International Journal of Latest Engineering and Management Research (IJLEMR)
ISSN: 2455-4847
www.ijlemr.com || Volume 03 - Issue 04 || April 2018 || PP. 26-34

Management of Household Solid Waste to Control Environmental Pollution

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Abstract: This paper discusses a mathematical model of household solid waste consisting of garbage and rubbish to study the impact of environmental pollutants. Household solid waste is collected to storage where they are categorised into hazardous household solid waste and non-hazardous household solid waste. Appropriate action is given to hazardous and non-hazardous household solid waste in terms of treatment and composting, respectively. All these factors increase the environmental pollutants resulting an environmental pollution. We have developed a system of non-linear differential equations to control the level of environmental pollution using stability analysis and numerical simulation. Threshold is computed using validated data for the transmission of the household solid waste.

Keywords: Household solid waste, Environmental pollutants, System of non-linear differential equations, Threshold, Local stability, Global stability

1. Introduction

Solid waste is the branch of garbage discarded as unwanted or useless from animals and human activities. Solid waste is arising from industries, households and commercial activities. In this paper, we will be focusing on environmental pollution caused by household activities. Any household activities create solid waste called household solid waste. In simple words, household solid waste is the solid waste generated only from private homes or apartments as in bottles, clothes, food packages, newspapers, magazines, batteries, e-waste, etc. Household solid waste may injurious to humans' daily life if it is storedeven for few hours. This may blowout many diseases. These wastes are collected to storage area. Storage is the place where waste is get separated by their categories like hazardous household solid waste and non-hazardous household solid waste. Hazardous household solid waste is the solid waste originated during household activities which contains contaminated rudiments. Other are called nom-hazardous household solid waste. Both may directly harmful to the environment. Thus, treatment plant and compost plant are constructed. In the treatment plant, treatment is given to hazardous waste and in the compost plant, composting is provided to non-hazardous waste. Thus, plants and storage also may create environmental pollutants. Hence, increase in household solid waste causes environmental pollution.

Some researchers have studies the household waste management for the betterment of environment. Chongwoo and Iain surveyed an economic analysis of household waste managementin 1999. Asa *et al.* calculated life cycle assessment of energy from solid waste – part 2: landfilling compared to other treatment methods in 2005. Some researchers formulated model on household waste. In 2005, Brian and Chang formed forecasting municipal solid waste generation in a fast-growing urban region with system dynamics modeling. In 2015, mathematical modeling of household wastewater treatment was expressed by duckweed batch reactor by Udaya and Achyuth. In 2008, Sara *et al.*settled mathematical modeling to predict residential solid waste generation. Some researchers have constructed mathematical models on environment to control natural difficulties raised by human activities. In 2017, Nita *et al.* have consideredoptimum control for spread of pollutants through forest resources and also formulated optimal control on depletion of green belt due to industries.

In this paper, parametric notation along with its values and a system of non-linear differential equationsare formulated in section 2. In section 3, stability is defined. Numerical simulation is calculated using validated data in section 4.

2. Mathematical Modeling

Household activities include disposing of garbage, cleaning, dusting and vacuuming which all generate waste. Household solid waste is kept in area known as storage where it declared as either hazardous or non-hazardous. Different plants are built for the betterment of these waste. But then again, this chain creates environment pollution. Therefore, seven different compartments are considered here to study the effect of household solid waste on environment pollutants viz. the tons of household solid waste (H_W), the tons of storage of household solid waste (S), the tons of hazardous household solid waste (H_Z), the tons of non-hazardous household solid waste (N_Z), the treatment plant (T), the compost plant (C) and the environmental pollutants (E_P).

The notation along with its parametric values of each parameters used in household solid waste model is given in the table 1.

Table 1: Notations and its parametric values

B	:	Recruitment rate of household solid waste	0.60
β	:	The rate at which household solid waste is stored	0.72
δ_1	:	The rate of hazardous household solid waste kept in storage	0.30
δ_2	:	The rate of non-hazardous household solid waste kept in storage	0.60
ε_1	:	The rate at which hazardous household solid waste goes to treatment plant	0.55
ε_2	:	The rate at which hazardous household solid waste creates environmental pollutants	0.25
η_1	:	The rate at which non-hazardous household solid waste goes to compost plant	0.80
η_2	:	The rate at which non-hazardous household solid waste creates environmental pollutants	0.40
α_1	:	The rate of environmental pollutants caused by storage	0.10
α_2	:	The rate of environmental pollutants caused by treatment plant	0.30
α_3	:	The rate of environmental pollutants caused by compost plant	0.25
μ	:	Reduction rate of household solid waste from each compartment	0.30

The mathematical model is constructed via above parameters and some necessary assumptions. The transmission diagram of proposed model is as figure 1.

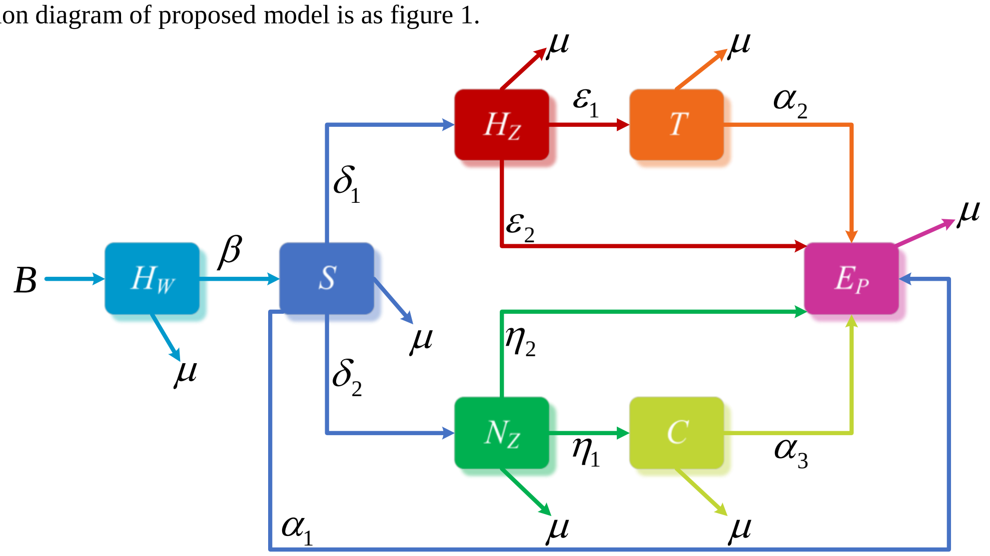

Figure 1: Transmission of household solid waste

The system of non-linear differential equation of transmission of household solid waste is specified below:

$$\frac{dH_W}{dt} = B - \beta H_W S - \mu H_W$$

$$\frac{dS}{dt} = \beta H_W S - \delta_1 S - \delta_2 S - \alpha_1 S E_P - \mu S$$

$$\frac{dH_Z}{dt} = \delta_1 S - \varepsilon_1 H_Z - \varepsilon_2 H_Z - \mu H_Z$$

$$\frac{dN_Z}{dt} = \delta_2 S - \eta_1 N_Z - \eta_2 N_Z - \mu N_Z \qquad (1)$$

$$\frac{dT}{dt} = \varepsilon_1 H_Z - \alpha_2 T E_P - \mu T$$

$$\frac{dC}{dt} = \eta_1 N_Z - \alpha_3 C E_P - \mu C$$

$$\frac{dE_P}{dt} = \varepsilon_2 H_Z + \eta_2 N_Z + \alpha_1 S E_P + \alpha_2 T E_P + \alpha_3 C E_P - \mu E_P$$

where $H_W + S + N_Z + N_Z + T + C + E_P \le N$ and $H_W > 0; S, H_Z, N_Z, T, C, E_P \ge 0$

Adding above differential equations of system (1), we get

$$\frac{d}{dt}\left(H_W + S + H_Z + N_Z + T + C + E_P\right) = B - \mu\left(H_W + S + H_Z + N_Z + T + C + E_P\right) \ge 0$$

$$\Rightarrow \limsup_{t \to \infty}\left(H_W + S + H_Z + N_Z + T + C + E_P\right) \le \frac{B}{\mu}$$

Therefore, the feasible region of the model is

$$\Lambda = \left\{\left(H_W, S, H_Z, N_Z, T, C, E_P\right) \in \mathbb{R}^7 : H_W + S + H_Z + N_Z + T + C + E_P \le \frac{B}{\mu}\right\}.$$

Now, replacing $\delta_1 + \delta_2 + \mu = q_1, \varepsilon_1 + \varepsilon_2 + \mu = q_2$ and $\eta_1 + \eta_2 + \mu = q_3$ in the system (1) then we get the following new system of non-linear differential equations:

$$\frac{dH_W}{dt} = B - \beta H_W S - \mu H_W$$

$$\frac{dS}{dt} = \beta H_W S - q_1 S - \alpha_1 S E_P$$

$$\frac{dH_Z}{dt} = \delta_1 S - q_2 H_Z$$

$$\frac{dN_Z}{dt} = \delta_2 S - q_3 N_Z \qquad (2)$$

$$\frac{dT}{dt} = \varepsilon_1 H_Z - \alpha_2 T E_P - \mu T$$

$$\frac{dC}{dt} = \eta_1 N_Z - \alpha_3 C E_P - \mu C$$

$$\frac{dE_P}{dt} = \varepsilon_2 H_Z + \eta_2 N_Z + \alpha_1 S E_P + \alpha_2 T E_P + \alpha_3 C E_P - \mu E_P$$

The dynamical behaviour of system (1) is equivalent to the system (2). Therefore, in the rest of the paper, we will study the system (2) in the feasible region noted as Λ which can be shown to be an invariant set for the system (2).

Now, the equilibrium point of the household solid waste model is $E^*\left(H_W^*, S^*, H_Z^*, N_Z^*, T^*, C^*, L_f^*, E_P^*\right)$:

$$E_P^* = r, H_W^* = \frac{1}{\beta}\left(q_1 + \alpha_1 r\right), S^* = \frac{B\beta - q_1\mu - \alpha_1 r\mu}{\left(q_1 + \alpha_1 r\right)\beta}, H_Z^* = \frac{\delta_1\left(B\beta - q_1\mu - \alpha_1 r\mu\right)}{q_2\left(q_1 + \alpha_1 r\right)\beta}$$

$$N_Z^* = \frac{\delta_2\left(B\beta - q_1\mu - \alpha_1 r\mu\right)}{q_3\left(q_1 + \alpha_1 r\right)\beta}, T^* = \frac{\delta_1\varepsilon_1\left(B\beta - q_1\mu - \alpha_1 r\mu\right)}{q_2\left(q_1 + \alpha_1 r\right)\left(\alpha_2 r + \mu\right)\beta}, C^* = \frac{\delta_2\eta_1\left(B\beta - q_1\mu - \alpha_1 r\mu\right)}{q_3\left(q_1 + \alpha_1 r\right)\left(\alpha_3 r + \mu\right)\beta}$$

where

$$\begin{aligned} r = Root\,of\, \Big(& \left(q_2 q_3 \alpha_1 \alpha_2 \alpha_3 \beta\mu + q_2 q_3 \alpha_1^2 \alpha_2 \alpha_3 \mu\right) Z^4 + \Big(q_1 q_2 q_3 \alpha_2 \alpha_3 \mu + q_1 q_2 q_3 \alpha_1 \alpha_2 \alpha_3 \mu + q_2 \alpha_1 \alpha_2 \alpha_3 \delta_2 \eta_1 \mu + q_2 q_3 \alpha_1 \beta \alpha_3 \mu^2 \\ & + q_2 q_3 \alpha_1^2 \alpha_2 \mu^2 + q_2 q_3 \alpha_1 \alpha_2 \beta\mu^2 + q_3 \alpha_1 \alpha_2 \alpha_3 \delta_1 \varepsilon_1 \mu + q_2 \alpha_1 \alpha_2 \alpha_3 \delta_2 \eta_2 \mu - q_2 q_3 \alpha_1 \alpha_2 \alpha_3 B\beta + q_3 \alpha_1 \alpha_2 \alpha_3 \delta_1 \varepsilon_2 \mu + q_2 q_3 \alpha_1^2 \alpha_3 \mu^2 \Big) Z^3 \\ & + q_3 \alpha_1 \alpha_3 \delta_1 \varepsilon_2 \mu^2 + q_1 q_2 q_3 \alpha_3 \beta\mu^2 - q_2 q_3 \alpha_1 \alpha_3 B\beta\mu + q_1 q_3 \alpha_2 \alpha_3 \delta_2 \varepsilon_1 \mu - q_2 \alpha_2 \alpha_3 B\beta\delta_2 \eta_1 + q_1 q_2 q_3 \alpha_1 \alpha_2 \mu^2 + q_2 \alpha_1 \alpha_2 \delta_2 \eta_2 \mu^2 \\ & - q_2 \alpha_2 \alpha_3 B\beta\delta_2 \eta_2 + q_1 q_2 q_3 \alpha_1 \alpha_3 \mu^2 + q_1 q_2 q_3 \alpha_2 \beta\mu^2 + q_1 q_2 \alpha_2 \alpha_3 \delta_2 \eta_1 \mu + q_2 q_3 \alpha_1^2 \mu^3 - q_3 \alpha_2 \alpha_3 B\beta\delta_1 \varepsilon_1 + q_2 \alpha_1 \alpha_3 \delta_2 \eta_2 \mu^2 \\ & - q_3 \alpha_2 \alpha_3 B\beta\delta_1 \varepsilon_2 + q_3 \alpha_1 \alpha_2 \delta_1 \varepsilon_2 \mu^2 - q_2 q_3 \alpha_1 \alpha_2 B\beta\mu + q_2 \alpha_1 \alpha_3 \delta_2 \eta_1 \mu^2 + q_2 q_3 \alpha_1 \beta\mu^3 + q_1 q_3 \alpha_2 \alpha_3 \delta_1 \varepsilon_2 \mu + q_3 \alpha_1 \alpha_2 \delta_1 \varepsilon_1 \mu^2 \\ & + q_1 q_2 \alpha_2 \alpha_3 \delta_2 \eta_2 \mu\Big) Z^2 + \Big(-q_2 \alpha_3 B\beta\delta_2 \eta_2 \mu + q_3 \alpha_1 \delta_1 \varepsilon_2 \mu^3 - q_3 \alpha_2 B\beta\delta_1 \varepsilon_1 \mu - q_2 \alpha_2 B\beta\delta_2 \eta_2 \mu + q_1 q_3 \alpha_3 \delta_1 \varepsilon_2 \mu^2 + q_1 q_3 \alpha_2 \delta_1 \varepsilon_2 \mu^2 \\ & - q_3 \alpha_3 B\beta\delta_1 \varepsilon_2 \mu - q_3 \alpha_2 B\beta\delta_1 \varepsilon_2 \mu + q_1 q_2 \alpha_3 \delta_2 \eta_2 \mu^2 + q_1 q_2 q_3 \alpha_1 \mu^3 + q_1 q_2 q_3 \beta\mu^3 + q_1 q_2 \alpha_3 \delta_2 \eta_1 \mu^2 - q_2 \alpha_3 B\beta\delta_2 \eta_1 \mu + q_2 \alpha_1 \delta_2 \eta_2 \mu^3 \\ & + q_1 q_2 \alpha_2 \delta_2 \eta_2 \mu^2 + q_1 q_3 \alpha_2 \delta_1 \varepsilon_1 \mu^3 - q_2 q_3 \alpha_1 B\beta\mu^2\Big) Z + q_1 q_3 \delta_1 \varepsilon_2 \mu^3 + q_1 q_2 \delta_2 \eta_2 \mu^3 - q_3 B\beta\delta_1 \varepsilon_2 \mu^2 - q_2 B\beta\delta_2 \eta_2 \mu^2 \Big) \end{aligned} \qquad (3)$$

Now, we derive the basic reproduction number R_0 by the method of next generation matrix.

Let us take $X' = \left(H_W, S, H_Z, N_Z, T, C, E_P\right)'$, where dash denotes derivative. So,

$$X' = \frac{dX}{dt} = f\left(X\right) - v\left(X\right)$$

where $f\left(X\right)$ is the rate of appearance of new individual in component and $v\left(X\right)$ is the rate of transfer of household solid waste. They are given by

$$f = \begin{bmatrix} \beta H_W S \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_1 S E_P + \alpha_2 T E_P + \alpha_3 C E_P \\ 0 \end{bmatrix} \text{ and } v = \begin{bmatrix} q_1 S + \alpha_1 S E_P \\ -\delta_1 S + q_2 H_Z \\ -\delta_2 S + q_3 N_Z \\ -\varepsilon_1 H_Z + \alpha_2 T E_P + \mu T \\ -\eta_1 N_Z + \alpha_3 C E_P + \mu C \\ -\varepsilon_2 H_Z - \eta_2 N_Z + \mu E_P \\ -B + \beta H_W S + \mu H_W \end{bmatrix}$$

Now, $Df\left(E^*\right) = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix}$ and $Dv\left(E^*\right) = \begin{bmatrix} V & 0 \\ J_1 & J_2 \end{bmatrix}$

where F and V are 7×7 matrices defined as

$$F = \left[\frac{\partial f_i\left(E^*\right)}{\partial X_j}\right] \text{ and } V = \left[\frac{\partial v_i\left(E^*\right)}{\partial X_j}\right].$$

Finding F and V , we get

$$F = \begin{bmatrix} \beta H_W^* & 0 & 0 & 0 & 0 & 0 & \beta S^* \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_1 E_P^* & 0 & 0 & \alpha_2 E_P^* & \alpha_3 E_P^* & \alpha_1 S^* + \alpha_2 T^* + \alpha_3 C^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$V = \begin{bmatrix} q_1 + \alpha_1 E_P^* & 0 & 0 & 0 & 0 & \alpha_1 S^* & 0 \\ -\delta_1 & q_2 & 0 & 0 & 0 & 0 & 0 \\ -\delta_2 & 0 & q_3 & 0 & 0 & 0 & 0 \\ 0 & -\varepsilon_1 & 0 & \alpha_2 E_P^* + \mu & 0 & \alpha_2 T^* & 0 \\ 0 & 0 & -\eta_1 & 0 & \alpha_3 E_P^* + \mu & \alpha_3 C^* & 0 \\ 0 & -\varepsilon_2 & -\eta_2 & 0 & 0 & \mu & 0 \\ \beta H_W^* & 0 & 0 & 0 & 0 & 0 & \beta S^* + \mu \end{bmatrix}$$

Here, above matrix V is non-singular matrix.

Therefore, the expression of basic reproduction number R_0 is as following

$R_0 =$ spectral radius of matrix FV^{-1}

$$\Rightarrow R_0 = \frac{q_2 q_3 \beta \mu^2 H_W^*}{\left(q_1 q_2 q_3 \mu + q_2 q_3 \alpha_1 \mu E_P^* + q_3 \alpha_1 \delta_1 \varepsilon_2 S^* + q_2 \alpha_1 \delta_2 \eta_2 S^*\right)\left(\beta S^* + \mu\right)} \qquad (4)$$

In the next section, stability analysis of the household solid waste model will be conversed.

3. Stability Analysis

In the current section, the equilibrium of local stability and global stability is covered for the transmission household solid waste.

3.1. Local Stability

Here, we have projected the local stability of the household solid waste model at equilibrium point $E^*\left(H_W^*, S^*, H_Z^*, N_Z^*, T^*, C^*, L_f^*, E_P^*\right)$.

Now, the Jacobian matrix J at the equilibrium point E^* is given by

$$J = \begin{bmatrix} -\beta S^* - \mu & -\beta H_W^* & 0 & 0 & 0 & 0 & 0 \\ \beta S^* & \beta H_W^* - q_1 - \alpha_1 E_P^* & 0 & 0 & 0 & 0 & -\alpha_1 S^* \\ 0 & \delta_1 & -q_2 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & -q_3 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_1 & 0 & -\alpha_2 E_P^* - \mu & 0 & -\alpha_2 T^* \\ 0 & 0 & 0 & \eta_1 & 0 & -\alpha_3 E_P^* - \mu & -\alpha_3 C^* \\ 0 & \alpha_1 E_P^* & \varepsilon_2 & \eta_2 & \alpha_2 E_P^* & \alpha_3 E_P^* & \alpha_1 S^* + \alpha_2 T^* + \alpha_3 C^* - \mu \end{bmatrix}$$

Now, taking $a_{11} = \beta S^* + \mu, a_{22} = -\beta H_W^* + q_1 + \alpha_1 E_P^*, a_{55} = \alpha_2 E_P^* + \mu, a_{66} = \alpha_3 E_P^* + \gamma_2 + \mu$ and $a_{77} = -\alpha_1 S^* - \alpha_2 T^* - \alpha_3 C^* + \mu$, we get

$$J' = \begin{bmatrix} -a_{11} & -\beta H_W^* & 0 & 0 & 0 & 0 & 0 \\ \beta S^* & -a_{22} & 0 & 0 & 0 & 0 & -\alpha_1 S^* \\ 0 & \delta_1 & -q_2 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & -q_3 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_1 & 0 & -a_{55} & 0 & -\alpha_2 T^* \\ 0 & 0 & 0 & \eta_1 & 0 & -a_{66} & -\alpha_3 C^* \\ 0 & \alpha_1 E_P^* & \varepsilon_2 & \eta_2 & \alpha_2 E_P^* & \alpha_3 E_P^* & -a_{77} \end{bmatrix}$$

The characteristics polynomial of the Jacobian matrix J' at E^* is as below:

$$\lambda^7 + x_1\lambda^6 + x_2\lambda^5 + x_3\lambda^4 + x_4\lambda^3 + x_5\lambda^2 + x_6\lambda + x_7$$

where

$$x_1 = a_{77} + a_{66} + a_{55} + q_3 + q_2 + a_{22} + a_{11}$$

$$\begin{aligned} x_2 = {} & a_{77}\left(a_{66} + a_{55} + q_3 + q_2 + a_{22} + a_{11}\right) + a_{66}\left(a_{55} + q_3 + q_2 + a_{22} + a_{11}\right) + a_{55}\left(q_2 + q_2 + a_{22} + a_{11}\right) + q_3\left(q_2 + a_{22} + a_{11}\right) \\ & + q_2\left(a_{22} + a_{11}\right) + a_{22}\left(a_{11}\right) + \left(\alpha_1^2 S^* + \alpha_2^2 T^* + \alpha_3^2 C^*\right)E_P^* + \beta^2 S^* H_W^* \end{aligned}$$

$$\begin{aligned} x_3 = {} & a_{77}\left(a_{66}\left(a_{55} + q_3 + q_2 + a_{22} + a_{11}\right) + a_{55}\left(q_3 + q_2 + a_{22} + a_{11}\right) + q_3\left(q_2 + a_{22} + a_{11}\right) + q_2\left(a_{22} + a_{11}\right) + a_{22}\left(a_{11}\right)\right. \\ & \left. + \beta^2 S^* H_W^*\right) + a_{66}\left(a_{55}\left(q_3 + q_2 + a_{22} + a_{11}\right) + q_3\left(q_2 + a_{22} + a_{11}\right) + q_2\left(a_{22} + a_{11}\right) + a_{22}\left(a_{11}\right) + \left(\alpha_1^2 S^*\right.\right. \\ & \left.\left. + \alpha_2^2 T^*\right)E_P^* + \beta^2 S^* H_W^*\right) + a_{55}\left(q_3\left(q_2 + a_{22} + a_{11}\right) + q_2\left(a_{22} + a_{11}\right) + a_{22}\left(a_{11}\right) + \left(\alpha_1^2 S^* + \alpha_3^2 C^*\right)E_P^*\right. \\ & \left. + \beta^2 S^* H_W^*\right) + q_3\left(q_2\left(a_{22} + a_{11}\right) + a_{22}\left(a_{11}\right) + \left(\alpha_1^2 S^* + \alpha_2^2 T^* + \alpha_3^2 C^*\right)E_P^* + \beta^2 S^* H_W^*\right) + q_2\left(a_{22}\left(a_{11}\right) + \left(\alpha_1^2 S^*\right.\right. \\ & \left.\left. + \alpha_2^2 T^* + \alpha_3^2 C^*\right)E_P^* + \beta^2 S^* H_W^*\right) + a_{22}\left(\alpha_2^2 T^* + \alpha_3^2 C^*\right)E_P^* + a_{11}\left(\alpha_1^2 S^* + \alpha_2^2 T^* + \alpha_3^2 C^*\right)E_P^* + \alpha_1\left(\delta_1\varepsilon_2\right. \\ & \left. + \delta_2\eta_2\right)S^* \end{aligned}$$

$$x_4 = a_{77}\left(a_{66}\left(a_{55}\left(q_3+q_2+a_{22}+a_{11}\right)+q_3\left(q_2+a_{22}+a_{11}\right)+q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)+a_{55}\left(q_3\left(q_2+a_{22}\right.\right.\right.$$
$$\left.+a_{11}\right)+q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)+q_3\left(q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)+q_2\left(a_{22}\left(a_{11}\right)\right.$$
$$\left.\left.+\beta^2 S^* H_W^*\right)\right)+a_{66}\left(a_{55}\left(q_3\left(q_2+a_{22}+a_{11}\right)+q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\alpha_1^2 S^* E_P^*+\beta^2 S^* H_W^*\right)+q_3\left(q_2\left(a_{22}+a_{11}\right)\right.\right.$$
$$+a_{22}\left(a_{11}\right)+\left(\alpha_1^2 S^*+\alpha_2^2 T^*\right)E_P^*+\beta^2 S^* H_W^*\right)+q_2\left(a_{22}\left(a_{11}\right)+\left(\alpha_1^2 S^*+\alpha_2^2 T^*\right)E_P^*+\beta^2 S^* H_W^*\right)+a_{22}\alpha_2^2 T^* E_P^*$$
$$+a_{11}\left(\alpha_1^2 S^*\,\alpha_2^2 T^*\right)E_P^*+\alpha_1\left(\delta_1\varepsilon_2+\delta_2\eta_2\right)S^*\right)+a_{55}\left(q_3\left(q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\left(\alpha_1^2 S^*+\alpha_3^2 C^*\right)E_P^*\right.\right.$$
$$+\beta^2 S^* H_W^*\right)+q_2\left(a_{22}\left(a_{11}\right)+\left(\alpha_1^2 S^*+\alpha_3^2 C^*\right)E_P^*+\beta^2 S^* H_W^*\right)+a_{22}\alpha_3^2 C^* E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_3^2 C^*\right)E_P^*+\alpha_1\left(\delta_1\varepsilon_2\right.$$
$$\left.+\delta_2\eta_2\right)S^*\right)+q_3\left(q_2\left(a_{22}\left(a_{11}\right)+\left(\alpha_1^2 S^*+\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+\beta^2 S^* H_W^*\right)+a_{22}\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+a_{11}\left(\alpha_1^2 S^*\right.\right.$$
$$\left.+\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+\alpha_1\delta_2\eta_2 S^*\right)+q_2\left(a_{22}\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+\alpha_1\delta_2\eta_2 S^*\right)$$
$$+a_{22}a_{11}\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+a_{11}\alpha_1\left(\delta_1\varepsilon_2+\delta_2\eta_2\right)S^*+\left(\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)\beta^2 S^* H_W^*+\alpha_1\left(\alpha_2\delta_1\varepsilon_1+\alpha_3\delta_2\eta_1\right)S^*\right)E_P^*$$

$$x_5 = a_{77}\left(a_{66}\left(a_{55}\left(q_3\left(q_2+a_{22}+a_{11}\right)+q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)+q_3\left(q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)\right.\right.$$
$$+q_2\left(a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)\right)++a_{55}\left(q_3\left(q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)+q_2\left(a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)\right)$$
$$+q_3 q_2\left(a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)\right)+a_{66}\left(a_{55}\left(q_3\left(q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\alpha_1^2 S^* E_P^*+\beta^2 S^* H_W^*\right)+q_2\left(a_{22}+\left(a_{11}\right)\right.\right.\right.$$
$$\left.+\alpha_1^2 S^* E_P^*+\beta^2 S^* H_W^*\right)+a_{11}\alpha_1^2 S^* E_P^*+\alpha_1\left(\delta_1\varepsilon_2+\delta_2\eta_2\right)S^*\right)+q_3\left(q_2\left(a_{22}\left(a_{11}\right)+\left(\alpha_1^2 S^*+\alpha_2^2 T^*\right)E_P^*\right.\right.$$
$$\left.+\beta^2 S^* H_W^*\right)+a_{22}\alpha_2^2 T^* E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_2^2 T^*\right)E_P^*+\alpha_1\delta_1\varepsilon_2 S^*\right)+q_2\left(a_{22}\alpha_2^2 T^* E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_2^2 T^*\right)E_P^*\right.$$
$$\left.+\alpha_1\delta_2\eta_2 S^*\right)+a_{22}a_{11}\alpha_2^2 T^* E_P^*+a_{11}\alpha_1\left(\delta_1\varepsilon_2+\delta_2\eta_2\right)S^*+\left(\alpha_2^2 T^*\beta^2 S^* H_W^*+\alpha_1\alpha_2\delta_1\varepsilon_1 S^*\right)E_P^*\right)$$
$$+a_{55}\left(q_3\left(q_2\left(a_{22}\left(a_{11}\right)+\left(\alpha_1^2 S^*+\alpha_3^2 C^*\right)E_P^*+\beta^2 S^* H_W^*\right)+a_{22}\alpha_3^2 C^* E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_3^2 C^*\right)E_P^*+\alpha_1\delta_1\varepsilon_2 S^*\right)\right.$$
$$+A_2\left(a_{22}\alpha_3^2 C^* E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_3^2 C^*\right)E_P^*+\alpha_1\delta_2\eta_2 S^*\right)+a_{22}a_{11}\alpha_3^2 C^* E_P^*+a_{11}\alpha_1\left(\delta_1\varepsilon_2+\delta_2\eta_2\right)S^*\right)$$
$$+\left(\alpha_3^2 C^*\beta^2 S^* H_W^*+\alpha_1\alpha_3\delta_2\eta_1 S^*\right)E_P^*\right)+q_3\left(q_2\left(a_{22}\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*\right)\right.$$
$$+a_{22}a_{11}\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+a_{11}\alpha_1\delta_1\varepsilon_2 S^*+\left(\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)\beta^2 S^* H_W^*+\alpha_1\alpha_2\delta_1\varepsilon_1 S^*\right)E_P^*\right)$$
$$+q_2\left(a_{22}a_{11}\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+a_{11}\alpha_1\delta_2\eta_2 S^*+\left(\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)+\alpha_3^2 C^*\right)\beta^2 S^* H_W^*+\alpha_1\alpha_3\delta_2\eta_1 S^*\right)E_P^*\right)$$
$$+a_{11}\alpha_1\left(\alpha_2\delta_1\varepsilon_1+\alpha_3\delta_2\eta_1\right)S^* E_P^*$$

$$x_6 = a_{77}\left(a_{66}\left(a_{55}\left(q_3\left(q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)+q_2\left(a_{22}+a_{11}\right)+a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)+q_2\left(a_{22}\left(a_{11}\right)\right.\right.\right.$$
$$\left.\left.+\beta^2 S^* H_W^*\right)\right)+q_3 q_2\left(a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)\right)+a_{55}q_3 q_2\left(a_{22}\left(a_{11}\right)+\beta^2 S^* H_W^*\right)\right)+a_{66}\left(a_{55}\left(q_3\left(q_2\left(a_{22}\left(a_{11}\right)\right.\right.\right.\right.$$
$$\left.+\alpha_1^2 S^* E_P^*+\beta^2 S^* H_W^*\right)+a_{11}\alpha_1^2 S^* E_P^*+\alpha_1\delta_1\varepsilon_2 S^*\right)+q_2\left(a_{11}\alpha_1^2 S^* E_P^*+\alpha_1\delta_2\eta_2 S^*\right)+a_{11}\alpha_1\left(\delta_1\varepsilon_2+\delta_2\eta_2\right)S^*\right)$$
$$+q_3\left(q_2\left(a_{22}\alpha_2^2 T^* E_P^*+a_{11}\left(\alpha_1^2 S^*+\alpha_2^2 T^*\right)E_P^*\right)+a_{22}a_{11}\alpha_2^2 T^* E_P^*+a_{11}\alpha_1\delta_1\varepsilon_2 S^*+\left(\alpha_2^2 T^*\beta^2 S^* H_W^*\right.\right.$$
$$\left.\left.+\alpha_1\alpha_2\delta_1\varepsilon_1 S^*\right)E_P^*\right)+q_2\left(a_{22}a_{11}\alpha_2^2 T^* E_P^*+a_{11}\alpha_1\delta_2\eta_2 S^*+\alpha_2^2 T^*\beta^2 S^* H_W^* E_P^*\right)+a_{11}\alpha_1\alpha_2\delta_1\varepsilon_1 S^* E_P^*\right)$$
$$+a_{55}\left(q_3\left(q_2\left(a_{22}\alpha_3^2 C^* E_p^*+a_{11}\left(\alpha_1^2 S^*+\alpha_3^2 C^*\right)E_P^*\right)+a_{22}a_{11}\alpha_3^2 C^* E_P^*+a_{11}\alpha_1\delta_1\varepsilon_2 S^*\right.\right.$$
$$\left.+\alpha_3^2 C^*\beta^2 S^* H_W^* E_P^*\right)+a_{11}\alpha_1\delta_2\eta_2 S^*+\left(\alpha_3^2 C^*\beta^2 S^* H_W^*+\alpha_1\alpha_3\delta_2\eta_1 S^*\right)E_P^*\right)+a_{11}\alpha_1\alpha_3\delta_2\eta_1 S^* E_P^*$$
$$+q_3\left(q_2\left(a_{22}a_{11}\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)E_P^*+\left(\alpha_2^2 T^*+\alpha_3^2 C^*\right)\beta^2 S^* H_W^* E_P^*\right)+a_{11}\alpha_1\alpha_2\delta_1\varepsilon_1 S^* E_P^*\right)$$
$$+q_2 a_{11}\alpha_1\alpha_3\delta_2\eta_1 S^* E_P^*$$

$$x_7 = a_{77}a_{66}a_{55}q_3 q_2\left(a_{22}\left(a_{11}\right)+\beta^2 S^* H_w^*\right)+a_{66}\left(a_{55}\left(q_3\left(a_{11}\alpha_1^2 S^* E_P^*+a_{11}\alpha_1\delta_1\varepsilon_2 S^*\right)+q_2 a_{11}\alpha_1\delta_2\eta_2 S^*\right)\right.$$
$$+q_3\left(q_2\left(a_{22}a_{11}\alpha_2^2 T^* E_P^*+\alpha_2^2 T^*\beta^2 S^* H_W^* E_P^*\right)+a_{11}\alpha_1\alpha_2\delta_1\varepsilon_1 S^* E_P^*\right)\right)+a_{55}\left(q_3 q_2\left(a_{22}a_{11}\alpha_3^2 C^* E_P^*\right.\right.$$
$$\left.+\alpha_3^2 C^*\beta^2 S^* H_W^* E_P^*\right)+q_2 a_{11}\alpha_1\alpha_3\delta_2\eta_1 S^* E_P^*\right)$$

Here, all the coefficients are positive and satisfy the condition of Routh-Hurwitz criterion (Routh E.J. 1877).

Theorem 1: The unique positive equilibrium point E^* is locally asymptotically stable with the condition that $a_{77} > 0$ if and only if $\mu > \alpha_1 S^* + \alpha_2 T^* + \alpha_3 C^*$.

3.2. Global Stability

Here, we have studied the global stability of the household solid waste model at equilibrium point $E^*\left(H_W^*, S^*, H_Z^*, N_Z^*, T^*, C^*, L_f^*, E_P^*\right)$.

Consider the Lyapunov function

$$L(t) = \frac{1}{2}\left[\left(H_W - H_W^*\right) + \left(S - S^*\right) + \left(H_Z - H_Z^*\right) + \left(N_Z - N_Z^*\right) + \left(T - T^*\right) + \left(C - C^*\right) + \left(E_P - E_P^*\right)\right]^2$$

$$\begin{aligned} L'(t) &= \left[\left(H_W - H_W^*\right) + \left(S - S^*\right) + \left(H_Z - H_Z^*\right) + \left(N_Z - N_Z^*\right) + \left(T - T^*\right) + \left(C - C^*\right) + \left(E_P - E_P^*\right)\right] \\ &\quad \left[H_W' + S' + H_Z' + N_Z' + T' + C' + L_f' + E_P'\right] \\ &= \left[\left(H_W - H_W^*\right) + \left(S - S^*\right) + \left(H_Z - H_Z^*\right) + \left(N_Z - N_Z^*\right) + \left(T - T^*\right) + \left(C - C^*\right) + \left(E_P - E_P^*\right)\right] \\ &\quad \left[B - \mu H_W - \mu S - \mu H_Z - \mu N_Z - \mu T - \mu C - \mu L_f - \mu E_P\right] \\ &= \left[\left(H_W - H_W^*\right) + \left(S - S^*\right) + \left(H_Z - H_Z^*\right) + \left(N_Z - N_Z^*\right) + \left(T - T^*\right) + \left(C - C^*\right) + \left(E_P - E_P^*\right)\right] \\ &\quad \left[\mu H_W^* + \mu S^* + \mu H_Z^* + \mu N_Z^* + \mu T^* + \mu C^* + \mu E_P^* - \mu H_W - \mu S - \mu H_Z - \mu N_Z - \mu T - \mu C - \mu E_P\right] \\ &= -\mu\left[\left(H_W - H_W^*\right) + \left(S - S^*\right) + \left(H_Z - H_Z^*\right) + \left(N_Z - N_Z^*\right) + \left(T - T^*\right) + \left(C - C^*\right) + \left(E_P - E_P^*\right)\right]^2 \le 0 \end{aligned}$$

Where $B = \mu H_W^* + \mu S^* + \mu H_Z^* + \mu N_Z^* + \mu T^* + \mu C^* + \mu L_f^* + \mu E_P^*$.

Theorem 2: The unique positive equilibrium point E^* is globally asymptotically stable.
Next, to support the analytical result we have considered the optimal control numerically.

4. Numerical Simulation

In this section, we analyse the numerical simulation using the parametric values mentioned in Table 1.

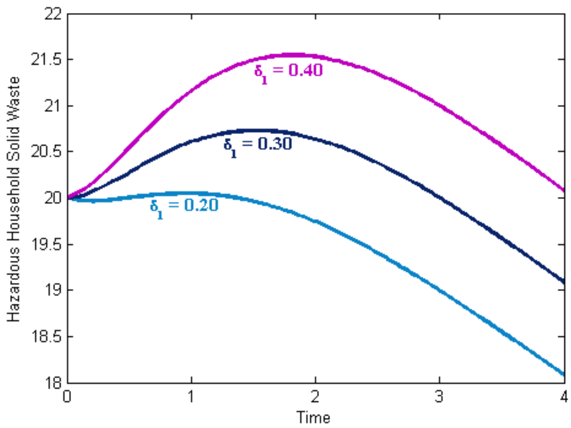

Figure 2: The effect of the rate of hazardous household solid waste kept in storage

This figure 2 represents the effect of different rates of δ_1 on hazardous household solid waste. Here, if δ_1 is varied from 20% to 40% then hazardous household solid waste increases with the time which indicates that if one will increase the rate of hazardous household solid waste kept in storage by 20% then environment pollution grows approximately by 11%.

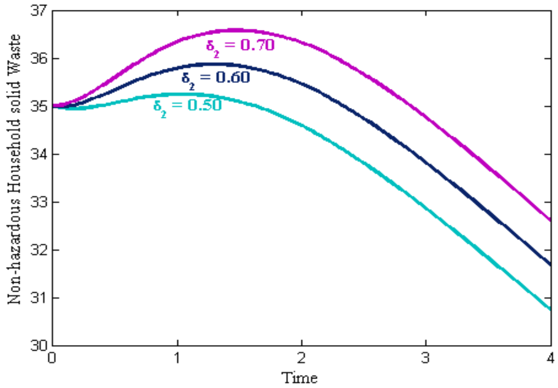

Figure 3: The effect of the rate of non-hazardous household solid waste kept in storage

From figure 3, it can be analysed that with the rate of δ_2, non-hazardous household solid waste is increasing. If one will increase the rate of non-hazardous household solid waste kept in storage by 20% then environmental pollution grows by 6%.
From figure 2 and 3, one can say that hazardous household solid waste is dangerous for environmental pollution in compared to non-hazardous household solid waste.

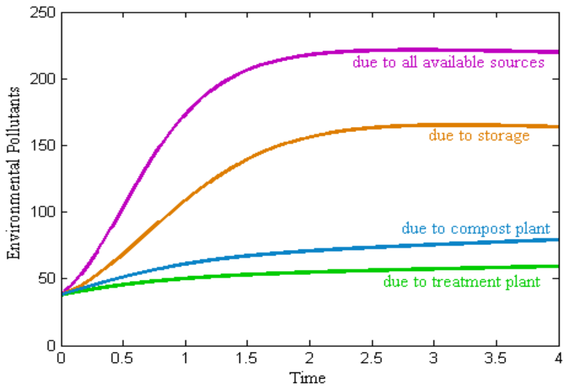

Figure 4: The effect of garbage related to different sources on environmental pollutants

Above figure 4 shows the effect of garbage related to different sources on environmental pollutants. It can be seen that environmental pollutants are increasing much more due to all sources viz. storage, treatment plant and compost plant. On the other hand, due to only storage, environmental pollutants rise approximately by 220 ppm with the time. Similarly, for the treatment plant and compost plant, environment pollutants are increasing by 60 ppm and 80 ppm, respectively which indicates that garbage should not be stored for a longer time; and in subject to revive environmental pollution, treatment plant and compost plant should be constructed.

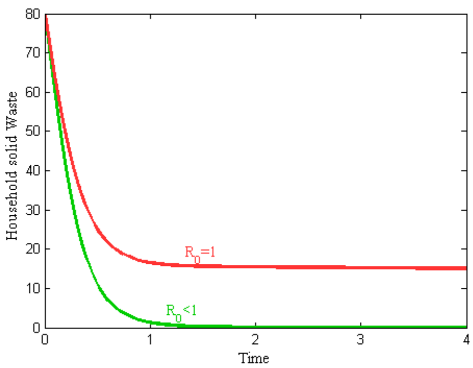

Figure 5 (a): Transmission of household solid waste with R_0

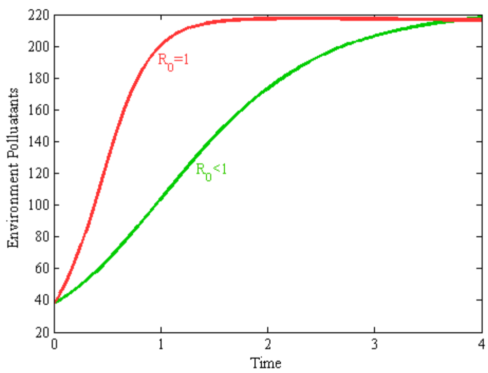

Figure 5 (b): Transmission of environmental pollutants with R_0

Figure 5 (a) and 5 (b) indicates the case of R_0 for the behaviour of household solid waste and environmental pollutants, respectively. If the threshold achieves the value one, the household solid waste and environmental pollutants increases with time. This advocates that if 15% household solid waste is added then environmental pollution will be 89%.

5. Conclusion

In the proposed paper, a mathematical model of household solid waste is formulated to study the transmission of household solid waste. Using parametric values given in the table 1, we concluded that 83% pollution exists due to household solid waste. The aim of the preparing model is to control environmental pollutants. Pollutants may revive when household waste kept in appropriate containers. For this, household solid waste can be divided into several class:

i. Vegetable waste like fruit and vegetable peelings and scraps.
ii. Metal waste like tins, glass, bottles and vessels.
iii. Paper waste like newspapers, magazine and more.
iv. Hazardous waste like batteries,plastics, old medicines, used motor oils, used kerosene and fuel.

Keeping waste in different containers will be good for human health. These containers always should be a with a tight-fitting closure. Container should be get emptied regularly and do not let overflowed. When container get emptied, it should be cleaned using soap and water.

Acknowledgement

The authors thank DST-FIST file # MSI-097 for technical support to the department.

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