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Evaluation of quality of expert pairwise comparison judgements in decision-making techniques

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Abstract: The paper deals with a problem of evaluation of quality of expert judgements, given in integer-valued scales using the pairwise comparison method. These judgements are applied in popular decision-making techniques, such as the analytic hierarchy and network processes and other. The notion of consistency is traditionally used to evaluate the contradiction level of expert pairwise comparison judgements. Consistency coefficients and criteria are known, which helps to estimate the acceptability of inconsistency level. However, different consistency coefficients may lead to different results and we do not know exactly whether quality of expert pairwise comparison judgements is acceptable or not. Some sets of the expert judgements may have cycles, which mean that there is no vector of weights, which satisfies all these judgements. The traditional criteria of acceptable inconsistency, unfortunately, may identify ordinal intransitive judgements with a cycle as acceptably inconsistent. To solve these problems a more efficient method for evaluation and improvement of consistency level of expert multiplicative pairwise comparison judgements without participation of an expert is proposed. Influence of requirement of the expert judgements' consistency level on an accuracy of resulting weights is estimated.

Keywords: decision support systems, efficient improvement of consistency, pairwise comparisons, simulation, the most inconsistent judgments

I. INTRODUCTION

Expert pairwise comparison judgements are input data for several modern decision-making techniques, such as the analytic hierarchy process [1] and its generalization – the analytic network process [2], the ELECTRE [3] and PROMETHEE [4]. These techniques are used to solve semistructured and unstructured decision-making problems, such as problems of choice of the best decision alternative (DA), multiple criteria and multiple goal evaluation of DAs, resource allocation problems, risks evaluation, scenarios of development evaluation, foresight [5] and analytical planning. They are applied in various application areas: in economics, finances and banking, ecology, industry, politics, medicine and health protection and many other [6].

Methods of the type "line" [7 – 9] and "triangle" [1, 2, 10, 11] for calculation of relative weights of DAs on basis of expert pairwise comparison judgements are a part of the decision-making techniques. A reference DA or a set of DAs is chosen in the "line" methods, with which all DAs are compared. The "line" methods assume consistency of expert knowledge. The "triangle" methods, such as the eigenvector method (EM) [1, 2], the RGMM [10], the AN [11] and other [12] require redundant quantity of $n(n\text{-}1)/2$ expert pairwise comparison judgements for DAs weights calculation, where n is a number of DAs. In the "triangle" methods all pairs of DAs have to be compared with each other. This redundant quantity of estimates allows evaluation of the consistency level of expert knowledge. Then a pairwise comparison matrix (PCM) $A=\{a_{ij} \mid i, j=1,...,n\}$ is formed, which satisfies the property of inverse symmetry, where a_{ij} is a result of comparison of i-th DA over the j-th one. Consistency coefficients and criteria of acceptable inconsistency are used to estimate the consistency level of a PCM [1, 2, 9, 10, 11, 13, 14].

However, different consistency coefficients may lead to different results and we do not know exactly whether quality of expert pairwise comparison judgements is acceptable or not.

The purpose of the paper is to investigate the accuracy of weights, calculated using the "line" and "triangle" methods, and to propose an efficient method for evaluation and improvement of consistency level of a multiplicative PCM without participation of an expert. The paper is organized as follows. In Section 1 definitions are given, which are used throughout the paper. A comparative analysis of the "line" [7 – 9] and "triangle" [1, 2, 10, 11] methods for calculation of relative weights of DAs on basis of expert pairwise comparison judgements are given in Section 2. A method for evaluation and improvement of consistency level of multiplicative expert pairwise comparison judgements without participation of an expert is proposed in Section 3. Conclusions are given in Section 4.

II. BACKGROUND

Let $D = \{d_{ij} \mid i, j = 1,...,n\}$ be a multiplicative PCM of *n* DAs, such that $d_{ij} > 0$ and $d_{ji} = 1/d_{ij}$ (the property of inverse symmetry). The notion of consistency of a PCM is used to estimate quality of expert pairwise comparison judgments.

PCM *D* is called *consistent (strongly consistent)* if cardinal transitivities $d_{ij} = d_{ik}d_{kj}$ are held for all $i, j, k = 1,...,n$ [1].

PCM *D* is called *weak or ordinal consistent* if ordinal transitivities $(d_{ij} > 1) \wedge (d_{jk} > 1) \Rightarrow (d_{ik} > 1)$, $(d_{ij} = 1) \wedge (d_{jk} > 1) \Rightarrow (d_{ik} > 1)$, $(d_{ki} > 1) \wedge (d_{ij} = 1) \Rightarrow (d_{kj} > 1)$ and $(d_{ij} = 1) \wedge (d_{jk} = 1) \Rightarrow (d_{ik} = 1)$ are held [15].

Several consistency coefficients are used to estimate the contradiction level of multiplicative PCM, such as the consistency ratio *CR* [1], the geometric consistency index *GCI* [10], the harmonic consistency ratio *HCR* [11], the consistency index of transitivities CI^{tr} [13], the spectral coefficient of consistency k_y [9] and other. Several criteria of acceptable inconsistency are known [1, 9, 10, 11]. In these criteria the inconsistency coefficients are compared with their threshold values and a decision is made about the acceptability of inconsistency level and the quality of expert judgements. The results obtained using the "triangle" methods are theoretically justified only for acceptably inconsistent PCMs [1]. The next criterion is widely applied in practical decision-making problems.

Criterion of acceptable inconsistency: PCM *D* is acceptably inconsistent if *CR* value of *D* does not exceed the threshold value: $CR(D) < 0.08$ for *n*=4, $CR(D) < 0.1$ for $n \geq 5$ [1].

III. A COMPARATIVE ANALYSIS OF "LINE" AND "TRIANGLE" METHODS

The "line" and "triangle" pairwise comparison methods are used for calculation of relative weights or priorities of DAs in terms of a qualitative criterion on basis of expert judgements. The "line" method [9] requires *n*-1 expert judgements for *n* DA's weights calculation, assuming that these judgements are totally consistent. However, if an expert gives consistent judgements, there is no guarantee that they represent real weights of compared DAs. The "triangle" methods require redundant quantity of *n*(*n*-1)/2 expert pairwise comparisons of all pairs of DAs. This redundant quantity of estimates allows evaluation of the consistency level of expert knowledge.

A computer simulation is performed to find estimates of errors of weights, which are calculated using the "line" and "triangle" methods. A process of DAs evaluation by a highly competent expert is modelled when he/she provides pairwise comparison judgements in the Saaty's integer-valued scale. A set $\{w^*(l) \mid l = 1,..,M\}$ of normalized vectors $w^*(l) = w^*$ of weights of *n* DAs is generated on a first stage of simulation, where *M* is a number of experiments. Suppose that each w^* is close to some vector of real weights. A PCM D^* is calculated on basis of each w^*, where element d^*_{ij} equals to a value of Saaty's scale, which is the closest one to the ratio (w^*_i / w^*_j). Using the "triangle" method EM vectors of weights w^{EM} and also values of consistency coefficient *CR* of PCM D^* are further calculated. Using the "line" method vectors of weights w^e, $e = 1,...,n$ are found. On the next stage of simulation values of the Euclidean and Chebyshev norms of deviation of vectors w^{EM} and w^e, $e = 1,...,n$ from corresponding vector w^* are found. Average value $\overline{CR}$ of consistency coefficients *CR* and average values $\overline{Eucl}$ and $\overline{Cheb}$ of Euclidean and Chebyshev norms, respectively, which are calculated on basis of results of all experiments are shown in the **Table**.

Average values $\overline{Eucl}$ and $\overline{Cheb}$ for weights, calculated using the "triangle" method are smaller than those, calculated using the "line" method (**Table**). Therefore, the requirement of consistency of expert pairwise comparison judgements in "line" methods adds additional error in calculated weights.

TABLE: Average values $\overline{Eucl}$ and $\overline{Cheb}$ of Euclidean and Chebyshev norms of deviation of calculated weights from real weights depending on dimension *n* of PCM

	n	3	4	5	6	7	8	9
The "triangle" method	$\overline{CR}$	0.008	0.009	0.009	0.009	0.009	0.009	0.009
	$\overline{Eucl}$	0.045	0.037	0.029	0.025	0.022	0.019	0.017
	$\overline{Cheb}$	0.034	0.028	0.022	0.019	0.016	0.013	0.012
The "line" method	$\overline{Eucl}$	0.057	0.066	0.067	0.065	0.062	0.059	0.057
	$\overline{Cheb}$	0.056	0.060	0.063	0.061	0.060	0.058	0.052

Values in the **Table** show that when an expert provides pairwise comparisons in an integer-valued Saaty's scale, the relatively small inconsistency level of his/her judgements, such that the consistency coefficient *CR* is nearly equal 0.01, is not only acceptable but is desirable.

IV. A METHOD FOR EVALUATION AND IMPROVEMENT OF CONSISTENCY LEVEL OF MULTIPLICATIVE PCM

Based on Saaty's criterion [1], a generalized criterion for evaluation of acceptable inconsistency level of multiplicative PCM $A = \{a_{ij} \mid i, j = 1,...,n\}$ is developed. Suppose that $ConsInd(A)$ is one of the following consistency coefficients of PCM *A*: *CR* [1], *GCI* [10], *HCR*[11]or CI^{tr} [13]. A generalized c*riterion of acceptable inconsistency of multiplicative PCM A is formulated as follows*:

- PCM *A* is strongly consistent (consistent), if and only if $ConsInd(A) = 0$,

- PCM *A* is acceptably inconsistent and an improvement of PCM *A* is not required if $ConsInd(A) \le \delta^k$, where δ^k is the threshold value of the respective consistency coefficient,

- PCM *A* contains information, but it is inacceptably inconsistent and is required an improvement if the consistency coefficients exceed their threshold values,

- PCM *A* is an information noise, if coefficients $CR(A) \ge 1$ or $HCR(A) \ge 1$.

The inacceptable inconsistency level of a PCM means that PCM's elements are mutually contradictory. Therefore, such PCM cannot be used for weights calculation and requires correction. One of the methods for increasing consistency level of a set of pairwise comparison judgements is a feedback with an expert when he/she overviews all or the most inconsistent his/her judgments. The overview procedure is repeated until the acceptable inconsistency of a PCM is achieved. Methods for finding the most inconsistent elements, unusual and false observations in a PCM are proposed in [15 – 17]. In practical decision-making problems when a feedback with an expert is not possible another approach for increasing the consistency level of a PCM is applied – to change PCM's elements using the multiplicative or additive rules [18] without participation of an expert.

The next statement shows that usage of the multiplicative and additive rules results in improvement of the PCM's consistency level in terms of several different consistency coefficients.

Statement. Let $A = \{a_{ij} \mid i, j = 1,...,n\}$ be a multiplicative PCM of *n* DAs in terms of some qualitative criterion. Suppose that the *CR*, *GCI*, CI^{tr} or *HCR* consistency coefficients are used to measure the consistency level of a PCM *A*. Let $B^{mult} = \{b_{ij}^{mult} \mid i, j = 1,...,n\}$ and $B^{ad} = \{b_{ij}^{ad} \mid i, j = 1,...,n\}$ be improved PCMs:

$b_{ij}^{mult} = (a_{ij})^x (w_i / w_j)^{1-x}$ (the multiplicative rule),

$b_{ij}^{ad} = x a_{ij} + (1-x)(w_i / w_j)$ if $i < j$ and

$b_{ij}^{ad} = (x a_{ji} + (1-x)(w_j / w_i))^{-1}$ if $i \ge j$ (the additive rule),

where $x \in (0,1)$ is a parameter of improvement, *w* – vector of weights, calculated on basis of PCM *A*. Then the following inequalities hold:

$$CI(B^{mult}) \le CI(A),\ GCI(B^{mult}) \le GCI(A),\ HCI(B^{mult}) \le HCI(A) \text{ and } CI^{tr}(B^{mult}) \le CI^{tr}(A),$$

$$CI(B^{ad}) \le CI(A),\ GCI(B^{ad}) \le GCI(A),\ HCI(B^{ad}) \le HCI(A) \text{ and } CI^{tr}(B^{ad}) \le CI^{tr}(A).$$

These inequalities become equalities if and only if a PCM *A* is consistent. ▪

It is worth noting that an application of the multiplicative or additive rules results in changing of all elements of a PCM. If majority of PCM's elements corresponds to real weights of DAs and only few of PCM's elements are considerable disturbances of real weights of DAs, i.e. are outliers or unusual and false observations, and a PCM is inacceptably inconsistent, then an application of the above multiplicative or additive rules results in distortion of true information in a PCM. Therefore, the multiplicative and additive rules seem to be efficient for improving consistency level of a weak consistent PCM without cycles.

Efficiency of the multiplicative and additive rules depends on the percent of elements in a PCM, which are significant disturbances of real values [19]. This percent may be estimated using the Transitiv method [17]. The rules are efficient for improving the consistency level of PCM, which elements are slightly disturbed.

Examples in [19] show, that the known criteria of acceptable inconsistency [1, 10, 11] may define weak inconsistent PCM as acceptably inconsistent in terms of coefficients CR, GCI and HCR, and these criteria do not identify ordinal intransitivity in a PCM.

The above considerations underlie a proposed method for evaluation and correction of a multiplicative PCM, which helps to increase consistency level of a PCM efficiently. The features of the method are:

- the traditional notion of consistent PCM and a new notion of weakly consistent PCM are used;
- evaluation and correction of a PCM are made using several consistency coefficients and criteria;
- methods Transitiv [17] and MOutflow [20] for finding the most inconsistent elements in a PCM are a component part of the proposed method;
- methods of PCM's correction without participation of an expert depending upon properties of PCM are a component part of the proposed method.

Steps of the method for evaluation and improvement of consistency level of a multiplicative PCM $A=\{a_{ij}\mid i,j=1,...,n\}$ without participation of an expert are as follows:

1) To check whether a PCM *A* has a cycle or cycles.

2) If PCM *A* has no cycles, then:

2.1) a consistency coefficient of PCM *A* is calculated and the above criterion of acceptable inconsistency is checked,

2.2) if the criterion is accepted, then quality of PCM *A* is considered to be satisfactory,

2.3) otherwise, a correction of PCM *A* is performed. PCM's *A* elements are iteratively changed using the above multiplicative or additive rules until PCM *A* becomes an acceptably inconsistent. Number of iterations depends on the value of parameter $x\in(0,1)$, namely greater values of x result in greater number of iterations. If correction is efficient, the method is finished, otherwise, go to the Step 3.1.

3) If PCM *A* has a cycle, then:

3.1) the most inconsistent element (MIE) of PCM *A* is founded using the Transitiv or the MOutflow methods,

3.2) a correction of the MIE without participation of an expert is performed. Correction means a finding of a new proper value for the MIE from the set of values according to the scale. The new value for the MIE should remove a cycle or cycles in a PCM *A* and ensure the minimum possible value of consistency coefficient of this matrix,

3.3) go to the Step 1.

The Transitiv method is based on an analysis of transitivities of a PCM [17]. On the first stage a set of transitivities $\Gamma=\{\Gamma_u\mid u=1,...,NT\}$ is calculated:

$$\Gamma_u=\{a_{ij},a_{jk},a_{ik}\},\ i,j,k=1,...,n,\ i<j<k,\ NT=n!/(n-3)!/3!,\ n\geq 3.$$

On the second stage values of determinants of these transitivities are calculated:

$$Det=\{\det(\Gamma_u)\},\ \det(\Gamma_u)=\frac{a_{ij}a_{jk}}{a_{ik}}+\frac{a_{ik}}{a_{ij}a_{jk}}-2.$$

On the third stage values $S_{i,j}=\sum_{k=1}^{n}\left(\frac{a_{ij}a_{jk}}{a_{ik}}+\frac{a_{ik}}{a_{ij}a_{jk}}-2\right)$ for each $i,j=1,...,n$ are founded.

Then an element $a_{i^*j^*}$, which indexes (i^*,j^*) are defined as follows:

$$(i^*,j^*):\ \max_{i,j}S_{i,j}$$

is the most inconsistent one.

The MOutflow method contains several stages [20]. On the first stage the inflow Φ_i^- and outflow Φ_i^+ values are calculated for each decision alternative x_i, $i = 1,2,...,n$.

Φ_i^+ is a number of DAs x_j, such that x_i outperforms x_j, namely $a_{ij} > 1$, $j = 1,...,n$.

Φ_i^- is a number of DAs x_j, such that x_j outperforms x_i, namely $a_{ji} > 1$, $j = 1,...,n$.

On the second stage the maximum of differences $\Phi_j^+ - \Phi_i^+$ and $\Phi_i^- - \Phi_j^-$ are founded.

Then an element $a_{i^* j^*}$, which indexes (i^*, j^*) are defined as follows:

$$(i^*, j^*) : \max_{i,j}(\max(\Phi_j^+ - \Phi_i^+,\ \Phi_i^- - \Phi_j^-)), \text{ if } i \neq j \text{ and } a_{i,j} > 1.$$

is the most inconsistent one.

If several elements $\{a_{i^* j^*}\}$ satisfy the latter condition, then an element among them is found which results in the maximum value of the next expression, i.e. in more inconsistency:

$$(\tilde{i}, \tilde{j}) : \max_{i^*, j^*}\left(\frac{1}{n-2} \sum_{k=1}^{n} (\ln a_{i^* j^*} - \ln(a_{i^* k} a_{kj^*})) \right), \text{ where } k \neq i^* \neq j^*.$$

Then an element $a_{\tilde{i}\tilde{j}}$ is the most inconsistent one.

A comparative study of the known methods [15, 16] for finding the most inconsistent elements in a PCM is done. These methods are meant for increasing of consistency level of a PCM and removing cycles in this matrix. The comparative study is based on results of computer modeling of test sets of PCMs. In the beginning vectors of real weights and corresponding disturbed PCMs with cycles are generated. The most inconsistent elements of these PCMs are founded using several methods. The method is considered to be an efficient if the most inconsistent element of a PCM is founded correctly [20]. Then vector of weights, calculated on basis of improved matrix is closer to vector of real weights, which is known in a process of modeling, than vector of weights on basis of a PCM before the improvement.

The known methods [15, 16] for finding the most inconsistent elements of a PCM do not always remove cycles in a PCM. The problem of a PCM with a cycle is that there does not exist a vector of weights, which satisfies all elements of such matrix. Results of the modeling show that the Transitiv and MOutflow methods are more efficient in comparison with the methods, proposed in [15, 16] since they correctly founded the most inconsistent elements in a greater number of experiments and removed cycles in all test PCM under conditions of the modeling.

V. CONCLUSION

An actual problem nowadays is an investigation of reliability of weights calculated on basis of consistent, inconsistent and weak inconsistent expert pairwise comparison judgements. The problem is that different inconsistency coefficients and criteria in "triangle" type pairwise comparison methods lead to different results about acceptable inconsistency of expert PCM. Acceptable inconsistency of PCM, in its turn, does not guarantee that rankings of DAs on basis of weights, calculated using the EM, RGMM and AN methods coincide with each other. Another problem is that the traditional Saaty's criterion of acceptable inconsistency does not identify the ordinal transitivity violation or a cycle in a PCM.

In this paper simplified "line" type pairwise comparison methods are investigated, which require smaller number of expert judgements and assume consistency of expert knowledge. A computer modeling helps to obtain estimates of errors of weights, which are calculated using the "line" and the "triangle" methods. Accuracy of results is evaluated in terms of average values of the Euclidean and Chebyshev norms of deviation of calculated weights from real weights. Results reveal that the requirement of consistency of expert pairwise comparison judgements in "line" type methods may introduce additional error in a PCM and, consequently, in resulting weights.

A method for efficient evaluation and improvement of consistency level of a multiplicative PCM is proposed. In this method the acceptable quality of PCM is measured using a relatively new notion of weakly consistent PCM, evaluation and correction of PCM are made using several consistency coefficients. More efficient Transitiv [17] and MOutflow [20] methods for finding the most inconsistent element in a PCM and also methods for PCM's improvement without participation of an expert are a component part of the proposed method.

Usage of the proposed method for evaluation and improvement of consistency level of a multiplicative PCM helps to obtain PCMs of acceptable quality, which may be used further for calculation of reliable local weights of hierarchical and network decision-making models.

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