function is equal to

$$W_e(z) = \mathfrak{Z}\left[\left(1 - e^{-T_c s}\right)e^{-\lambda_1 s}\frac{M}{P\ s}\right] = \left(1 - z^{-1}\right)\mathfrak{Z}\left(\frac{M}{P}, \mu\right)\Bigg|_{\mu = 1 - \frac{\lambda_1}{T_c}} \tag{5}$$

where $\mathfrak{Z}$ is the symbol of the operation *z* - transformations; $\mathfrak{Z}(*,\mu)$ is the symbol of the modified *z*-transform.

The extension of the action of expression (5) to a multidimensional control object is carried out on the basis of the ideas given in books [11-13] on discrete object control systems without delay.

Since the matrix of discrete transfer functions of the control object determines the relationship between the vectors u_e and x_e with zero initial conditions, we can write that

$$x_e = W_e u_e \text{ or } x_e = P_e^{-1} M_e u_e \tag{6}$$

The connection between the input and the output of the equivalent control object is determined by the equation

$$x_e = \mathfrak{Z}\left[P^{-1}M\ D_z E_i\right]u_e \tag{7}$$

Substituting the expression E_i for equation (7) and taking into account the properties of the *z*-transform, we can, by comparing the relations (6) and (5), write down the following

$$W_e(z) = \left(1 - z^{-1}\right)\mathfrak{Z}\left(\frac{1}{s}P^{-1}M\ D_z\right) \tag{8}$$

Given the form of the matrix D_z, defined by the equation (1) and expression (5), the matrix of the discrete transfer functions of the equivalent object of management is appropriate to be defined as a matrix of modified *z*-transforms

$$W_e(z) = \left(1 - z^{-1}\right)\mathfrak{Z}\left(\frac{1}{s}P^{-1}M\ ,\mu\right)\Bigg|_{\mu = 1 - \frac{\lambda_i}{T_c}, \forall i \in 1,m} \tag{9}$$

From equation (9) it follows that each element of the matrix W_e is connected to the corresponding element of the matrix of the transfer functions of the source control object $P^{-1}M$ with the following relation

$$[W_e(z)]_{ij} = (1 - z^{-1})З\left(\frac{[P^{-1}M]_{ij}}{s}, \mu\right)\Bigg|_{\mu=1-\frac{\lambda_i}{T_c}} \tag{10}$$

if the condition is realized

$$\lambda_i < T_c \tag{11}$$

Where i is row number of the matrix indicated in square brackets; j is the column number of the specified matrix. In the case of non-fulfilment of condition (11) it is necessary [9] to present a delay λ_i in the form of a sum

$$\lambda_i = \left[\frac{\lambda_i}{T_c}\right]T_c + \left\{\frac{\lambda_i}{T_c}\right\}T_c \tag{12}$$

where [] is the symbol of the search function of the whole part of the relation; {} is the symbol for finding the fractional part of the relation, and rewrite the equation (10) in the form

$$[W_e(z)]_{ij} = (1 - z^{-1})z^{-\nu_{oi}}З\left(\frac{[P_0^{-1}M_0]_{ij}}{s}, \mu\right)\Bigg|_{\mu=1-\nu_{1i}} \tag{13}$$

in which

$$\nu_{oi} = \left[\frac{\lambda_i}{T_c}\right],\ \nu_{1i} = \left\{\frac{\lambda_i}{T_c}\right\} \tag{14}$$

The algorithm for determining the polynomial matrices M_e, P_e of an equivalent discrete model of minimal order depends on the properties of the matrix of discrete transfer functions (9).

To find the transposed matrix of discrete spectral densities $Z'_{\psi\psi}$, by analogy with [11-13], two-sided *z*-transforms were used.

Given that the vector y is determined only at discrete points in time, separated by a period T_c, and the block diagram of the object is of the form at fig. 2, the desired transposed matrix of discrete spectral densities is determined by the equation

$$Z'_{\psi\psi} = P_e(z^{-1})З\left[P^{-1}(s)S'_{\psi 0\psi 0}P_*^{-1}(s)\right]P_e(z) \tag{15}$$

If the components of the perturbation vector belong to the deterministic signals, then its *z*-transformation has the following form

$$\Psi_e(z) = P_e(z^{-1})З\left[P^{-1}(s)\psi_0(s)\right] \tag{16}$$

Thus, we obtain the relationships that determine the equivalent discrete model of the object with a delay in the control signals.

4.2 Synthesis of the optimal multidimensional stabilization system of a multidimensional object with a delay in control signals at random influences

In terms of the work [14], the synthesis problem is formulated as follows. Suppose that a continuous control object is stable, has a delay in control signals and is represented by an equivalent discrete generalized control object. In this case, the block diagram of the stabilization system (Fig. 3) consists of two elements: a generalized control object and a controller, whose dynamics is represented by two systems of difference equations in *z*-transformations

$$P_e(z^{-1})x_e(z^{-1}) = M_e(z^{-1})u_e(z^{-1}) + \psi_e(z^{-1}), \tag{17}$$

$$u_e(z^{-1}) = W(z^{-1})x_e(z^{-1}), \tag{18}$$

where P_e is a polynomial matrix of size $n \times n$, which characterizes the own dynamics of a discrete analogue of the control object with a delay; M_e is a polynomial $n \times m$ matrix characterizing the sensitivity of the control object to a change in the vector u_e; ψ_e is *n*-dimensional perturbation vector, whose components are discrete centred interconnected random functions, whose dynamics are determined by the matrix of discrete spectral densities $Z\psi\psi$;

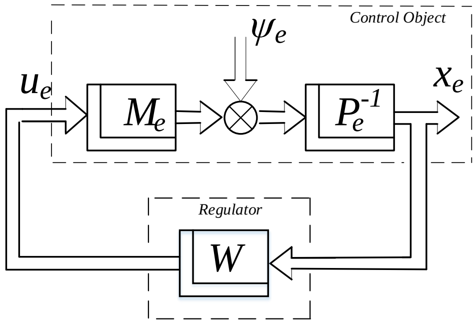

FIG. 3. BLOCK DIAGRAM OF THE STABILIZATION SYSTEM

W is the fractionally rational matrix of transfer functions of the $m \times n$ regulator.

The task of synthesis is formulated as follows. It is necessary with the known matrices P_e, M_e, $Z\psi\psi$ to find the matrix of transfer functions of the multidimensional regulator *W*, whose connection to the feedback circuit (Fig. 3) ensures the stability of the closed control system and delivers a minimum of the sum of the weighted dispersions of the components of the vectors $u_e$and x_e, represented as

$$J = \frac{1}{2\pi j} \oint_L tr\left(Z'_{xx} R + Z'_{uu} C\right) \frac{dz}{z} \tag{19}$$

where Z'_{xx} is the transposed matrix of discrete spectral densities of the output vector x_e; Z'_{uu} is the transposed matrix of discrete spectral densities of the control signal vector u_e.

Given the stability of the control object to solve the problem, the basic method for the synthesis of optimal stabilization systems from the monograph [14] is extended to the case of a discrete control object.

In accordance with the basic synthesis method based on the Wiener-Khinchin theorem, transposed matrices of discrete spectral densities Z'_{xx} and Z'_{uu} were found in the form

$$Z'_{xx} = F_x Z'_{\psi\psi} F_{x*} \tag{20}$$

$$Z'_{uu} = F_u Z'_{\psi\psi} F_{u*} \tag{21}$$

whereF_x is the matrix of transfer functions of the closed system (Fig. 3) from the perturbation vector ψ_e to the vector of output coordinates x_e; F_u is the matrix of transfer functions of a closed system from the disturbance vector ψ_e to the control signal vector u_e.

From several literature sources [7, 14] it is known that in the case of the fulfilment of relations (17) and (18) between the matrices of discrete transfer functions P_e, M_e, F_x, F_u, and *W* there is a relationship that characterizes the equation;

$$F_x = \left(P_e - M_e W\right)^{-1} \tag{22}$$

$$F_u = W\left(P_e - M_e W\right)^{-1} \tag{23}$$

$$P_e F_x - M_e F_u = E_n \tag{24}$$

When a stable control object is as a variable matrix of transfer functions Φ, it is advisable to choose the matrix (23). In this case, the stabilization quality functional (19), taking into account equations (20), (21), (24) and the properties of the matrix trace [17], is represented as

$$J = \frac{1}{2\pi j} \oint_L tr\Big(P_{e*}^{-1} R P_e^{-1} Z'_{\psi\psi} + \Phi_* M_{e*} P_{e*}^{-1} R P_e^{-1} Z'_{\psi\psi} + \Phi_* M_{e*} P_{e*}^{-1} R P_e^{-1} M_e \Phi Z'_{\psi\psi} +$$

$$+ P_{e*}^{-1} R P_e^{-1} M_e \Phi Z'_{\psi\psi} + \Phi_* C \Phi Z'_{\psi\psi}\Big) \frac{dz}{z} \tag{25}$$

and the relation between the matrices Φ and *W*, taking into account the expression (23), is determined by the equation

$$W = \left(E_m + \Phi M_e\right)^{-1} \Phi P_e \tag{26}$$

Thus, finding a physically feasible matrix of variable transfer functions Φ, which provides a minimum of functional (25), allows us to find a matrix of discrete transfer functions of the controller, the use of which ensures the stability of the stabilization system and the minimum of the quality indicator.

The search for the algorithm for determining the structure and parameters of the transfer function matrix *W*, as in [7, 14, 15], can be accomplished by minimizing the functional (25) in the class of stable and physically realizable variable matrices *Φ* using the Wiener – Kolmogorov procedure. In accordance with this procedure, the first variation of the functional (25) was found using the matrix *Φ*

$$\delta J = \frac{1}{2\pi j} \oint_L tr\left[\delta\Phi_* \frac{\partial(\ )}{\partial\Phi_*} + \frac{\partial(\ )_*}{\partial\Phi} \delta\Phi \right] \frac{dz}{z} \tag{27}$$

whereδΦ is an analytical variation in the unit circle;

$$\frac{\partial(\ )}{\partial\Phi_*} = M_{e*}P_{e*}^{-1}RP_e^{-1}Z_{\psi\psi}^{/} + M_{e*}P_{e*}^{-1}RP_e^{-1}Z_{\psi\psi}^{/} + C\Phi Z_{\psi\psi}^{/} \tag{28}$$

Let's denote

$$\Gamma_*\Gamma = M_{e*}P_{e*}^{-1}RP_e^{-1}M_e + C \tag{29}$$

where Γ is the result of the factorization on the right [18] of the sum of the matrices on the right side of the equation (29).

We define the matrix *D* as the result of the factorization on the left [18] of the transposed matrix of discrete spectral density disturbances

$$DD_* = Z_{\psi\psi}^{/} \tag{30}$$

We assume that the fractional-rational matrix *T* is:

$$T = \Gamma_*^{-1}M_{e*}P_{e*}^{-1}RP_e^{-1}D \tag{31}$$

Taking into account the notation (29) - (31) allows us to bring the partial derivative from expression (27) to the form

$$\frac{\partial(\ )}{\partial\Phi_*} = \Gamma_*(T + \Gamma\Phi D)D_* \tag{32}$$

Thus, the first variation of the functional quality (27) becomes equal

$$\delta e = \frac{1}{2\pi j} \oint_L tr\left[\delta\Phi_*\Gamma_*(T + \Gamma\Phi D)D_* + D(D_*\Phi_*\Gamma_* + T_*)\Gamma\delta\Phi\right]\frac{dz}{z} \tag{33}$$

As can be seen from [14], the matrix of variable functions *Φ*, which meets the conditions of stability and physical realizability and delivers the minimum of the functional (25), taking into account expression (33), should be determined on the basis of the following relation

$$\Phi = -\Gamma^{-1}(T_0 + T_+)D^{-1} \tag{34}$$

where$T_0 + T_+$ is the fractionally rational matrix, which is a stable part of the result of separation (splitting) [18-20] of the matrix *T*.

Thus, a new method of synthesizing optimal multidimensional discrete systems for stabilization of objects with delay in random effects was obtained, which prevents the emergence of transcendental functions in the synthesis of an optimal system for stabilizing an object with delay.

5 The results of the research

Below are the results of the implementation of the basic version of the calculations for which the variables have the following form

$C = 1$; R=1 $\alpha_1 = 0.13$; $\alpha_2 = 3.4$.

where $\alpha_1$is the coefficient which characterizes the ratio of the mean square deviation of the refrigerant temperature at the outlet of the evaporator, caused by the action of perturbations to the air temperature in the refrigerating chamber, caused by the action of perturbations; α_2 is the coefficient that characterizes the ratio of the mean square deviation of air temperature in the refrigerating chamber to the mean square deviation of noise and nonlinear distortions.

The matrix of continuous transfer functions of the refrigerating chamber W_a as

$$W_a = \begin{bmatrix} \dfrac{-3.09\cdot 10^{-3}(s-0.0067)(s-0.44)}{(s+0.136)(s+0.00115)} \\ \dfrac{0.0188(s+0.002)(s+0.102)}{(s+0.136)(s+0.00115)} \end{bmatrix} \tag{35}$$

Matrix of transfer functions of the object

$$W_{ob} = e^{-164s} W_a \tag{36}$$

Taking into account the sampling period T_c = 77.3 seconds, modified z-transforms were made and a matrix of discrete transfer functions of the control object was obtained.

$$W_e = z^{-3} \begin{bmatrix} \dfrac{0.0055z^2 - 0.01z - 9.4 \cdot 10^{-7}}{z^2 - 0.92z + 2.49 \cdot 10^{-5}} \\ \dfrac{0.014z^2 - 0.013z + 7.7 \cdot 10^{-7}}{z^2 - 0.92z + 2.49 \cdot 10^{-5}} \end{bmatrix} \tag{37}$$

In accordance with the above methodology, the polynomial matrices P_e, M_e are defined:

$$P_e = z^3 (z - 0.92)\left(z - 2.7 \cdot 10^{-5}\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{38}$$

$$M_e = \begin{bmatrix} 0.0055(z - 1.88)\left(z + 9.1 \cdot 10^{-5}\right) \\ 0.014(z - 0.86)\left(z - 6 \cdot 10^{-5}\right) \end{bmatrix} \tag{39}$$

The variable part of the models is the set of transposed matrices of discrete spectral densities of perturbations $Z'_{\psi\psi}$, which is found for all possible combinations of coefficients α_1, α_2.

Substitution of the base values of the coefficients α1 = 0.13; α2 = 3.4 allows to find the next transposed matrix of spectral densities of disturbances

$$S'_{\psi 0 \psi 0} = \begin{bmatrix} 6.32 \cdot 10^9 s^8 - 1.79 \cdot 10^6 s^6 + 182s^4 - 0.0052s^2 + 2.22 \cdot 10^{-9} & \begin{matrix} -4.7 \cdot 10^7 s^8 - 4.96 \cdot 10^6 s^7 - 1.96 \cdot 10^4 s^6 + 1430s^5 + 14s^4 + 0.0017s^3 + \\ + 6.9 \cdot 10^{-4} s^2 + 8.9 \cdot 10^{-7} s + 3.03 \cdot 10^{-10} \end{matrix} \\ \begin{matrix} -4.7 \cdot 10^7 s^8 + 4.96 \cdot 10^6 s^7 - 1.96 \cdot 10^4 s^6 - 1430s^5 + 14s^4 - 0.0017s^3 + \\ + 6.9 \cdot 10^{-4} s^2 - 8.9 \cdot 10^{-7} s + 3.03 \cdot 10^{-10} \end{matrix} & 3.5 \cdot 10^5 s^8 - 3490s^6 + 1.09s^4 - 8.35 \cdot 10^{-5} s^2 + 4.23 \cdot 10^{-11} \end{bmatrix} \times$$

$$\times \frac{\sigma_\varphi^2}{\pi \left| s^2 + 6.8 \cdot 10^{-4} s + 8.4 \cdot 10^{-7} \right|^2} \tag{40}$$

As a result of the two-side polar removal using the Davis algorithm [23], the fractional-rational matrix (40) obtained a polynomial matrix S_{00}

$$S_{00} = \begin{bmatrix} 6.32 \cdot 10^9 s^8 - 1.79 \cdot 10^6 s^6 + 182s^4 - 0.0052s^2 + 2.22 \cdot 10^{-9} & \begin{matrix} -4.7 \cdot 10^7 s^8 - 4.96 \cdot 10^6 s^7 - 1.96 \cdot 10^4 s^6 + 1430s^5 + 14s^4 + 0.0017s^3 + \\ + 6.9 \cdot 10^{-4} s^2 + 8.9 \cdot 10^{-7} s + 3.03 \cdot 10^{-10} \end{matrix} \\ \begin{matrix} -4.7 \cdot 10^7 s^8 + 4.96 \cdot 10^6 s^7 - 1.96 \cdot 10^4 s^6 - 1430s^5 + 14s^4 - 0.0017s^3 + \\ + 6.9 \cdot 10^{-4} s^2 - 8.9 \cdot 10^{-7} s + 3.03 \cdot 10^{-10} \end{matrix} & 3.5 \cdot 10^5 s^8 - 3490s^6 + 1.09s^4 - 8.35 \cdot 10^{-5} s^2 + 4.23 \cdot 10^{-11} \end{bmatrix} \tag{41}$$

and a fractional-rational function of a complex argument with stable poles

$$T_{00} = \frac{\sigma_\varphi}{\sqrt{\pi}\left(s^2 + 6.8 \cdot 10^{-4} s + 8.4 \cdot 10^{-7}\right)} \tag{42}$$

The factorization of the matrix (41) is performed using the Larin algorithm [25], allowed to find the following factor Q_c, which polynomial has zeros in the left half-plane of the complex variable s:

$$Q_c = \begin{bmatrix} -7.95\cdot 10^4 s^4 - 2094s^3 - 16.31s^2 - 0.021s - 1.16\cdot 10^{-5} & -8.89s^3 - 0.23s^2 + 0.072s + 4.57\cdot 10^{-5} \\ 591s^4 - 46.8s^3 - 1.11s^2 - 0.0015s - 2.57\cdot 10^{-6} & 0.066s^3 - 0.0053s^2 - 0.0093s + 5.98\cdot 10^{-6} \end{bmatrix} \quad (43)$$

Consequently, taking into account the results (42), (43) and the definition of factorization, the matrix $D_a$is defined as a product

$$D_a = P^{-1}T_{00}Q_c = \frac{\sigma_\varphi}{\sqrt{\pi}\left(s^2 + 6.8\cdot 10^{-4}s + 8.4\cdot 10^{-7}\right)(s + 0.0012)(s + 0.136)} \times \begin{bmatrix} -7.95\cdot 10^4 s^4 - 2094s^3 - 16.31s^2 - 0.021s - 1.16\cdot 10^{-5} & -8.89s^3 - 0.23s^2 + 0.072s + 4.57\cdot 10^{-5} \\ 591s^4 - 46.8s^3 - 1.11s^2 - 0.0015s - 2.57\cdot 10^{-6} & 0.066s^3 - 0.0053s^2 - 0.0093s + 5.98\cdot 10^{-6} \end{bmatrix} \quad (44)$$

By the known elements of the matrix D_a and the known sampling period T_c = 77.3 seconds, the matrix of discrete transfer functions of the forming filter of the random process ψ was found in the form

$$D_d = \frac{\sigma_\varphi}{\sqrt{\pi}(z - 0.92)\left(z - 2.72\cdot 10^{-5}\right)\left(z^2 - 1.94z + 0.95\right)} \times \begin{bmatrix} -7.95\cdot 10^4\left(z^2 - 1.89z + 0.89\right)\left(z^2 - 1.71z + 0.8\right) & 1.1\cdot 10^3(z + 1.77)(z - 0.95)(z - 0.012) \\ 591(z - 2.95)(z - 0.49)\left(z^2 - 1.89z + 0.91\right) & -162(z + 1.27)(z - 1.051)(z + 0.012) \end{bmatrix} \quad (45)$$

In turn, the found matrix (45) allowed us to determine the following transposed matrix of discrete spectral densities of perturbations

$$Z'_{\psi\psi} = \frac{\sigma_\varphi^2}{\pi z^2\left(z^2 - 2.05z + 1.054\right)\left(z^2 - 1.94z + 0.95\right)} \times \begin{bmatrix} 4.8\cdot 10^9(z - 1.05)(z - 0.95)(z - 0.77)(z - 1.3)\left(z^2 - 1.59z + 0.73\right)\times\left(z^2 - 2.17z + 1.37\right) & -6.46\cdot 10^7(z - 2.11)(z - 0.34)\left(z^2 - 1.9z + 0.91\right)\left(z^2 - 1.43z + 0.58\right)\times\left(z^2 - 2.3z + 1.48\right) \\ -3.6\cdot 10^7(z - 2.98)(z - 0.47)\left(z^2 - 2.1z + 1.1\right)\left(z^2 - 1.55z + 0.68\right)\times\left(z^2 - 2.46z + 1.72\right) & 4.8\cdot 10^5(z - 1.06)(z - 0.95)(z - 3.26)(z - 0.31)\left(z^2 - 1.09z + 0.34\right)\times\left(z^2 - 3.15z + 2.91\right) \end{bmatrix} \quad (46)$$

Thus, a set of equivalent discrete models of a refrigerating chamber with one dispenser is defined. It allows you to go to the search for structures and parameters of optimal regulators that correspond to certain variants of calculations. The synthesis of a set of matrices of discrete transfer functions of optimal multidimensional regulators of the system for stabilizing the air temperature in a refrigerating chamber with one SRW is carried out in accordance with the procedure given above.

The sum (29) found by the known matrices M_e, P_e, R and the base weight coefficient C is equal to

$$\Gamma_*\Gamma = \frac{1.03z^4 - 3.78\cdot 10^4 z^3 + 7.6\cdot 10^4 z^2 - 3.78\cdot 10^4 z + 1.03}{z^4 - 3.68\cdot 10^4 z^3 + 7.39\cdot 10^4 z^2 - 3.68\cdot 10^4 z + 1} \quad (47)$$

The factorization result for C = 1 has the form

$$\Gamma = \frac{1.02z^2 - 0.92z + 2.5 \cdot 10^{-5}}{z^2 - 0.91z + 2.5 \cdot 10^{-5}} \quad (48)$$

The result factorization of the matrix (49) looks like

$$D = \frac{\sigma_\varphi}{\sqrt{\pi}\left(z^2 - 1.94z + 0.95\right)} \begin{bmatrix} 7.9 \cdot 10^4 \left(z^2 - 1.84z + 0.85\right)\left(z^2 - 1.71z + 0.81\right) & -611\left(z - 0.95\right)\left(z + 14\right)\left(z^2 - 1.05z + 0.35\right) \\ -611\left(z - 0.48\right)\left(z - 2.7\right)\left(z^2 - 1.88z + 0.9\right) & 625\left(z - 1.04\right)\left(z + 0.095\right)\left(z^2 - 0.64z + 0.17\right) \end{bmatrix} \quad (49)$$

The formation of the matrix *T* by substituting the required data to the expression (31) and separating the result obtained relative to the unit circle allowed us to determine that the following equation is satisfied for the base coefficients C, α_1 and α_2

$$T_0 + T_+ = \frac{\sigma_\varphi}{\sqrt{\pi}} \left[\frac{-4.47z^3 + 16.7z^2 - 29.8z + 29.7 - 15.1z^{-1} + 2.85z^{-2}}{z^4 - 2.86z^3 + 2.73z^2 - 0.87z + 2.36 \cdot 10^{-5}} \quad \frac{6.73z^3 - 11.8z^2 + 4.27z + 1.43 - 0.787z^{-1} + 0.156z^{-2}}{z^4 - 2.86z^3 + 2.73z^2 - 0.87z + 2.36 \cdot 10^{-5}} \right] \quad (50)$$

The obtained data provided the possibility of finding the matrix of variable discrete transfer functions *Φ* for the base coefficients C, α_1 and α_2. After the reduction of F_u, the following matrix *Φ*is defined:

$$\Phi = \left[\frac{-0.29\left(z - 6.1\right)\left(z - 0.68\right)\left(z - 0.44\right)\left(z - 0.43\right)\left(z^2 - 1.91z + 0.92\right)}{z^2\left(z - 0.96\right)\left(z^2 - 1.88z + 0.89\right)\left(z^2 - 1.63z + 0.79\right)\left(z^2 - 0.56z + 0.22\right)} \quad \frac{-106\left(z + 0.15\right)\left(z - 0.93\right)\left(z^2 - 1.93z + 0.94\right)\left(z^2 - 1.56z + 0.66\right)}{z^2\left(z - 0.96\right)\left(z^2 - 1.88z + 0.89\right)\left(z^2 - 1.63z + 0.79\right)\left(z^2 - 0.56z + 0.22\right)} \right] \quad (51)$$

The degree of accuracy of the reduction is verified using the Bode diagram.
According to a certain matrix *Φ*, using equation (22), the matrix of transfer functions of the optimal controller is calculated, which after reduction has the form

$$W = \left[\frac{0.38\left(z - 1.051\right)\left(z^2 - 1.37z + 0.59\right)}{\left(z^2 - 1.87z + 0.88\right)\left(z^2 - 1.71z + 0.86\right)} \quad \frac{3.91\left(z - 0.95\right)\left(z^2 - 1.47z + 0.66\right)}{\left(z^2 - 1.87z + 0.88\right)\left(z^2 - 1.71z + 0.86\right)} \right] \quad (52)$$

Since the poles of the elements of the matrix *Φ* are in the circle of unit radius for all ratios of the coefficients C, α_1 and α_2, the controller with the transfer function matrix W ensures the stability of the closed stabilization system.

The quality of the optimal stabilization system with the same disturbances and noise depends on the structure and parameters of the matrix of discrete transfer functions *W*. The choice of the matrix *W*, which is advisable to implement in the controller, is based on the results of a study of the limits of improving the accuracy of stabilizing the air temperature in the refrigerating chamber.

6 Conclusion

The use of a modified *z*-transform allowed us to develop a new method for synthesizing optimal multidimensional systems for stabilizing objects with delays in control signals, which is distinguished by the ability to determine the matrix of discrete transfer functions of the controller under conditions that affect the system of multidimensional stationary disturbances and measurement noise, the dynamics of which are different from “white” noise, and prevents the emergence of mathematical operations with transcendental functions. The inclusion of such a regulator in the feedback circuit ensures the stability of a closed system and provides a minimum of the specified quality functionality.

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