

Lagrange interpolation polynomial and an application in agriculture

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Abstract: In this study, the interpolation method was studied theoretically and applied on field crops in agriculture. Sown area and production amount data of 2013-2017 period are used. Individual interpolation polynomials were formed for wheat, barley, corn, rye and oat plants. Generated polynomials are fourth order Lagrange polynomials. According to these polynomials, the production amount of wheat which cultivated interval 1950000-2450000 is between 573388-784572 ton. Likewise; 359802-605602 maize production amount corresponding to the range of 350000-600000 decares of corn. The amount of production of barley which cultivated at 2350000-2750000 decares is 711145-736948. The amount of production of dye which cultivated at 55000-80000 decares is 18015-22753. The amount of production of oats which cultivated at 46000-80000 decares is 9468-16789. The interpolation method has been a good model for forecasting production in agriculture.

Keywords: Interpolation, agriculture, production.

I. Introduction

Interpolation is a mathematical function that estimates the values at locations where no measured values are available [1]. Basic properties of the interpolation polynomial are the existence and the uniqueness of the interpolation polynomials [2].

The interpolation problem has extensive applications in different areas of science and engineering, so different procedures have been developed in this field [3].

Interpolation techniques are based on the principles of spatial autocorrelation, which supports that closer points are more alike compared to farther ones. The literature reveals a great deal of interpolation methods which are generally classified as local and global approaches [4].

Nowadays, Interpolation method is used, whether in agricultural production or defense advanced science and technology research, such as large and medium-sized electromechanical product optimization design, major project design, etc. Through measuring or experiment can only get finite discrete points in the interval of function values in (x, y), it's only a function table [5].

In this study, it is aimed to model the production amount of selected grains according to the area to be sown using Lagrange interpolations.

II. Material

The research material consists of 2013-2017 period is data taken from the www.gov.tr website of Turkey Statistics Institute (TSI) [6]. This data includes the data under the heading "Cereals and other crop products" in the "Plant Production Statistics" section.

Konya province in Turkey of wheat, maize, barley, rye and oats cultivated hectare and production quantity (ton) is given in Table 1.

Table 1. Cereals sowing area and production amount according to years

Years	Wheat		Maize		Barley		Rye		Oats	
	Sown	Produc.	Sown	Produc.	Sown	Produc.	Sown	Produc.	Sown	Produc.
2013	1860804	601691	341310	353552	2306009	761080	63631	20570	51022	12035
2014	2013178	553341	381002	382099	2403777	650101	56706	16991	45859	9267
2015	2330549	857930	548861	558190	2416454	853627	53288	19042	48068	12418
2016	2482681	766978	635269	638300	2766418	734012	81679	22779	49633	11275
2017	2468895	772968	637972	621884	2422416	795535	83840	28561	74290	16849

Produc.: Production

III. Method

It may be more meaningful to use the Lagrange interpolation as a rather than curve fitting operation from an interpolation process. A line or curve is fitted to the existing points. Then the values of the desired points are calculated on this equation. In this method, the polynomial grade changes depending on the number of points. For example, the polynomial grade to be fitted for n points is $n - 1$.

The problem of determining a polynomial of degree one that passes through the distinct points (x_0, y_0) and (x_1, y_1) is the same as approximating a function f for which $f(x_0)=y_0$ and $f(x_1)=y_1$ by means of a first-degree polynomial interpolating, or agreeing with, the values of f at the given points. Using this polynomial for approximation within the interval given by the endpoints is called polynomial interpolation [7].

The Lagrange interpolating polynomial is the polynomial $P(x)$ of degree less than $(n-1)$ that passes through the n points $(x_1, y_1 = f(x_1)), (x_2, y_2 = f(x_2)), \dots, (x_n, y_n = f(x_n))$, and is given by

$$f(x) = \sum_{j=1}^n f_j(x)$$

Where

$$f_j(x) = y_j \prod_{\substack{k=1 \\ k \neq j}}^n \frac{x - x_k}{x_j - x_k}$$

Written explicitly,

$$f(x) = \frac{(x - x_2)(x - x_3) \dots (x - x_n)}{(x_1 - x_2)(x_1 - x_3) \dots (x_1 - x_n)} y_1$$

$$+ \frac{(x - x_1)(x - x_3) \dots (x - x_n)}{(x_2 - x_1)(x_2 - x_3) \dots (x_2 - x_n)} y_2$$

⋮

$$+ \frac{(x - x_1)(x - x_2) \dots (x - x_{n-1})}{(x_n - x_1)(x_n - x_2) \dots (x_n - x_{n-1})} y_n$$

The formula was first published by Waring (1779) [8], rediscovered by Euler in 1783, and published by Lagrange in 1795 [9]. If the X_i data points are not equal discontinuous, Lagrange interpolations can be created at different ratios [10].

IV. Results

The production amount of cereals produced in Konya province was estimated by creating the interpolation polynomial according to the donation.

Wheat

Lagrange polynomial interpolation has been created for durum wheat varieties in the province of Konya in Turkey the amount of production (ton) according to per hectare between the years 2013-2017. Interpretations were estimated according to the obtained polynomial. The production quantity is estimated for the area sown values in the stated range. The program written in the MATLAB program is presented in Table 2.

Table 2. Wheat production MATLAB commands according to area sown.

```
x=[1860804 2013178 2330549 2482681 2468895];
y=[601691 553341 857930 766978 772968];
x1=1860804;x2=2013178;x3=2330549; x4=2482681;x5=2468895;
y1=601691;y2=553341;y3=857930;y4=766978;y5=772968;
k1=[1 -x1];k2=[1 -x2];k3=[1 -x3];k4=[1 -x4];k5=[1 -x5];
m11=conv(k2,k3);m12=conv(m11,k4);m13=conv(m12,k5);
payda1=(x1-x2)*(x1-x3)*(x1-x4)*(x1-x5);
fonk1=m13*y1/payda1
m21=conv(k1,k3);m22=conv(m21,k4);m23=conv(m22,k5);
```

```

payda2=(x2-x1)*(x2-x3)*(x2-x4)*(x2-x5);
fonk2=m23*y2/payda2
m31=conv(k1,k2);m32=conv(m31,k4);m33=conv(m32,k5);
payda3=(x3-x1)*(x3-x2)*(x3-x4)*(x3-x5);
fonk3=m33*y3/payda3
m41=conv(k1,k2);m42=conv(m41,k3);m43=conv(m42,k5);
payda4=(x4-x1)*(x4-x2)*(x4-x3)*(x4-x5);
fonk4=m43*y4/payda4
m51=conv(k1,k2);m52=conv(m51,k3);m53=conv(m52,k4);
payda5=(x5-x1)*(x5-x2)*(x5-x3)*(x5-x4);
fonk5=m53*y5/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
format long
poly2str(t,'x')
xg=1950000:50000:2450000
interp1(x,y,xg,'lagrange')
    
```

The Lagrange interpolation which consists of the coefficients of the function determined as t is as follows.

$$f(x) = 3.2194e - 17 x^4 - 2.8938e - 10 x^3 + 0.00097012 x^2 - 1436.9576 x + 793926608.77$$

According to this polynomial, the estimate of wheat production for the reservoir from 1950000 to 2450000 is presented in Table 3.

Table 3. Estimated amount of wheat production

Area sown	Production (Tonnes)
1950000	573388
2000000	557523
2050000	588680
2100000	636666
2150000	684653
2200000	732639
2250000	780625
2300000	828611
2350000	845985
2400000	815278
2450000	784572

According to the results obtained in Table 3, the average average wheat yield per card was 321.98 kg / day.

Maize

Lagrange interpolating polynomial was formed (tons) of the amount of production according to the decanter of corn plant. Interpretations have been estimated according to this polynomial. Estimation of the corresponding production amount in the given range was estimated in the MATLAB program (Table 4).

Table 4. MATLAB program that calculates the amount of maize production

```

x=[341310 381002 548861 635269 637972];
y=[353552 382099 558190 638300 621884];
x1=341310;x2=381002;x3=548861; x4=635269;x5=637972;
y1=353552;y2=382099;y3=558190;y4=638300;y5=621884;
k1=[1 -x1];k2=[1 -x2];k3=[1 -x3];k4=[1 -x4];k5=[1 -x5];
m11=conv(k2,k3);m12=conv(m11,k4);m13=conv(m12,k5);
payda1=(x1-x2)*(x1-x3)*(x1-x4)*(x1-x5);
fonk1=m13*y1/payda1
m21=conv(k1,k3);m22=conv(m21,k4);m23=conv(m22,k5);
    
```

```

payda2=(x2-x1)*(x2-x3)*(x2-x4)*(x2-x5);
fonk2=m23*y2/payda2
m31=conv(k1,k2);m32=conv(m31,k4);m33=conv(m32,k5);
payda3=(x3-x1)*(x3-x2)*(x3-x4)*(x3-x5);
fonk3=m33*y3/payda3
m41=conv(k1,k2);m42=conv(m41,k3);m43=conv(m42,k5);
payda4=(x4-x1)*(x4-x2)*(x4-x3)*(x4-x5);
fonk4=m43*y4/payda4
m51=conv(k1,k2);m52=conv(m51,k3);m53=conv(m52,k4);
payda5=(x5-x1)*(x5-x2)*(x5-x3)*(x5-x4);
fonk5=m53*y5/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
poly2str(t,'x')
xg=350000:25000:600000
interp1(x,y,xg,'lagrange')
    
```

Here the Lagrange interpolation can be written as

$$f(x) = -1.0005e - 15 x^4 + 1.903e - 09 x^3 - 0.0013241 x^2 + 401.8973 x - 44546355.4122$$

According to this polynomial, the maize production estimate is calculated according to the reservoir between 350000 and 600000 and it is presented in Table 5.

Table 5. Estimated amount of maize production according to the area sown

Area sown	Production amount (tons)
350000	359802
375000	377782
400000	402029
425000	428255
450000	454481
475000	480707
500000	506933
525000	533159
550000	559246
575000	582424
600000	605602

According to the results obtained in Table 5, the average corn yield per decare was 1012.60 kg / da.

Barley

Lagrange interpolation polynomials were generated (tons) of the amount of production of the barley vegetation according to the area sown (Table 6). The corresponding production amount in the given range was estimated in the MATLAB program (Table 7).

Table 6. User-defined functions source code of Lagrange interpolation Matlab 13 (for barley)

```

x=[2306009 2403777 2416454 2766418 2422416];
y=[761080 650101 853627 734012 795535];
x1=2306009;x2=2403777;x3=2416454; x4=2766418;x5=2422416;
y1=761080;y2=650101;y3=853627;y4=734012;y5=795535;
k1=[1 -x1];k2=[1 -x2];k3=[1 -x3];k4=[1 -x4];k5=[1 -x5];
m11=conv(k2,k3);m12=conv(m11,k4);m13=conv(m12,k5);
payda1=(x1-x2)*(x1-x3)*(x1-x4)*(x1-x5);
fonk1=m13*y1/payda1
m21=conv(k1,k3);m22=conv(m21,k4);m23=conv(m22,k5);
payda2=(x2-x1)*(x2-x3)*(x2-x4)*(x2-x5);
fonk2=m23*y2/payda2
m31=conv(k1,k3);m22=conv(m21,k4);m23=conv(m22,k5);
    
```

```

payda3=(x3-x1)*(x3-x2)*(x3-x4)*(x3-x5);
fonk3=m33*y3/payda3
m41=conv(k1,k2);m42=conv(m41,k3);m43=conv(m42,k5);
payda4=(x4-x1)*(x4-x2)*(x4-x3)*(x4-x5);
fonk4=m43*y4/payda4
m51=conv(k1,k2);m52=conv(m51,k3);m53=conv(m52,k4);
payda5=(x5-x1)*(x5-x2)*(x5-x3)*(x5-x4);
fonk5=m53*y5/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
poly2str(t,'x')
xg=2350000:50000:2750000
interp1(x,y,xg,'lagrange')
    
```

The obtained Lagrange interpolation equation

$$f(x) = 3.718e - 14 x^4 - 3.683e - 07 x^3 + 1.3656 x^2 - 2247058.379 x + 1384545405346.58$$

it shaped.

Table 7. Estimation of the amount of barley production according to the area sown

Ekilen dekar	Üretim miktarı (ton)
2350000	711145
2400000	654388
2450000	790602
2500000	781659
2550000	772717
2600000	763775
2650000	754833
2700000	745890
2750000	736948

According to the results obtained in Table 7, the average barley yield per carat was 292.94 kg / da.

Rye

Lagrange interpolating polynomials was established of the amount of production (tons) of the rye plant according to area sown (Table 8). The corresponding production amount in the given range was estimated in the MATLAB program (Table 9).

Tablo 8. User-defined functions source code of Lagrange interpolation Matlab13 (for rye)

```

x=[63631 56706 53288 81679 83840];
y=[20570 16991 19042 22779 28561];
x1=63631;x2=56706;x3=53288; x4=81679;x5=83840;
y1=20570;y2=16991;y3=19042;y4=22779;y5=28561;
k1=[1 -x1];k2=[1 -x2];k3=[1 -x3];k4=[1 -x4];k5=[1 -x5];
m11=conv(k2,k3);m12=conv(m11,k4);m13=conv(m12,k5);
payda1=(x1-x2)*(x1-x3)*(x1-x4)*(x1-x5);
fonk1=m13*y1/payda1
m21=conv(k1,k3);m22=conv(m21,k4);m23=conv(m22,k5);
payda2=(x2-x1)*(x2-x3)*(x2-x4)*(x2-x5);
fonk2=m23*y2/payda2
m31=conv(k1,k3);m32=conv(m21,k4);m33=conv(m22,k5);
payda3=(x3-x1)*(x3-x2)*(x3-x4)*(x3-x5);
fonk3=m33*y3/payda3
m41=conv(k1,k2);m42=conv(m41,k3);m43=conv(m42,k5);
payda4=(x4-x1)*(x4-x2)*(x4-x3)*(x4-x5);
fonk4=m43*y4/payda4
m51=conv(k1,k2);m52=conv(m51,k3);m53=conv(m52,k4);
    
```

```
payda5=(x5-x1)*(x5-x2)*(x5-x3)*(x5-x4);
fonk5=m53*y5/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
poly2str(t,'x')
xg=55000:5000:83000
interp1(x,y,xg,'lagrange')
```

The obtained Lagrange interpolation equation

$$f(x) = 3.14158e - 13 x^4 - 8.2442e - 08 x^3 + 0.00798 x^2 - 336.812 x + 5224527.211$$

it shaped.

Table 9. Estimation of the amount of rye production according to the area sown

Area sown	Production amount (tons)
55000	18015
60000	18693
65000	20738
70000	21350
75000	21962
80000	22573

According to the results reached in Table 9, the average rye yield was 306.36 kg / day.

Oats

Lagrange interpolating polynomials was established of the amount of production (tons) of the oats plant according to area sown (Table 10). The oats production amount in the given range was estimated in the MATLAB program (Table 11).

Table 10. User-defined functions source code of Lagrange interpolation Matlab13 (for oats)

```
x=[51022 45859 48068 49633 74290];
y=[12035 9267 12418 11275 16849];
x(1);x(2);x(3);x(4);x(5);
y(1);y(2);y(3);y(4);y(5);
k1=[1 -x(1)];k2=[1 -x(2)];k3=[1 -x(3)];k4=[1 -x(4)];k5=[1 -x(5)];
m1=conv(conv([1 -x(2)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda1=(x(1)-x(2))*(x(1)-x(3))*(x(1)-x(4))*(x(1)-x(5));
fonk1=m1*y(1)/payda1
m2=conv(conv([1 -x(1)],[1 -x(3)]),conv([1 -x(4)],[1 -x(5)]))
payda2=(x(2)-x(1))*(x(2)-x(3))*(x(2)-x(4))*(x(2)-x(5));
fonk2=m2*y(2)/payda2
m3=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(4)],[1 -x(5)]))
payda3=(x(3)-x(1))*(x(3)-x(2))*(x(3)-x(4))*(x(3)-x(5));
fonk3=m3*y(3)/payda3
m4=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(5)]))
payda4=(x(4)-x(1))*(x(4)-x(2))*(x(4)-x(3))*(x(4)-x(5));
fonk4=m4*y(4)/payda4
m5=conv(conv([1 -x(1)],[1 -x(2)]),conv([1 -x(3)],[1 -x(4)]))
payda5=(x(5)-x(1))*(x(5)-x(2))*(x(5)-x(3))*(x(5)-x(4));
fonk5=m5*y(5)/payda5
t=fonk1+fonk2+fonk3+fonk4+fonk5
xg=46000:1000:74000;
yg=interp1(x,y,xg,'lagrange')
plot(xg,yg,'*')
format long
poly2str(t,'x')
```

According to these results Lagrange interpolation equation

$$f(x) = -7.438e - 12 x^4 + 1.641e - 06 x^3 - 0.13404 x^2 + 4809.7877 x - 64111377.455$$

it shaped.

Table 11. Estimation of the amount of oats production according to the area sown

Area sown	Production amount (tons)
46000	9468
50000	11476
54000	12651
58000	13479
62000	14306
66000	15134
70000	15961
74000	16789

According to the results obtained in Table 11, the average oat yield was 227.12 kg / day.

V. Conclusion

In the study, wheat, barley, rye, maize and oats production amount was estimated by area sown per hectare in Turkey in Konya. A fourth-order Lagrange interpolation was established and the amount of produced cereals according to the sown area in which they were harvested. Lagrange interpolation has been found to be a very good method for predicting production in agriculture.

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