International Journal of Latest Engineering and Management Research (IJLEMR)
ISSN: 2455-4847
www.ijlemr.com || Volume 03 - Issue 04 || April 2018 || PP. 87-92

Comparative Analyses of the Investment Portfolio on the Basis of a Multi-Criterial Optimization Model in the Stock Market with Linear Convolution Method

Leyla M. Mamedova
Ph.D., Associate Professor, Baku State University/Department of Mathematical Economics / Azerbaijan, Baku city

Abstract: In this paper, a method for reducing the number of criteria for the multi objective optimization problem is proposed.The result is two really conflicting criteria in which the improvement of any of them inevitably leads to the deterioration of others.The Markowitz model, modified by the addition of two criteria, reduction of their one-criterion optimization problem by means of a linear convolution of the criteria is considered.The article analyzes the stock market papers of the American stock market in real period.
Key words: linear convolution, multi-objective optimization Markowitz model, efficient frontier, Lagrange method, covariance, portfolio yield, portfolio of securities

The fund market forms a mechanism for attracting investments to the economy, building relationships between those who need additional financial resources and those who want to invest surplus income.Portfolio investment allows you to plan, evaluate, and monitor the final outcomes of all investment activities in various sectors of the stock market.

Optimization of the structure of the securities portfolio is one of the most important tasks of making decisions in investing in the stock market.The purpose of securities portfolio optimization is the formation of a portfolio of securities that would satisfy the requirements of the investor, the enterprise, both in terms of profitability and possible risk, which is achieved through the distribution of securities in the portfolio.In general, portfolio optimization concerns not only the formation of a portfolio of investment projects, a loan portfolio, etc.The core of portfolio optimization is to select from a set of alternative objects the subset that, within a given period, will bring the optimal portfolio to the portfolio owner, that is, the best outcome.Criteria of optimization can be several; tendencies of their improvement can contradict each other.The optimal result in different issues is understood as either the maximum profit or the specified profit level under the minimum risk, possibly taking into account additional outer constraints and the preferences of the decision-maker.

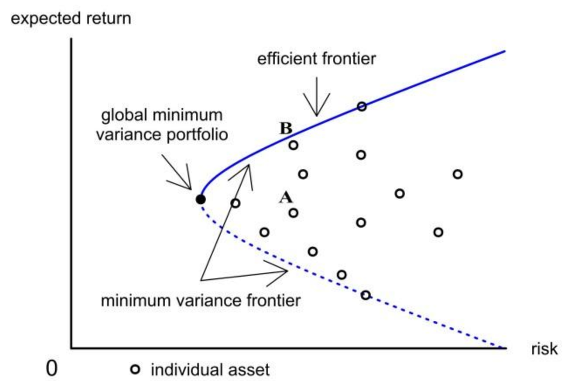

Figure 1. Efficient frontier

Each investor seeks to create such a portfolio of securities, which would provide the maximum possible income with minimal risk.There are two problems: how to forecast revenue based on statistical data and how to measure risk.

In the classical formulation of Markowitz, the problem of choosing the optimal portfolio is reduced to the theory of an effective set of portfolios, or the so-called effective boundary.The essence of the theory is that if there are n securities available to the investor, each with its expected return $E(r_i)$, where $i=1,2,\ldots, n,$ then there is one combination of securities in the portfolio that minimizes the portfolio risk at each set value expected return on the portfolio.Fig. 1 shows that whatever the value of the expected return is determined by the investor (for example, $E(r_m)$), always by scaling the securities we can find a portfolio in which the risk level reaches a minimum value (in Figure 1 - point B) [1].

The expected yield of a security in the Markowitz model is calculated as the mathematical expectation of its returns for the previous period of time, the risk is the standard deviation of these yields, and the covariance is given by the formula $\sigma=V_{ij}\sigma_i\sigma_j$, where V_{ij} is the coefficient of the pairwise linear correlation between the yields of the two assets [2].

The investor's task in Markowitz's model boils down to the following: from a set of portfolios with the expected rate of return $E(r_p)$, one must find one that would ensure a minimum level of risk. In other words, the investor's task can be reduced to solving the following system:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i\theta_j b_{ij} \rightarrow min, \quad b_{ij} = cov(R_i, R_j), \\ \sum_{i=1}^{n}\theta_i = 1 \\ \sum_{i=1}^{n} m_i\theta_i = m_p, \\ \theta_1 \geq 0, \dots, \theta_n \geq 0 \end{cases} \qquad (1)$$

where
m_p – the value of the portfolio efficiency selected by the investor;
θ_i – share of the i -th security in the portfolio;
m_i – mean of effectiveness of $R_i i$-th security
We pass from the mono objective model of Markowitz to the model of multicriteria optimization, that is, on our cases to the model two-criterion optimization[3] [4]:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i\theta_j b_{ij} \rightarrow min, \quad b_{ij} = cov(R_i, R_j), \\ \sum_{i=1}^{n} m_i\theta_i \rightarrow max \quad \theta = (\theta_1, \dots, \theta_n), \\ \sum_{i=1}^{n}\theta_i = 1 \end{cases} \qquad (2)$$

Here we will apply the method of linear convolution for multi-objective portfolio optimization.From the model with two criteria (2) by using the method of linear convolution, one can pass to a model with one criterion.The simplest and most frequently used method for reducing the multicriteria problem to single-criterion is linear convolution.Weighted nonnegative coefficients α_i are designated, denoting the importance of each criterion, and the linear combination of objective functions [5] [6] is maximized, i.e. the problem is solved:

$$g(x) = \sum_{i=1}^{m} \propto_i f_i(x)$$
$$x \in X$$
$$\propto_i \geq 0, i = 1, \dots, m, \sum_{i=1}^{m} \propto_i = 1$$

This task involves combining the criteria from the above problem by constructing a linear combination $f_i(x), i = 1,2, \dots, m$ (constructing a weighted sum of partial criteria) and passing to a single-objective problem:

$$\sum_{i=1}^{m} \propto_i f_i(x) \rightarrow \underbrace{min}_{x \in X} \qquad (3)$$
$$\propto_i = const > 0, i = 1,2, \dots, m, \sum_{i=1}^{m} \propto_i = 1$$

Where $\propto_i$ are determined by experts. However, this approach of determining $\propto_i$, based on the subjective opinion of experts, ultimately leads to the fact that the solution of problem (2), (3) will be largely subjective.In this paragraph, another way of determiningof $\propto_i, i = 1,2, \dots, m$. First we assume that all the criteria in (1) are not ranked.In this case, the following method of convolution of the criteria$f_i(x)$from (1).

Let there be given points$x^{(1)}, x^{(2)}, \dots, x^{(r)} \in X$.Let's calculate the values

$$y_i^{(k)} = f_i\left(x^{(k)}\right), i = 1,2, \dots, m, k = 1, \dots, r, \qquad (4)$$

We construct a linear combination:

$$y\left(\propto_1, \dots, \propto_m, x^{(k)}\right) = \propto_1 f_1\left(x^{(k)}\right) + \propto_2 f_2\left(x^{(k)}\right) + \dots + \propto_m f_m\left(x^{(k)}\right),$$
$$k = 1, \dots, r, \qquad (5)$$

Here it is proposed to choose nonlinear programming problems:

$$\sum_{i=1}^{m}\left[\left(y\left(\propto_1, \propto_2, \dots, \propto_m, x^{(1)}\right) - y_i^1\right)^2 + \left(y\left(\propto_1, \propto_2, \dots, \propto_m, x^{(2)}\right) - y_i^2\right)^2 + \dots + \left(y\left(\propto_1, \propto_2, \dots, \propto_m, x^{(r)}\right) - y_i^r\right)^2\right]$$
$$\rightarrow \underbrace{min}_{\propto_1, \propto_2, \dots, \propto_m} \qquad (6)$$
$$\sum_{i=1}^{m} \propto_i = 1$$
$$\propto_i \geq 0, i = 1, \dots, m.$$

For its numerical solution, you can use various tools, for example, an office application of Excel spreadsheets.

Now let all the criteria$f_i(x), i = 1,2, \dots, m$,ranked as follows:

$$f_1(x) \succ= f_2(x) \succ= \dots \succ= f_m(x) \qquad (7)$$

где

$$f_p(x) \succ= f_{p+1}(x), p = 1, \dots, m-1,$$

(7) means that the criterion$f_p(x)$is not less preferable than the criterion$f_{p+1}(x)$. However, the degree of preference of $f_p(x)$for$f_{p+1}(x)$isn't marked. In such case, obviously$\propto_i, i = 1, \dots, m$,must satisfy the additional condition

$$\propto_1 \geq \propto_2 \geq \dots \geq \propto_n \qquad (8)$$

.

Then the problem of approximate calculation of$\propto_i, i = 1,2, \dots, m$,in the case of their ranking according to (7) with the solution [7] [8], reduces to the solution of the optimization problem of(4)-(6), (8).

For solving the problem of optimization of the investment portfolio, a holistic review of all indicators of the portfolio should be made.In a holistic view, it must be taken into account that maximizing the values of some indicators can be accompanied by minimizing the values of others.Particular criteria for multi-criteria optimization of the investment portfolio are:

- Maximization of the predicted return on the securities portfolio;
- Minimizing the risk of the formed portfolio;

Taking into account the above two goals, our task is reduced to two-criterion optimization. After determining the approximate values of $\propto_1, \propto_2$, the quadratic programming problem is solved [9]:

$$\begin{cases} \propto_1 \left(\sum_{i=1}^{n}\sum_{j=1}^{n} \theta_i \theta_j b_{ij}\right) - \propto_2 \left(\sum_{i=1}^{n} m_i \theta_i\right) \rightarrow min \\ i, j = 1, \dots, n, \ b_{ij} = cov(R_i, R_J) \\ \sum_{i=1}^{n} \theta_i = 1 \\ \theta_1 \geq 0, \dots, \theta_n \geq 0 \quad \propto_1 \geq 0, \propto_2 \geq 0, \propto_1 + \propto_2 = 1 \end{cases} \qquad (9)$$

Let us give an example of the portfolio problem of Markowitz with shares of the American stock market at the beginning of 2016: Chevron, Walt Disney, Caterpillar, AT&TиAdobe System. In calculating the expected return of the portfolio we will use real data reflecting the value of the indices for the period 01/01/2016 - 01/01/2018 (106 trading weeks) [10]:

The yield of each security can be calculated according to the rules:

$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100\%$ (yield as a percentage of the invested amount).

Here P_t is the price of the security in period t.

The average yield $E(r_i)$ is defined as the arithmetic mean of historical returns for 106 weeks.Next, we find the variances and standard deviations of these indices. As a result, we obtain 5-dimensional vectors:

r= {0.449, 0.141, 0.956, 0.153, 0.765}
σ^2 = {4.463, 5.316, 9.896, 5.495, 9.078}
σ= {2.112, 2.306, 3.146, 2.344, 3.013}

Let's make the covariance matrix of these shares:

	First Solar	Walt Disney	Caterpillar	AT&T	Adobe Syst
First Solar	4.4197	0.9294	1.0937	1.0357	-0.4246
Walt Disney	0.9294	5.2651	2.4346	1.5159	1.9907
Caterpillar	1.0937	2.4346	9.8007	1.3168	-0.0531
AT&T	1.0357	1.5159	1.3168	5.4424	-0.5424
Adobe System	-0.4246	1.9907	-0.0531	-0.5424	8.9908

Table 1. Covariance

Using the method of determining ofα_1, α_2, described above$\theta_1 = 2{,}23$, $\theta_2 = -0{,}60$, $\theta_3 = -0{,}67$, $\theta_4 = -0{,}31$, $\theta_1 = 0{,}35$we find$\alpha_1 = 0.5$; $\alpha_2 = 0.5$.

We define stationary points. Let us find the extreme of the function [14]:

$$\mathrm{F}(\theta) = 0.5 * (4.463 * \theta_1^2 + 5.316 * \theta_2^2 + 9.896 * \theta_3^2 + 5.495 * \theta_4^2 + 9.078 * \theta_5^2 + 1.859 * \theta_1 * \theta_2 + 2.188 * \theta_1 * \theta_3 + 2.071 * \theta_1 * \theta_4 - 0.849 * \theta_1 * \theta_5 + 4.869 * \theta_2 * \theta_3 + 3.032 * \theta_2 * \theta_4 + 3.981 * \theta_2 * \theta_5 + 2.634 * \theta_3 * \theta_4 + (-0.106) * \theta_3 * \theta_5 + (-1.085) * \theta_4 * \theta_5) - 0.5 * (0.449 * \theta_1 + 0.141 * \theta_2 + 0.956 * \theta_3 + 0.153 * \theta_4 + 0.765 * \theta_5)$$

We rewrite the restriction of the problem in an implicit form:

$$\varphi_1(\theta) = 1 - (\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5) = 0$$

Let us compose the auxiliary Lagrange function:

$$L(\theta, \lambda, \mu) = 0.5 * (4.463 * \theta_1^2 + 5.316 * \theta_2^2 + 9.896 * \theta_3^2 + 5.495 * \theta_4^2 + 9.078 * \theta_5^2 + 1.859 * \theta_1 * \theta_2 + 2.188 * \theta_1 * \theta_3 + 2.071 * \theta_1 * \theta_4 - 0.849 * \theta_1 * \theta_5 + 4.869 * \theta_2 * \theta_3 + 3.032 * \theta_2 * \theta_4 + 3.981 * \theta_2 * \theta_5 + 2.634 * \theta_3 * \theta_4 + (-0.106) * \theta_3 * \theta_5 + (-1.085) * \theta_4 * \theta_5) - 0.5 * (0.449 * \theta_1 + 0.141 * \theta_2 + 0.956 * \theta_3 + 0.153 * \theta_4 + 0.765 * \theta_5) + \lambda_1 * \left(1 - (\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5)\right)$$

Differentiating the function, we formulate the system of equations:

$$\begin{cases} 4{,}463 * \theta_1 + 0{,}929 * \theta_2 + 1{,}094 * \theta_3 + 1{,}036 * \theta_4 - 0{,}425 * \theta_5 - \lambda_1 - 0{,}225 = 0 \\ 0{,}929 * \theta_1 + 5{,}316 * \theta_2 + 2{,}435 * \theta_3 + 1{,}516 * \theta_4 + 1{,}991 * \theta_5 - \lambda_1 - 0{,}071 = 0 \\ 1{,}094 * \theta_1 + 2{,}435 * \theta_2 + 9{,}896 * \theta_3 + 1{,}317 * \theta_4 - 0.053 * \theta_5 - \lambda_1 - 0{,}478 = 0 \\ 1{,}036 * \theta_1 + 1{,}516 * \theta_2 + 1{,}317 * \theta_3 + 5{,}495 * \theta_4 - 0{,}542 * \theta_5 - \lambda_1 - 0{,}077 = 0 \\ -0{,}425 * \theta_1 + 1{,}991 * \theta_2 - 0.053 * \theta_3 - 0{,}542 * \theta_4 + 9{,}078 * \theta_5 - \lambda_1 - 0{,}383 = 0 \\ 1 - (\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5) = 0 \end{cases}$$

Solving the system of equations by the inverse matrix method, we finally obtain:
$\theta^{(1)}$=(0.3475, 0.0431, 0.1368, 0.2278, 0.2448), λ_1=1.6477. This point satisfies all conditions.The function value:$F(\theta)$=0.6951

$$\sum_{i=1}^{5} \theta_i r_i = 0.5154$$

In the portfolio, the decision maker will obtain the following combination of shares:
$\theta_1 = 34{,}75$
$\theta_2 = 4{,}31\%$
$\theta_3 = 13{,}68\%$
$\theta_4 = 22{,}78\%$
$\theta_4 = 24{,}48\%$

And the profitability of the entire portfolio$r_p = 51.54\%$

Compared with the previous decision, given the subjective decisions of the decision maker, you can specify other combinations of the criteria and, solving the problems, we get the following results:
If$\propto_1 = 0{,}75$ $\propto_2 = 0{,}25$then:
$\theta^{(1)}$=(0.3424, 0.0795, 0.1098, 0.2421, 0.2262), λ_1=2.7028.This point satisfies all conditions. The function value:$F(\theta)$=1.4117

$$\sum_{i=1}^{5} \theta_i r_i = 0.4803$$

In the portfolio, the decision maker will obtain the following combination of shares:
$\theta_1 = 34.24\%$
$\theta_2 = 7.95\%$
$\theta_3 = 10.98\%$
$\theta_4 = 24.21\%$
$\theta_4 = 22.62\%$

And the profitability of the entire portfolio$r_p = 48.03\%$

Let's examine the situation with$\propto_1 = 0{,}4$ $\propto_2 = 0{,}6$. Then we will obtain the following result:
$\theta^{(1)}$=(0.3512, 0.01601, 0.157, 0.217, 0.2588), λ_1=1.2257. This point satisfies all conditions. The function value:$F(\theta)$=0,0.4505

$$\sum_{i=1}^{5} \theta_i r_i = 0.5416$$

In the portfolio, the decision maker will obtain the following combination of shares:
$\theta_1 = 35{,}12\%$
$\theta_2 = 1{,}60\%$
$\theta_3 = 15.70\%$
$\theta_4 = 21{,}70\%$
$\theta_4 = 25{,}88\%$

And the profitability of the entire portfolio$r_p = 54.16\%$

Summary

As can be seen from the three cases when the decision maker makes a rational decision (i.e., gives more preferring to risk criteria), the portfolio returns less. With aggressive choice (preference for high yield), the profitability of the entire portfolio is increased.

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