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Equivalence Resolving Partition of a Graph

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Abstract: Started with the concept of locating sets and locating dimension ([17], [18]), the terms where rechristened as resolving sets and metric dimension [8]. Resolving partition and partition dimension were introduced in [11]. Varieties of resolving partition were studied, one among them being resolving independent partition. A set S of vertices in a graph G is an equivalence set if the components of <S> are complete ([1], [2], [3], [4]). Equivalence set is a generalization of independent set. A partition of the vertex set V(G) of a graph G into equivalence sets is called an equivalence partition. Trivial partition of V(G) into singleton sets is an equivalence partition. An equivalence partition II = $\{V_1, V_2, \ldots, V_K\}$ in a connected graph G is resolving if the code $c_\Pi(u) = (d(u, V_1), d(u, V_2),.., d(u, V_k))$ of a vertex u of G is difference for different u. The trivial partition is an equivalence resolving partition. The minimum cardinality of an equivalence resolving partition of a graph G is called the equivalence partition dimension of G and is denoted by $pd_{eq}(G)$. The equivalence partition dimension of some well-known graphs is determined and characterization of connected graphs of order n having equivalence partition dimension n, 2, n – 1 are established. We present bounds for the equivalence partition dimension of a graph in terms of other graphical parameters.
Keywords: Resolving set, partition dimension, equivalence set, equivalence resolving partition.

1. Introduction

Definition 1.1. [8] Let G be a simple, finite and undirected graph. A subset S of V(G) is called a resolving set of G if for any vertex u in V(G), $c_S(u) = (d(u, x_1), d(u, x_2),\ldots, d(u, x_k))$ are distinct for distinct vertices u where $S = \{x_1, x_2, \ldots, x_k\}$. V(G) is a resolving set of G. Thus, existence of a resolving set is guaranteed in any graph. The minimum cardinality of a resolving set of G is called the metric dimension of G and is denoted by dim (G).

Definition 1.2. [11] A partition $\Pi = \{V_1, V_2, \ldots, V_k\}$ of V(G) is called a resolving partition of G if the code $c_\Pi(u) = (d(u, V_1), d(u, V_2),\ldots, d(u, V_k))$ is different for different $u \in V(G)$ where $d(u, V_i) = \min \{d(u, x) / x \in Vi\}$. The minimum cardinality of a resolving partition of a graph G is called the partition dimension of G and is denoted by pd(G).

Remark 1.3. The property of a resolving set is super hereditary.

Remark 1.4. $\dim(K_n) = n – 1, n \geq 2$ and $\dim(K_{1,n}) = n – 1, n \geq 2$.

Definition 1.5. [1] A subset S of the vertex set V(G) of a graph G is said to be an equivalence set of G if the components of < S > are complete. A graph G is called an equivalence graph if all the components of G are complete.

Example 1.6. A complete graph, a totally disconnected graph are some examples of an equivalence graph.

Definition 1.7. A resolving set which is also an equivalence set is called an equivalence resolving set. The minimum cardinality of an equivalence resolving set of G is called the equivalence dimension of G and is denoted by $\dim_{eq}(G)$.

Remark 1.8. If a graph G has an independent resolving set, then it has an equivalence resolving set.

Definition 1.9. ([6], [7]) A partition $\Pi = \{V_1, V_2, \ldots,V_k\}$ of V(G) is called an independent resolving partition of G if each V_i , $1 \leq i \leq k$ is independent and Π is a resolving partition of G. The minimum cardinality of an independent resolving partition of a graph G is called the independent partition dimension of G and is denoted by ipd(G).

Definition 1.10. Let $\Pi = \{V_1, V_2, \ldots, V_k\}$ be a partition of V(G). Π is called an equivalence resolving partition of G if each V_i is an equivalence set and Π is a resolving partition. Let $V(G) = \{u_1, u_2, \ldots u_n\}$. Then $\Pi = \{\{u_1\}, \{u_2\}, \ldots, \{u_n\}\}$ is an equivalence resolving partition of G. Hence the existence of an equivalence resolving partition is guaranteed in any graph. The minimum cardinality of an equivalence resolving partition of a graph G is called the equivalence partition dimension of G and is denoted by $pd_{eq}(G)$.

Theorem 1.11. Let G be a non-trivial connected graph. Then $pd_{eq}(G) \leq s + t$ where dim(G) = s and t is the minimum cardinality of partition of V – S into an equivalence sets for different S.

Proof. Let dim(G) = k. Let $S = \{u_1, u_2, \ldots, u_k\}$ be a basis of G. Let $S_i = \{u_i\}$, $1 \leq i \leq k$. Let T = V(G) – S. Consider < T >. Let $T_1, T_2, \ldots, T_r$ be a minimum partition of T into an equivalence sets. Let $\Pi = \{\{u_1\}, \{u_2\}, \ldots \{u_k\}, T_1, T_2, \ldots, T_r\}$. Clearly, Π is an equivalence partition of V(G).
Let $x, y \in V(G)$. Suppose $x, y \in T_i$, $1 \leq i \leq r$. Clearly, $x, y \in V(G) - S$. Therefore, $d(x, u_j) \neq d(y, u_j)$ for some $u_j \in S$ (since S is a resolving set of G). Therefore, x, y are resolved by $\{u_j\}$ when $x \in T_i$ and $y \in T_j$, $i \neq j$ then $d(x,T_i) = 0$ and $d(x, T_j) > 0$. $d(y, T_i) > 0$ and $d(y, T_j) = 0$. Therefore x and y are resolved by Π. Therefore, Π is an equivalence resolving partition of G.
Let t be the smallest cardinality of partitions of V – S into an equivalence sets for different basis S. Then, $pd_{eq}(G) \leq |\Pi|$. Let Π_1 be the partition corresponding to a basis S such that the number of elements in the partition of V – S into an equivalence sets is the minimum t. Then $|\Pi| = s + t$. Therefore $pd_{eq}(G) \leq |\Pi| = s + t$.

Remark 1.12. $pd_{eq}(K_n) = n$. That is, $\Pi = \{\{u_1\}, \{u_2\}, \ldots, \{u_n\}\}$ is a minimum partition dimension of K_n. Π is also an equivalence partition. Therefore, $pd_{eq}(K_n) = n = \dim(K_n) + 1$. Here $s = \dim(K_n) = n - 1$ and t = 1. Therefore, $pd_{eq}(K_n) = s + t$. Hence the bound in the above theorem is sharp.

2. $Pd_{eq}(G)$ For Well Known Graphs

Theorem 2.1. $pd_{eq}(P_n) = \begin{cases} 2 & \text{when } n = 2,3,4 \\ 3 & \text{when } n \geq 5 \end{cases}$

Proof. Let $V(P_n) = \{u_1, u_2, \ldots, u_n\}$.
Case (i): n is even, say n = 2k
Let $\Pi = \{\{u_1\}, \{u_2, u_4, u_6, \ldots, u_n\}, \{u_3, u_5, u_7, \ldots, u_{n-1}\}\}$. Clearly Π is an equivalence resolving partition.
Case (ii): n is odd, say n = 2k + 1
Let $\Pi_1 = \{\{u_1\}, \{u_2, u_4, \ldots, u_{n-1}\}, \{u_3, u_5, u_7, \ldots, u_n\}\}$. Clearly Π_1 is an equivalence resolving partition. Therefore, $pd_{eq}(P_n) \leq 3$ when n = 2, 3, 4. Therefore, $pd_{eq}(P_n) = 2$.
Let $n \geq 5$. Suppose $pd_{eq}(P_n) = 2$. Let $\Pi = \{V_1, V_2\}$ be an equivalence resolving partition. Let $u_1 \in V_1$.
Let $x, y \in V_1$ such that $d(x, y) \geq 3$ with respect to V_1. Then x, y will have 1 in the second place of the code with respect to Π. Therefore, $d(x, y) \leq 2$ with respect to V_1.
Suppose d(x, y) = 2. Then also x and y have 1 in the second place of the code with respect to Π.
Suppose d(x, y) = 1 and let $x = u_i$, $y = u_{i+1}$. Then $u_{i-1}, u_{i+2} \in V_2$ provided $i \geq 2$. In this case, u_i and u_{i+1} have 1 in the second place of the code with respect to Π.
Suppose i = 1. Then $u_1, u_2 \in V_1$. Since $n \geq 5$, there exists at least one $u_i \in V_1$ such that $i \geq 4$. Then $d(u_2, u_i) \geq 2$.
In this case, u_2 and u_i will have 1 in the second place of the code with respect to Π. Therefore, $pd_{eq}(P_n) \geq 3$, when $n \geq 5$. Therefore, $pd_{eq}(P_n) = 3$, for $n \geq 5$.

Theorem 2.2. $pd_{eq}(C_n) = 3$, for $n \geq 3$.

Proof. Case (i): Let n be even and n = 2k.
Let $V(C_n) = \{u_1, u_2, \ldots, u_{2k}\}$. Let $\Pi = \{\{u_1\}, \{u_2, u_{2k-1}, u_4, u_{2k-3}, \ldots, u_k, u_{2k-(k-3)}\}, \{u_3, u_{2k}, u_5, u_{2k-2}, \ldots, u_{k+1}, u_{k+2}\}\}$ where the second element of the partition contains (k – 1) elements and the third element contains k-elements. In the second element, the even suffixes occur one more than the odd suffixes. Π is an equivalence resolving partition. Therefore, $pd_{eq}(C_n) \leq 3$ where n is a multiple of 4.
Case (ii): Let n be even and n = 2k, where k is odd.

Let $V(C_n) = \{u_1, u_2,\ldots, u_{2k}\}$. Let $\Pi = \{\{u_1\}, \{u_2, u_{2k-1}, u_4, u_{2k-3}, u_6, u_{2k-5},\ldots,u_k, u_{2k+1}\}, \{u_3,u_{2k}, u_5, u_{2k-2},\ldots, u_k, u_{k+3}\}\}$ where the second element of the partition contains k elements and the third element contains (k – 1) elements. In the second element, the even suffixes occur one more than the odd suffixes. Π is an equivalence resolving partition. Therefore, $pd_{eq}(C_n) \leq 3$ where n is a multiple of 2.

Case (iii): Let n be odd. Let $n = 2k + 1$.

Let $\Pi = \{\{u_1\}, \{u_2, u_4,\ldots, u_{2k}\},\{u_3, u_5, \ldots, u_{2k+1}\}\}$. Clearly, Π is an equivalence resolving partition of C_n. Therefore, $pd_{eq}(C_n) \leq 3$ when n is odd.

Suppose, $pd_{eq}(C_n) = 2$. Let $\Pi = \{V_1, V_2\}$ be an equivalence resolving partition of C_n. When n = 3, 4, $pd_{eq}(C_n) = 3$.

Let $n \geq 5$. Then at least one of V_1, V_2 has cardinally greater than or equal to 3. Let Without loss of generality $|V_1| \geq 3$. Let $x, y \in V_1$. Suppose $d_{V1}(x, y) \geq 3$. Then the adjacent vertices of x and y are in V_2. Therefore x and y have the same code with respect to Π, which is a contradiction.

Suppose $d_{V1}(x, y) = 2$. Then the common adjacent point of x and y belongs to V_2. Therefore, x and y have the same code with respect to Π, which is a contradiction. Suppose $d_{V1}(x, y) = 1$. Since G is a cycle, the previous adjacent point of x and the succeeding adjacent point of y are in V_2. Hence x and y have the same code with respect to Π, which is a contradiction. Therefore, $pd_{eq}(C_n) \geq 3$. Therefore $pd_{eq}(C_n) = 3$.

Theorem 2.3. $pd_{eq}(K_{1,n}) = \begin{cases} n & \text{if } n \geq 2 \\ 2 & \text{if } n = 1 \end{cases}$

Proof. Let $n \geq 2$. Let $V(K_{1,n}) = \{u, v_1, v_2,\ldots, v_n\}$. Let $\Pi = \{\{u, v_1\}, \{v_2\}, \{v_3\},\ldots, \{v_n\}\}$. Π is clearly an equivalence resolving partition of $K_{1,n}$. Therefore, $pd_{eq}(K_{1,n}) \leq n$. Since two pendant vertices cannot belong to the same element of an equivalence resolving partition $pd_{eq}(K_{1,n}) \geq n$. Therefore, $pd_{eq}(K_{1,n}) = n$.

When n = 1, $K_{1,n} = K_2$ and $pd_{eq}(K_{1,n}) = 2$.

Theorem 2.4. $pd_{eq}(K_{m,n}) = \begin{cases} m+1 & \text{if } m = n \\ m & \text{if } m > n \end{cases}$

Proof. Case (i): $m = n$ and $m, n \geq 2$.

Let V_1, V_2 be the bipartite sets of $K_{m,n}$. Let $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. Let $\Pi = \{\{u_1, v_1\}, \{u_2, v_2\},\ldots, \{u_{m-1}, v_{m-1}\}, \{u_m\}, \{v_m\}\}$. Clearly, Π is an equivalence resolving partition of $K_{m,n}$. Therefore, $pd_{eq}(K_{m,n}) \leq m + 1$.

Let $\Pi_1 = \{W_1, W_2, \ldots,W_k\}$ be an equivalence resolving partition of $K_{m,n}$, where $k \leq m$. $|W_i| \leq 2$, since if $|W_i| \geq 3$, then W_i contains atleast 3 vertices of V_1 or 3 vertices of V_2. Then every vertex of W_i will have the same code with respect to Π_1, which is a contradiction. If W_i contains two vertices of the partite set, then also we get a contradiction.

Suppose $|W_i| = 1$for some i. Then $|V(K_{m,n})| = |W_1| + |W_2| +\ldots\ldots+ |W_k| \leq 1 + (k - 1)2 = 2k - 1$. That is $2m \leq 2k - 1$. Therefore, $k \geq m + 1$. Therefore, $pd_{eq}(K_{m,n}) = |\Pi_1| = k \geq m + 1$. Therefore, $pd_{eq}(K_{m,n}) = m + 1$.Suppose $|W_i| = 2$ for all i. Then W_i contains exactly one vertex from V_1 and one vertex from V_2. Let Without loss of generality $W_i = \{u_i, v_i\}$, $1 \leq i \leq k$. Clearly, k = m. $c_{\Pi 1}(u_1) = (0, 1,1, 1,\ldots\ldots 1) = c_{\Pi 1}(v_1)$, a contradiction. Therefore, $pd_{eq}(K_{m,n}) = m + 1$.

Case (ii): $m > n$ and $n \geq 2$.

Let V_1 and V_2 be the partite sets of $K_{m,n}$. Let $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. Let $\Pi = \{\{u_1, v_1\}, \{u_2, v_2\},\ldots, \{u_n, v_n\}, \{u_{m-1}, v_{m-1}\}, \{u_m\}\}$. Clearly, Π is an equivalence resolving partition of $K_{m,n}$. Therefore, $pd_{eq}(K_{m,n}) \leq n + (m - n) = m$.

Let $\Pi_1 = \{W_1, W_2, \ldots,W_k\}$ be an equivalence resolving partition of $K_{m,n}$, where $k \leq m - 1$. $|W_i| \leq 2$, since if $|W_i| \geq 3$, then W_i contains atleast 3 vertices of V_1 or 3 vertices of V_2. Then every vertex of W_i will have the same code with respect to Π_1, a contradiction. If W_i contains two vertices of the partition set, then also we get a contradiction.

If $|W_i| = 2$, then W_i contains exactly one vertex from V_1 and one vertex from V_2. Let $W_1, W_2,\ldots,W_r$ be doubletons and the remaining are singletons. Then $|V(K_{m,n})| = 2r + (k - r) + 1 = k + r$. That is, $m + n = k + r$. Therefore, $k = m + n - r$. But $m - 1 \geq k$. Therefore, $m - 1 \geq m + n - r$. $n \leq r - 1$. But $n \geq r$, since each W_i, $(1 \leq i \leq r)$ contains a vertex of V_2, producing a contradiction. Therefore, $|\Pi_i| \geq m$. That is $pd_{eq}(K_{m,n}) \geq m$. Therefore, $pd_{eq}(K_{m,n}) = m$.

3. On Graphs with Prescribed Order and Equivalence Partition Dimension

Theorem 3.1. Let G be a connected graph of order n. Then $pd_{eq}(G) = n$ if and only if $G = K_n$.

Proof. If $G = K_n$, then $pd_{eq}(G) = n$.
Conversely, let $pd_{eq}(G) = n$. Let $V(G) = \{u_1, u_2, \ldots, u_n\}$. Suppose $G \neq K_n$. Then, there exist u_i, u_j such that u_i and u_j are not adjacent. Therefore, we assume that there exist vertices u_1, u_2, u_3 such that $d(u_1, u_2) = 1$, $d(u_2, u_3) = 1$ and $d(u_1, u_3) = 2$. Let $\Pi = \{S_1, S_2, \ldots, S_{n-1}\}$ where $S_1 = \{u_1, u_2\}$, $S_2 = \{u_3\}$, $S_3 = \{u_4\}$, …, $S_{n-1} = \{u_n\}$. Then Π is an equivalence resolving partition. Therefore, $pd_{eq}(G) \leq |\Pi| = n - 1$, which is a contradiction. Therefore $G = K_n$.

Theorem 3.2. Let G be a connected graph of order n. Then $pd_{eq}(G) = 2$ if and only if G is a path of order 2, 3 or 4.

Proof. When G is a path of order 2, 3 or 4, $pd_{eq}(G) = 2$.
Conversely, let $pd_{eq}(G) = 2$. Let $\Pi = \{S_1, S_2\}$ be a minimum equivalence resolving partition of G. Since G is connected, there exist vertices $u \in S_1$ and $v \in S_2$ such that u and v are adjacent. Suppose $u_1, u_2 \in S_1$ are adjacent with a vertex $v \in S_2$. Then u_1 and u_2 have the same code with respect to Π namely (0, 1). Similarly, no two points of S_2 are adjacent with the same point of S_1. Also there exists a unique vertex in S_1 which is adjacent with a unique vertex in S_2. Let $u \in S_1$and $v \in S_2$ be the unique vertices in S_1 and S_2 respectively which are adjacent. Let $S_1^{'} = S_1 - \{u\}$. Suppose $S_1^{'} \neq \phi$. Any vertex in $S_1^{'}$ is adjacent with a vertex of S_1. If v is adjacent with two or more vertices say $w_1, w_2 \in S_1$, then code of w_1, w_2 with respect to S_2 is 2, which is a contradiction. Therefore, we can assume that w is the unique vertex of S_1 that is adjacent with u. Similarly, w is adjacent to at most one vertex in S_1, that is different from u. Proceeding in this way, we get that S_1 is a path. Similarly S_2 is a path. Since S_1 and S_2 are equivalence sets, $|S_1| \leq 2$ and $|S_2| \leq 2$.
If $|S_1| = 1$ and $|S_2| = 1$, then $G = P_2$.
If $|S_1| = 1$ and $|S_2| = 2$ or $|S_1| = 2$ and $|S_2| = 1$, then $G = P_3$.
If $|S_1| = 2$ and $|S_2| = 2$, then $G = P_4$. Therefore, G is a path of order 2, 3 or 4.

Theorem 3.3. Let G be a connected graph of order $n \geq 2$. Then $pd_{eq}(G) = n - 1$ if and only if $G = C_4$, $K_{1,n-1}$, $K_n - e$ and $K_1 + (K_1 \cup K_s)$ where $s = n - 2$.

Proof. Let $pd_{eq}(G) = n - 1$. Let $V(G) = \{u_1, u_2, \ldots, u_n\}$. Let $\Pi = \{S_1, S_2, \ldots, S_{n-1}\}$be a minimum equivalence resolving partition of G. Therefore, exactly one S_i is a doubleton and all S_j's are singleton. Let Without loss of generality S_1 be a doubleton and let $S_1 = \{u_1, u_2\}$. Then $S_2 = \{u_3\}, \ldots, S_{n-1} = \{u_n\}$. Therefore, there exist u_i, $(3 \leq i \leq n)$ such that $d(u_1, u_i) \neq d(u_2, u_i)$. Suppose $diam(G) \geq 3$. Let $diam(G) = t \geq 3$. Let $\{u_1, u_2, \ldots, u_{t+1}\}$ be a diametrical path. Let $T_1 = \{u_1, u_2\}$, $T_2 = \{u_3, u_4\}$, $T_3 = \{u_5\}, \ldots, T_{n-2} = \{u_n\}$. Let $\Pi = \{T_1, T_2, \ldots, T_{n-2}\}$. Then $c_\Pi(u_1) = (0, 2, \ldots)$, $c_\Pi(u_2) = (0, 1, \ldots)$, $c_\Pi(u_3) = (1, 0, \ldots)$, $c_\Pi(u_4) = (2, 0, \ldots)$. Therefore, Π is an equivalence resolving partition of G. Therefore, $pd_{eq}(G) \leq n - 2$, which is a contradiction. Therefore, $diam(G) \leq 2$. If $diam(G) = 1$, then $G = K_n$ and $pd_{eq}(G) = n$, a contradiction. Therefore $diam(G) = 2$.
Case (i): Suppose G is a bipartite.

Since $diam(G) = 2$, $G = K_{m,n}$. If $m, n \geq 2$, then $pd_{eq}(G) = \begin{cases} m+1 & \text{if } m = n \\ m & \text{if } m > n \end{cases}$

$pd_{eq}(G) = m + n - 1$ if and only if $m + n - 1 = m + 1$ if $m = n$ or $m + n - 1 = m$ if $m > n$. That is, if and only if $G = C_4$ or G is a star.
Case (ii): Suppose G is not bipartite.
Let Y be the vertex set of a maximum clique in G. since G is not bipartite, G contains an odd cycle. Since $diam(G) = 2$, G contains either C_3 or C_5. Suppose G contains C_5. Let $V(C_5) = \{u_1, u_2, u_3, u_4, u_5\}$. Let $\Pi = \{\{u_1\}, \{u_2, u_3\}, \{u_4, u_5\}, \{u_6\}, \ldots, \{u_n\}\}$. Then $c_\Pi(u_2) = (1, 0, 2, \ldots)$, $c_\Pi(u_3) = (2, 0, 1, \ldots)$, $c_\Pi(u_4) = (2, 1, 0, \ldots)$, $c_\Pi(u_5) = (1, 2, 0, \ldots)$. Therefore, Π is an equivalence resolving partition of G. Therefore $pd_{eq}(G) \leq n - 2$, a contradiction. Therefore, G contains C_3. Therefore $|Y| \geq 3$. Let $U = V(G) - Y$. Since G is not complete, $|U| \geq 1$.
Subcase (i): $|U| = 1$.
Let $U = \{u_i\}$. Let $Y = V - \{u_i\}$. Let $u_j \in Y$, $j \neq i$, such that u_j is adjacent only with u_i. Then $G = \{u_j\} + (\{u_i\} \cup (\langle V(G) - \{u_i, u_j\} \rangle))$. That is $G = K_1 + (K_1 \cup K_s)$ where $s = n - 2$. Suppose u_i is adjacent with more than one vertex of Y. Let $u_{i1}, u_{i2}, \ldots, u_{ir}$, $r \geq 2$, $r \leq n - 3$ be the vertices of Y adjacent with u_i.

Then G = $\{u_{i_1}, u_{i_2}, u_{i_3}, ..., u_{i_r}\} + (\{u_i\} \cup (< V(Y - \{u_{i_1}, u_{i_2}, ..., u_{i_r}\}) >)$.

Let Π= {{u_{ir+1},u_{i1}}, {$u_{i\,r+2}$,u_{i2}}, {u_i}, {$u_{i\,r+3}$}, …….{u_n}, {u_{i3}},….., {u_{ir}}}.Then $c_\Pi(u_{i_r+1}) = (0,1,2,...)$, $c_\Pi(u_{i_1}) = (0,1,1,...)$, $c_\Pi(u_{i_r+2}) = (1,0,2,...)$, $c_\Pi(u_{i_2}) = (1,0,1,...)$, .Therefore, Π is an equivalence resolving partition of G. Therefore $pd_{eq}(G) \le n - 2$, a contradiction. Therefore, G = $K_1 + (K_1 \cup K_s)$, where s = n – 2.

Suppose r = n – 2. Then u_i is adjacent with n – 2 vertices of Y and not adjacent with exactly one vertex of Y. Therefore, G = K_n– e.

Subcase (ii): |U| ≥ 2.

Let U be non-independent. Then, there exit vertices u, w ∈ U such that u and w are adjacent. Proceeding as in Theorem 3.3 in [11], and taking the partition same as in Fig. 3, Fig. 4 and an alternate partition Π = {S_1, S_2, .., S_{n-2}} in Fig. 5 where S_1 = {u, y}, S_2 = {v, v′}, S_3 = {w}, S_4,…, S_{n-2} are singletons, we get $pd_{eq}(G) \le n - 2$, a contradiction. Therefore U is independent. Proceeding as in the same theorem, and taking the same partition as in Fig. 6 and an alternate partition Π = {S_1, S_2, …, S_{n-2}} in Fig.7, where S_1 = {x, y, z}, S_2 = {u}, S_3 = {w}, S_4,…, S_{n-2} are singletons, we get $pd_{eq}(G) \le n - 2$, which is a contradiction. Finally we get as in Theorem 3.3 in [11], G = $K_s + \overline{K_t}$, where s = |Y| ≥ 3 and t = |U| ≥ 2. If t = 2, then G = $K_{n-2} + \bar{K}_2$,G = K_n – e. If t ≥ 3, as in Theorem 3.3, we get that $pd_{eq}(G) \le n - 2$, a contradiction. Therefore, Subcase (ii) rise to K_n– e. Therefore G = C_4, K_1, $_{n-1}$, K_n – e and $K_1 + (K_1 \cup K_s)$, where s = n – 2. The converse is obvious.

4. Bounds for Equivalence Partition Dimension of A Graph

Definition 4.1. [9] Let Π = {V_1, V_2, …,V_k} of V(G) of a graph G. Let for any u ∈V(G), $c_\Pi(u)$ = (d(u, V_1), d(u, V_2),…, d(u, V_k)). Π is called a metric coloring of G if $c_\Pi(u) \ne c_\Pi(v)$ for any two adjacent vertices u, v in V(G). The minimum cardinality of a metric coloring of G is called the metric chromatic number of G and is denoted by μ(G).

Clearly, 2 ≤ μ(G) ≤ χ (G) ≤ n.

(i) 2 ≤ $pd_{eq}(G)$ ≤ n.

(ii) Since any independent resolving partition is an equivalence resolving partition, $pd_{eq}(G)$ ≤ipd(G). Since any equivalence resolving partition is a resolving partition, pd(G) ≤$pd_{eq}(G)$. Therefore, pd(G) ≤$pd_{eq}(G)$ ≤ipd(G).

(iii) $1 + \lceil \log_2(\omega(G)) \rceil \le \mu(G)$ ([9]).

Therefore, $1 + \lceil \log_2(\omega(G)) \rceil \le \mu(G) \le pd(G) \le pd_{eq}(G) \le ipd(G)$, where ω(G) is the clique number of G.

5. Graphs with Prescribed $Pd_{eq}(G)$ and Ipd(G)

Theorem 5.1. Given a pair of positive integers a, b with a ≤ b and 2a > b there exists a connected graph G such that $pd_{eq}(G)$ = a and ipd(G) = b.

Proof. Case (i): Let a < b and 2a > b.

Let G = $K_{a,b-a}$. ipd(G) = a + b – a = b. $pd_{eq}(G)$ = a, since a > b – a.

Case (ii): a = b. Let G = K_a. ipd(G) = $pd_{eq}(G)$ = a = b.

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