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On Interval Valued Vague Almost Resolvable Spaces

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Abstract: This paper aims to study the different vague spaces based on their characterizations. The linkage between the interval valued vague almost resolvable and other spaces are also examined.
Keywords: Interval valued vague almost resolvable and irresolvable space, interval valued vague Baire space, interval valued vague hyper connected space, interval valued vague nodec space.

1. Introduction:

Major part of the existing mathematical tools for modeling, reasoning and computing, deterministic are explicit in character. The fuzzy set which has been introduced by Zadeh[14] solves majority of these problems. Actually Vague set theory is an elongation of fuzzy set theory and also vague sets are considered as a particular case of context- dependent fuzzy sets. The basic concept and the extension of vague set theory has been defined by Gau and Buehrer[3]. Interval valued vague sets has been applied in different area and it is one of the higher order fuzzy sets. E. Hewitt [4] and A.G. El'kin [2] has studied the concept of resolvability and irresolvability in topological spaces and gave a new approach to open hereditarily irresolvable spaces in classical topology and Richard Bolstein[10] induced the concept of Almost resolvable spaces. Thangaraj and Balasubramanian[12] introduced the concept of fuzzy resolvable and irresolvable space. The notion of almost resolvable spaces in fuzzy topological space was introduced and investigated by Thangaraj and Vijayan[13]. The purpose of this paper is to further extend the idea of vague set theory by introducing the concept of an interval valued vague set in almost resolvable and irresolvable space and derived some of their basic relations with other spaces.

2. Preliminaries:

Definition 2.1: [5] Let [I] be the set of all closed subintervals of the interval [0,1] and $\mu = [\mu_L, \mu_U] \in [I]$, where μ_L and μ_U are the lower extreme and the upper extreme, respectively. For a set X, an IVFS A is given by equation $A = \{< x, \mu_A(x) > / x \in X\}$ where the function $\mu_A : X \to [I]$ defines the degree of membership of an element x to A, and $\mu_A(x) = [\mu_{AL}(x), \mu_{AU}(x)]$ is called an interval valued fuzzy number.

Definition 2.2: [3] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and
(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$. The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.3:[11] An interval valued vague sets $\widetilde{A}^V$ over a universe of discourse X is defined as an object of the form $\widetilde{A}^V = \{< x_i, [T_{\widetilde{A}^V}(x_i), F_{\widetilde{A}^V}(x_i)] >, x_i \in X\}$ where $T_{\widetilde{A}^V} : X \to D([0,1])$ and $F_{\widetilde{A}^V} : X \to D([0,1])$ are called " truth membership function" and "false membership function" respectively and where D[0,1] is the set of all intervals within [0,1], or in other word an interval valued vague set can be represented by $\widetilde{A}^V = <[(x_i), [\mu_1, \mu_2], [v_1, v_2]]>, x_i \in X$ where $0 \le \mu_1 \le \mu_2 \le 1$ *and* $0 \le v_1 \le v_2 \le 1$. For each interval valued vague set $\widetilde{A}^V$, $\pi_{1\widetilde{A}^V}(x_i) = 1 - \mu_{1\widetilde{A}^V}(x_i) - v_{1\widetilde{A}^V}(x_i)$ are called degree of hesitancy of x_i in $\widetilde{A}^V$ respectively.

Definition 2.4:[6] An interval valued vague topology (IVT in short) on X is a family τ of interval valued vague sets(IVS) in X satisfying the following axioms.

(i) $0, 1 \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an interval valued vague topological space (IVTS in short) and any IVS in τ is known as a Interval valued vague open set(IVOS in short) in X.

The complement $\overline{A}$ of a IVOS A in a IVTS (X, τ) is called an interval valued vague closed set (IVCS in short) in X.

Definition 2.5:[6] Let $A = \{\langle x, [t_A^L(x), t_A^U(x)], [1 - f_A^L(x), 1 - f_A^U(x)]\rangle\}$ and $B = \{\langle x, [t_B^L(x), t_B^U(x)], [1 - f_B^L(x), 1 - f_B^U(x)]\rangle\}$ be two interval valued vague sets then their union, intersection and complement are defined as follows:

(i) $A \cup B = \{< x, [t_{A\cup B}^L(x), t_{A\cup B}^U(x)], [1 - f_{A\cup B}^L(x), 1 - f_{A\cup B}^U(x)] > / x \in X\}$ where

$t_{A\cup B}^L(x) = \max\{t_A^L(x), t_B^L(x)\}$, $t_{A\cup B}^U(x) = \max\{t_A^U(x), t_B^U(x)\}$ *and*

$1 - f_{A\cup B}^L(x) = \max\{1 - f_A^L(x), 1 - f_B^L(x)\}$, $1 - f_{A\cup B}^U(x) = \max\{1 - f_A^U(x), 1 - f_B^U(x)\}$

(ii) $A \cap B = \{< x, [t_{A\cap B}^L(x), t_{A\cap B}^U(x)], [1 - f_{A\cap B}^L(x), 1 - f_{A\cap B}^U(x)] > / x \in X\}$ where

$t_{A\cap B}^L(x) = \min\{t_A^L(x), t_B^L(x)\}$, $t_{A\cap B}^U(x) = \min\{t_A^U(x), t_B^U(x)\}$ *and*

$1 - f_{A\cap B}^L(x) = \min\{1 - f_A^L(x), 1 - f_B^L(x)\}$, $1 - f_{A\cap B}^U(x) = \min\{1 - f_A^U(x), 1 - f_B^U(x)\}$

(iii) $\overline{A} = \{< x, [f_A^L(x), f_A^U(x)], [1 - t_A^L(x), 1 - t_A^U(x)] > / x \in X\}$.

Definition 2.6:[6] Let (X, τ) be an interval valued vague topological space and $A = \{\langle x, [t_A^L, t_A^U], [1 - f_A^L, 1 - f_A^U]\rangle\}$ be a IVS in X. Then the interval valued vague interior and an interval valued vague closure are defined by

$IV\,\mathrm{int}(A) = \cup$ {G/G is an IVOS in X and G $\subseteq$ A}

$IVcl(A) = \cap$ {K/K is an IVCS in X and A $\subseteq$ K}

Note that for any IVS A in (X, τ), we have $IVcl(\overline{A}) = \overline{IV\,\mathrm{int}(A)}$ and $V\,\mathrm{int}(\overline{A}) = \overline{IVcl(A)}$.and $IVcl(A)$ is an IVCS and $IV\,\mathrm{int}(A)$ is an IVOS in X. Further we have, if A is an IVCS in X then IVcl(A)=A and if A is an IVOS in X then IVint(A)=A.

Definition 2.7:[6] An interval valued vague set A in an interval valued vague topological space (X, τ) is called an interval valued vague dense if there exists no interval valued vague closed set B in (X, τ) such that $A \subset B \subset 1$.

Definition 2.8:[6] An interval valued vague set A in an interval valued vague topological space (X, τ) is called an interval valued vague nowhere dense set if there exists no interval valued vague open set B in (X, τ) such that $B \subset IVcl(A)$. That is, $IV\,\mathrm{int}(IVcl(A)) = 0$.

Definition 2.9:[6] An interval valued vague topological space (X, τ) is called an interval valued vague first category set if $A = \bigcup_{i=1}^{\infty}(A_i)$, where A_i's are interval valued vague nowhere dense sets in (X, τ). Any other interval valued vague set in (X, τ) is said to be of interval valued vague second category.

Definition 2.10:[6] An interval valued vague set A in an interval valued vague topological space (X, τ) is called an interval valued vague G_δ-sets in (X, τ) if $A = \bigcap_{i=1}^{\infty}(A_i)$ where $A_i \in \tau$, for $i \in I$.

Definition 2.11:[6] An interval valued vague set A in an interval valued vague topological space (X,τ) is called an interval valued vague F_σ -sets in (X,τ) if $A=\bigcup_{i=1}^{\infty}(A_i)$ where $\overline{A_i}\in\tau$,for $i\in I.$

Definition 2.12:[6] An interval valued vague topological space (X,τ) is called an interval valued vague Volterra space if $IVcl(\bigcap_{i=1}^{N}A_i)=1$, where A_i's are interval valued vague dense and interval valued vague G_δ -sets in (X,τ).

Definition 2.13:[6] An interval valued vague topological space (X,τ) is called an interval valued vague weakly Volterra space if $IVcl(\bigcap_{i=1}^{N}A_i)\neq 0$, where A_i's are interval valued vague dense and interval valued vague G_δ -set in (X,τ).

Definition 2.14:[6] Let (X,τ) be an interval valued vague topological space. Then (X,τ) is called an interval valued vague baire space if $IV\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0$ where A_i's are interval valued vague nowhere dense sets in (X,τ).

Definition 2.15:[9] Let (X,τ) be an interval valued vague topological space. An interval valued vague set A in (X,τ) is called an interval valued vague σ - nowhere dense set if A is an interval valued vague F_σ set in (X,τ) such that $IV\operatorname{int}(A)=0$.

Definition 2.16:[9] Let (X,τ) be an interval valued vague topological spaces. Then (X,τ) is called an interval valued vague σ - Baire space if $IV\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0$, where $A_i's$ are interval valued vague σ - nowhere dense set (X,τ).

Definition 2.17:[9] An interval valued vague topological space (X,τ) is called an interval valued vague submaximal space if for each interval valued vague set A in (X,τ) such that $IVcl(A)=1$, then $A\in\tau$.

Definition 2.18:[9] Let (X,τ) be an interval valued vague topological space. An interval valued vague set A in (X,τ) is called interval valued vague σ - first category if $A=\bigcup_{i=1}^{\infty}A_i$ where A_i 's are interval valued vague σ - nowhere dense set in (X,τ). Any other interval valued vague set in (X,τ) is said to be interval valued vague σ - second category in (X,τ).

Definition 2.19:[9] An interval valued vague topological space (X,τ) is an interval valued vague σ - first category space if $1=\bigcup_{i=1}^{\infty}A_i$, where A_i 's are interval valued vague σ - nowhere dense set in (X,τ). (X,τ) is called an interval valued vague vague σ - second category space if it is not an interval valued vague σ - first category space.

Definition 2.20:[6] An interval valued vague topological space (X, τ) is said to be an interval valued vague open hereditarily irresolvable if $IV\operatorname{int}(IVcl(A))\neq 0$ then $IV\operatorname{int}(A)\neq 0$ for any interval valued vague set A in (X, τ).

Theorem 2.21:[6] Let (X, τ) be an interval valued vague topological space. If (X, τ) is an interval valued vague open hereditarily irresolvable then $IV\operatorname{int}(A)=0$ for any nonzero interval valued vague dense set A in (X, τ) implies that $IV\operatorname{int}(IVcl(A))=0.$

Theorem 2.22:[7] Let (X,τ) be an interval valued vague topological space. Then the following are equivalent
(i) (X,τ) is an interval valued vague Baire space.

(ii) $IV\operatorname{int}(A)=0$, for every interval valued vague first category set A in (X,τ).
(iii) $IVcl(B)=1$, for every interval valued vague residual set B in (X,τ).

Definition 2.23:[7] Let A be a vague first category set in an interval valued vague topological space (X,τ). Then $\bar{A}$ is called an interval valued vague residual sets in (X,τ).

Definition 2.24:[8] An interval valued vague topological space (X,τ) is called an interval valued vague p-space if countable intersection of interval valued vague open sets in (X,τ) is an interval valued vague open in (X,τ).

Interval valued vague almost resolvable and irresolvable space:

Definition 3.1: An interval valued vague topological space (X,τ) is an interval valued vague almost resolvable space if $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague set, A_i 's in (X,τ) are such that $IV\operatorname{int}(A_i)=0$. Otherwise, (X,τ) is called an interval valued vague almost irresolvable.

Theorem 3.2: If $\bigcap_{i=1}^{\infty}(A_i)=0$, where the interval valued vague sets A_i 's are interval valued vague dense sets in an interval valued vague topological space (X,τ), then (X,τ) is an interval valued vague almost resolvable space.

Proof: Suppose that $\bigcap_{i=1}^{\infty} A_i = 0$, where $IVcl(A_i)=1$ in (X,τ). Then we have $(\bigcap_{i=1}^{\infty} A_i)^c = 0^c = 1$, where $(IVcl(A_i))^c = 0$. This implies that $\bigcup_{i=1}^{\infty}(A_i^c)=1$, where $IV\operatorname{int}(A_i^c)=0$. Let $A_i^c = B_i$. Then, we have $\bigcup_{i=1}^{\infty} B_i = 1$, where $IV\operatorname{int}(B_i)=0$, in (X,τ). Hence (X,τ) is an interval valued vague almost resolvable space.

Theorem 3.3: If each interval valued vague set A_i is an interval valued vague F_σ-set in an interval valued almost resolvable space (X,τ), then $\bigcap_{i=1}^{\infty} A_i^c = 0$, where A_i^c 's are interval valued vague dense sets in (X,τ).

Proof: Let (X,τ) be an interval valued vague almost resolvable space. Then, $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i 's in (X,τ) are such that $IV\operatorname{int}(A_i)=0$. This implies that $\bigcap_{i=1}^{\infty}(A_i^c)=0$ and $IVcl(A_i^c)=1$. Since the interval valued vague sets A_i 's in (X,τ) are interval valued vague F_σ-sets, A_i^c 's are interval valued vague G_δ sets in (X,τ). Hence we have $\bigcap_{i=1}^{\infty}(A_i^c)=0$, where A_i^c 's are interval valued vague dense sets in (X,τ).

Theorem 3.4: If $\bigcap_{i=1}^{\infty} A_i = 0$, where the interval valued vague sets A_i 's are interval valued vague residual sets in an interval valued vague Baire space (X,τ), then (X,τ) is an interval valued vague almost resolvable space.

Proof: Let (X,τ) be an interval valued vague Baire space in which $\bigcap_{i=1}^{\infty} A_i = 0$, where the interval valued vague sets A_i 's are interval valued vague residual sets in (X,τ). Then by Theorem 2.22 , $IVcl(A_i)=1$. Then,

$(IVcl(A_i))^c = 0$ and hence $IV\operatorname{int}(A_i^c) = 0$. Now, $\bigcap_{i=1}^{\infty} A_i = 0$, implies that $\bigcup_{i=1}^{\infty}(A_i^c) = 1$. Let $A_i^c = B_i$. Hence $\bigcup_{i=1}^{\infty}(B_i) = 1$, where $IV\operatorname{int}(B_i) = 0$. Therefore (X,τ) is an interval valued vague almost resolvable space.

Theorem 3.5: If the interval valued vague topological space (X,τ) is an interval valued vague Baire space, then (X,τ) is an interval valued vague almost irresolvable space.

Proof: Suppose (X,τ) is an interval valued vague Baire space then $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0$, where A_i's are interval valued vague nowhere dense sets in (X,τ). This implies that $IV\operatorname{int}(IVcl(A_i)) = 0$. Suppose that (X,τ) is an interval valued vague almost resolvable space. Then $\bigcup_{i=1}^{\infty} A_i = 1$, where $IV\operatorname{int}(A_i) = 0$. Now $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = IV\operatorname{int}(1) = 1$, which implies that 0=1, a contradiction. Hence we must have $\bigcup_{i=1}^{\infty} A_i \neq 1$, where $IV\operatorname{int}(A_i) = 0$. Therefore (X,τ) is an interval valued vague almost irresolvable space.

Theorem 3.6: If $IVcl(\bigcap_{i=1}^{\infty} A_i) = 1$, where the interval valued vague sets A_i's are interval valued vague dense and interval valued vague G_δ- sets in an interval valued vague submaximal space in (X,τ), then (X,τ), is an interval valued vague almost irresolvable space.

Proof: Suppose that $IVcl(\bigcap_{i=1}^{\infty} A_i) = 1$, where the interval valued vague sets A_i's are interval valued vague dense and interval valued vague G_δ- sets in an interval valued vague submaximal space in (X,τ). Now $IVcl(\bigcap_{i=1}^{\infty} A_i) = 1$ implies that $IV\operatorname{int}(\bigcup_{i=1}^{\infty}(A_i^c) = 0$. Since A_i's are interval valued vague dense in an interval valued vague submaximal space, A_i's are interval valued vague open sets in (X,τ) and hence A_i^c 's are interval valued vague closed sets in (X,τ). Then $IVcl(A_i^c) = A_i^c$. Since A_i's are interval valued vague dense in (X,τ), $IVcl(A_i) = 1$. Then $(IVcl(A_i))^c = 0$ this implies that $IV\operatorname{int}(A_i^c) = 0$. Now $IV\operatorname{int}(IVcl(A_i^c)) = IV\operatorname{int}(A_i^c)$ implies that $IV\operatorname{int}(IVcl(A_i^c)) = 0$. Then A_i^c 's are interval valued vague nowhere dense sets in (X,τ). Hence we have, $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i^c) = 0$ where A_i^c 's are interval valued vague nowhere dense sets in (X,τ). Hence (X,τ) is an interval valued vague Baire apace and so by Theorem 3.4, (X,τ) is an interval valued vague almost irresolvable space.

Theorem 3.7: If each interval valued vague set is an interval valued vague F_σ- set in an interval valued vague almost resolvable space (X,τ), then (X,τ) is an interval valued vague σ-first category space.

Proof: Let (X,τ) be an interval valued vague almost resolvable space such that each interval valued vague set in (X,τ) is an interval valued vague F_σ- set. Then we have, $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's in (X,τ) are such that $IV\operatorname{int}(A_i) = 0$. Hence the interval valued vague sets A_i's in (X,τ) are interval valued vague F_σ- sets such that $IV\operatorname{int}(A_i) = 0$ $(i = 1\ to\ \infty)$. This implies that A_i's are interval valued vague σ-nowhere dense sets in (X,τ). Thus, we have $\bigcup_{i=1}^{\infty} A_i = 1$, where the A_i's are interval valued vague σ-nowhere dense sets in (X,τ). Therefore (X,τ) is an interval valued vague σ-first category space.

Theorem 3.8: If $IVcl(IV\text{int}(A)) = 1$, for each interval valued vague dense set A in an interval valued vague almost resolvable space (X, τ), then (X, τ) is an interval valued vague first category space

Proof: Let (X, τ) be an interval valued vague almost resolvable space such that $IVcl(IV\text{int}(A)) = 1$, for each interval valued vague dense set A in (X, τ). Since (X, τ) is an interval valued vague almost resolvable, $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's in (X, τ) are such that $IV\text{int}(A_i) = 0$. Now $(IV\text{int}(A_i))^c = 1$, implies that $IVcl(A_i^c) = 1$. Then, by hypothesis, $IVcl(IV\text{int}(A_i^c)) = 1$, for the interval valued vague dense set A_i^c in (X, τ). This implies that $IV\text{int}(IVcl(A_i)) = 0$. Hence A_i's are interval valued vague nowhere dense sets in (X, τ). Therefore $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's are interval valued vague nowhere dense sets in (X, τ), implies that (X, τ) is an interval valued vague first category space.

Theorem 3.9: If the interval valued vague topological space (X, τ) is an interval valued vague σ - second category space, then (X, τ) is an interval valued vague almost irresolvable space.

Proof: Let (X, τ) be an interval valued vague σ - second category space. Then, $\bigcup_{i=1}^{\infty} A_i \neq 1$, where the interval valued vague sets A_i's are interval valued vague σ - nowhere dense sets in (X, τ) .Therefore A_i's are interval valued vague F_σ - sets in (X, τ) such that $IV\text{int}(A_i) = 0$. Hence we have $\bigcup_{i=1}^{\infty} (A_i) \neq 1$, where $IV\text{int}(A_i) = 0$ implies (X, τ) is an interval valued vague almost irresolvable space.

Theorem 3.10: If $IVcl(IV\text{int}(A)) = 1$ for each interval valued vague dense set in an interval valued vague almost resolvable space (X, τ), then (X, τ) is not an interval valued vague Baire space.

Proof: Let (X, τ) be an interval valued vague almost resolvable space such that $IVcl(IV\text{int}(A)) = 1$, for each interval valued vague dense set A in (X, τ). Then by Theorem 3.7, (X, τ) is an interval valued vague first category space. This implies that $IV\text{int}(\bigcup_{i=1}^{\infty} A_i) = IV\text{int}(1) = 1 \neq 0.$ Hence (X, τ) is not an interval valued vague Baire space.

Theorem 3.11: If $\bigcup_{i=1}^{\infty} (A_i) = 1$, where the interval valued vague sets A_i's are non-zero interval valued vague open sets in an interval valued vague topological space (X, τ), then (X, τ) is an interval valued vague almost irresolvable space.

Proof: Suppose that $\bigcup_{i=1}^{\infty} (A_i) = 1$, where the interval valued vague sets A_i's are non-zero interval valued vague open sets in (X, τ), $IV\text{int}(A_i) = A_i \neq 0$. Therefore (X, τ) is an interval valued vague almost irresolvable space.

Definition 3.12: An interval valued vague topological space (X, τ) is called an interval valued vague hyper connected space if every interval valued vague open set is an interval valued vague dense in (X, τ).

Theorem 3.13: If $\bigcap_{i=1}^{\infty} (A_i) = 0$, where the interval valued vague sets A_i's are interval valued vague open sets in an interval valued vague hyper connected space in (X, τ), then (X, τ) is an interval valued vague almost resolvable space.

Proof: Suppose that $\bigcap_{i=1}^{\infty} A_i = 0$, where $A_i \in \tau$ and A_i's are interval valued vague open sets in an interval valued vague hyper connected space (X, τ) then A_i are interval valued vague dense set in (X, τ) for each i. Then by Theorem 3.2, (X, τ) is an interval valued vague almost resolvable space.

Theorem 3.14: If $\bigcup_{i=1}^{\infty} B_i = 1$, where the interval valued vague sets B_i's are interval valued vague σ-nowhere dense sets in an interval valued vague topological space (X, τ), then (X, τ) is an interval valued vague almost resolvable space.

Proof: Let B_i's be an interval valued vague σ-nowhere dense sets in (X, τ). Then B_i's are interval valued vague F_σ-sets with $IV\text{int}(B_i) = 0$. Now $\bigcup_{i=1}^{\infty} B_i = 1$, where $IV\text{int}(B_i) = 0$, implies that (X, τ) is an interval valued vague almost resolvable space.

Theorem 3.15: If $\bigcap_{i=1}^{\infty} (A_i) = 0$, where the interval valued vague sets A_i's are interval valued vague G_δ sets in an interval valued vague hyper connected and interval valued vague P-space, then (X, τ) is an interval valued vague almost resolvable space.

Proof: Let A_i's be an interval valued vague G_δ sets in an interval valued vague P- space in (X, τ). Then A_i's are interval valued vague open sets in (X, τ). Hence, we have $\bigcap_{i=1}^{\infty} (A_i) = 0$, where the interval valued vague sets A_i's are interval valued vague open sets in an interval valued vague hyper connected space (X, τ). Therefore, by Theorem 3.12, (X, τ) is an interval valued vague almost resolvable space.

Definition 3.16: An interval valued vague topological space (X, τ) is called an interval valued vague nodec space, if every non-zero interval valued vague nowhere dense set in (X, τ), is an interval valued vague closed set in (X, τ).

Theorem 3.17: If (X, τ) is an interval valued vague first category space and interval valued vague nodec space, then (X, τ) is an interval valued vague almost resolvable space.

Proof: Let (X, τ) be an interval valued vague first category space. Then $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's are interval valued vague nowhere dense sets in (X, τ). Since (X, τ) is an interval valued vague nodec space, That is, $IVcl(A_i) = A_i$. Now $IV\text{int}(IVcl(A_i)) = 0$, implies that $IV\text{int}(A_i) = 0$. Hence (X, τ) is an interval valued vague almost resolvable space.

Theorem 3.18: If the interval valued vague topological space (X, τ) is an interval valued vague second category space, then (X, τ) is an interval valued vague almost irresolvable space.

Proof: Let (X, τ) be an interval valued second category space. Then, $\bigcup_{i=1}^{\infty} A_i \neq 1$, where the interval valued vague sets A_i's are interval valued vague nowhere dense sets in (X, τ). That is, $\bigcup_{i=1}^{\infty} A_i \neq 1$, where $IV\text{int}(IVcl(A_i)) = 0$. Now $IV\text{int}(A_i) \subseteq IV\text{int}(IVcl(A_i))$, implies that $IV\text{int}(A_i) = 0$. Hence $\bigcup_{i=1}^{\infty} A_i \neq 1$, where $IV\text{int}(A_i) = 0$ and therefore (X, τ) is an interval valued vague almost irresolvable space.

Theorem 3.19: If the interval valued vague almost resolvable space (X, τ) is an interval valued vague submaximal space, then (X, τ) is an interval valued vague first category space.

Proof: Let (X, τ) be an interval valued vague almost resolvable space. Then $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's in (X, τ) are such that $IV\text{int}(A_i) = 0$. Then we have $\bigcap_{i=1}^{\infty} (A_i^c) = 0$, where $IVcl(A_i^c) = 1$. Since the space (X, τ) is an interval valued vague submaximal space, the interval valued vague dense sets A_i^c's are interval valued vague open sets in (X, τ). Then A_i's are interval valued vague closed sets in (X, τ) and hence $IVcl(A_i) = A_i$. Now $IV\operatorname{int}(IVcl(A_i)) = IV\operatorname{int}(A_i) = 0$. Then A_i's are interval valued vague nowhere dense sets in (X, τ). Hence $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's are interval valued vague nowhere dense sets in (X, τ) implies that (X, τ) is an interval valued vague first category space.

Theorem 3.20: If the interval valued vague almost irresolvable space (X, τ) is an interval valued vague submaximal space, then (X, τ) is an interval valued vague second category space.

Proof: Let (X, τ) be an interval valued vague almost irresolvable space. Then $\bigcup_{i=1}^{\infty} A_i \neq 1$, where the interval valued vague sets A_i's are such that $IV\operatorname{int}(A_i) = 0$. Now $IV\operatorname{int}(A_i) = 0$, implies that $IVcl(A_i^C) = 1$. That is, A_i^C 's are interval valued vague dense sets in (X, τ). Since (X, τ) is an interval valued vague submaximal space, the interval valued vague dense sets A_i^C 's are interval valued vague open sets in (X, τ). Then A_i's are interval valued vague closed sets in (X, τ). That is, $IVcl(A_i) = A_i$. Now $IV\operatorname{int}(A_i) = 0$, implies that $IV\operatorname{int}(IVcl(A_i)) = 0$. Then A_i's are interval valued vague nowhere dense sets in (X, τ). Hence we have $\bigcup_{i=1}^{\infty} A_i \neq 1$, where the interval valued vague sets A_i's are interval valued vague nowhere dense sets in (X, τ). Therefore (X, τ) is an interval valued vague second category space.

Theorem 3.21: If the interval valued vague almost resolvable space (X, τ) is an interval valued vague submaximal space, then (X, τ) is not an interval valued vague Baire space.
Proof: Let the interval valued vague almost resolvable space (X, τ) be an interval valued vague submaximal space. Then by Theorem 3.19 , (X, τ) is an interval valued vague first category space and hence $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's are interval valued vague nowhere dense sets in (X, τ). Now $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = IV\operatorname{int}(1) = 1 \neq 0$. Hence (X, τ) is not an interval valued vague Baire space.

Theorem 3.22: If the interval valued vague almost resolvable space (X, τ) is an interval valued vague open hereditarily irresolvable space, then (X, τ) is not an interval valued vague Baire space.

Proof: Let (X, τ) be an interval valued vague almost resolvable space. Then $\bigcup_{i=1}^{\infty} A_i = 1$, where the interval valued vague sets A_i's in (X, τ) are such that $IV\operatorname{int}(A_i) = 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable space, $IV\operatorname{int}(A_i) = 0$ implies that $IV\operatorname{int}(IVcl(A_i) = 0$. Now $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = IV\operatorname{int}(1) = 1 \neq 0$. Hence (X, τ) is not an interval valued vague Baire space.

Theorem 3.23: If the interval valued vague almost irresolvable space (X, τ) is an interval valued vague open hereditarily irresolvable space, then (X, τ) is an interval valued vague second category space.

Proof: Let (X, τ) be an interval valued vague almost irresolvable space. Then $\bigcup_{i=1}^{\infty} A_i \neq 1$, where the interval valued vague sets A_i's in (X, τ) are such that $IV\operatorname{int}(A_i) = 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable space, $IV\operatorname{int}(A_i) = 0$ implies that $IV\operatorname{int}(IVcl(A_i) = 0$. Then A_i's are interval valued vague nowhere dense sets in (X, τ). Hence $\bigcup_{i=1}^{\infty} A_i \neq 1$, where the interval valued vague sets A_i's are interval valued vague nowhere dense sets in (X, τ), implies that (X, τ) is an interval valued vague second category space.

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