*International Journal of Latest Engineering and Management Research (IJLEMR)*
*ISSN: 2455-4847*
*www.ijlemr.com || Volume 02 - Issue 09 || September 2017 || PP. 36-40*

# Quasi-Static Approach of A Transient Heat Conduction Problem of Semi-Infinite Solid Elliptical Cylinder And Its Thermal Deflection

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**Abstract:** This paper deals with a transient heat conduction problem and determination of quasi-static thermal deflection of a semi-infinite solid elliptical cylinder subjected to arbitrary initial heat supply on the lower surface with the curved surface having zero heat flux. The numerical calculations have been carried out for a copper cylinder and illustrated graphically.
**Keywords:** Elliptical plate, Heat conduction, Thermal deflection, Thermal stresses.

## I. INTRODUCTION

The solution of heat conduction problem gives the possible measures and thermoelastic behavior of the metallic bodies of any shape. Fatigue cracks have been one of the main sources of structural failures in machines for two centuries. The application of fracture mechanics to engineering design has led to more efficient use of structures and components, which leads to great economic benefits by avoiding premature retirement of serviceable machines. However, there are still some aspects of fatigue that remain partially understood, such as the crack closure effect. This lack of understanding arises principally from the difficulties associated in quantifying the phenomenon and measuring its effect on the crack driving force [1].

Unfortunately, there are only few studies concerned with steady and transient state heat conduction problems in elliptical objects. McLachlan [8,9] obtained mathematical solution of the heat conduction problem for elliptical cylinder in the form of an infinite Mathieu function series considering special case with neglecting surface resistance. Choubey [2] also introduced a finite Mathieu transform whose kernel is given by Mathieu function to solve heat conduction in a hollow elliptic cylinder with radiation. Sugano et al. [3] dealt with transient thermal stress in a confocal hollow elliptical structures with both face Transient Thermoelastic Problem in a Confocal Elliptical Disc with Internal Heat Sources surfaces insulated perfectly and obtained the analytical solution with couple-stresses. Sato [4] subsequently obtained heat conduction problem of an infinite elliptical cylinder during heating and cooling considering the effect of the surface resistance. However, there aren't many investigations done or studied to successfully eliminate thermoelastic problems Dhaba[5] studied a problem of plane, uncoupled linear thermoelasticity for an infinite, elliptical cylinder by a boundary integral method

## II. FORMULATION OF THE PROBLEM

Consider a semi-infinite solid elliptical cylinder occupying the space with radius $\xi = \xi_0$ and $0 \le z \le \infty$ . Let the lower surface be subjected to arbitrary initial temperature and a curved boundary surface $\xi = \xi_0$ is at zero heat flux. Under these more realistic prescribed conditions the quasi-static thermal deflection in the cylinder are required to be determined.

The heat conduction equation and boundary conditions are given as

$$h^2\left\{\frac{\partial^2\theta}{\partial\xi^2}+\frac{\partial^2\theta}{\partial\eta^2}\right\}+\frac{\partial^2\theta}{\partial z^2}=\frac{1}{\alpha}\,\frac{\partial\theta}{\partial t} \tag{1}$$

with boundary conditions

$$\theta(\xi,\eta,z,t)=0 \text{ for } z=\infty \tag{2}$$

$$\theta = f(\xi,\eta) \text{ for z = 0 at t=0} \tag{3}$$

$$\frac{\partial\theta}{\partial\xi}=0 \qquad \text{at } \xi=\xi_0 \tag{4}$$

Where, α is the thermal diffusivity of the material of the cylinder

The most general form of the equation of equilibrium for a plate element is expressed in terms of the partial derivatives of the deflection is found to satisfy the differential equations as

$$D\nabla^4\omega + \frac{\nabla^2 M_\theta}{1-\upsilon} = 0 \tag{5}$$

Where $D$ is the stiffness coefficient of the plate and denoted as

$$D = \frac{E\ell^3}{12(1-\upsilon^2)} \tag{6}$$

and $M_\theta$ is bending of the plate due to change in the temperature and expressed as

$$M_\theta = \alpha E \int_0^\infty z\,\theta\, dz \tag{7}$$

in which $\nabla^2$ denotes the two-dimensional Laplacian operator in $(\xi, \eta)$, $\upsilon$ denotes Poisson's ratio, $\alpha$ and $E$ denoting coefficient of linear thermal expansion and Young's Modulus of the material of the plate respectively.

In order to complete the formulation of the problem, it is necessary to introduce suitable boundary conditions. The plate edgeis assumed to be fixed and clamped, that is

$$\omega(\xi,\eta,0) = 0 \tag{8}$$

$$\omega(a,\eta,t) = \frac{\partial\omega(a,\eta,t)}{\partial\xi} = 0 \tag{9}$$

The equations (1) to (9) constitute the mathematical formulation of the problem under consideration.

## III. SOLUTION OF PROBLEM

To obtain the expression for temperature $\theta(\xi,\eta,z,t)$ one assumes that,

$$\theta(\xi,\eta,z,t) = e^{-z}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} f_{n,m}(t) Ce_{2n}(\xi, q_{2n,m}) ce_{2n}(\eta, q_{2n,m}) . \tag{10}$$

where $q_{2n,m}$ are the positive roots of the transcendental equation $Ce_{2n}(\xi_0, q_{2n,m})$ . Also $ce_{2n}(\eta, q_{2n,m})$ and $Ce_{2n}(\xi, q_{2n,m})$ are Mathieu and Modified Mathieu functions respectively given by [7]

Equation (1) and (10), gives

$$f_{n,m}(t) = A_{n,m}\exp[-(\gamma_{2n,m}^2 - 1)t] \tag{11}$$

Where, $A_{n,m}$ is a constant which can be found from the nature of the temperature at the lower surface of the cylinder and $\gamma_{2n,m}^2 = \dfrac{-4q_{2n,m}}{c^2}$.

Thus, the expression of the temperature becomes,

$$\theta(\xi,\eta,z,t) = e^{-z}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} A_{n,m}\exp[-(\gamma_{2n,m}^2 - 1)t] Ce_{2n}(\xi, q_{2n,m}) ce_{2n}(\eta, q_{2n,m}) \tag{12}$$

Hence by the theory of the Mathieu Function [11] we get

$$A_{n,m} = \frac{\int_0^{\xi_o}\int_0^{2\pi} C_{e_{2n}}(\xi, q_{n,m})\, c_{e_{2n}}(\eta, q_{n,m})\, f(\xi,\eta)(\cosh 2\xi - \cos 2\eta)}{\pi\int_0^{\xi_o} C^2{}_{e_{2n}}(\xi, q_{n,m})[\cosh 2\xi - \Theta_{n,m}]} \tag{13}$$

where

$$\pi\,\Theta_{n,m} = \int_0^{2\pi} c^2{}_{e_{2n}}(\eta, q_{n,m})\cos 2\eta\, d\eta \tag{14}$$

Using temperature distribution from equation (12) in equation (7) the expression for thermal bending moment can be obtained as

$$M_\theta = \alpha E\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} A_{n,m}\exp[-(\gamma_{2n,m}^2 - 1)t] Ce_{2n}(\xi, q_{2n,m}) ce_{2n}(\eta, q_{2n,m}) \tag{15}$$

Thus substituting equation (15) in (5), one obtains

$$\omega(\xi,\eta,t)=\frac{c^2\alpha E}{4D(1-\upsilon)}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{A_{n,m}}{q_{2n,m}}\exp[-(\gamma_{2n,m}^2-1)t]$$
$$\times[Ce_{2n}(\xi,q_{2n,m})-Ce_{2n}(\xi_0,q_{2n,m})]ce_{2n}(\eta,q_{2n,m}) \tag{16}$$

## IV. NUMERICAL CALCULATIONS

The numerical calculations have been carried out for a copper cylinder with the following material properties.
Then, setting

$$f(\xi,\eta)=\delta(\eta)(\xi-\xi_0)^2 \quad T_0=0 \tag{17}$$

Substituting the value of equation (17) in equations (12), (15) and (16), we obtained the expressions for the temperature, thermal moment and thermal deflection respectively for our numerical discussion. The numerical computations have been carried out for Aluminum metal with parameter a = 2.65 cm, l = 6.00 cm, Modulus of Elasticity E = 6.9 × 106 N/cm2, Shear modulus G = 2.7 × 106 N/cm2, Poisson ratio υ = 0.281, Thermal expansion coefficient, αt = 25.5 × 10-6 cm/cm-0C, Thermal diffusivity κ = 0.86 cm2/sec, Thermal conductivity λ = 0.48 cal sec-1/cm 0C with $q_{n,m} =$ 0.0986, 0.3947, 0.8882, 1.5791, 2.4674, 3.5530, 4.8361, 6.3165, 7.9943, 9.8696, 11.9422, 14.2122, 16.6796, 19.3444, 22.2066, 25.2661, 28.5231, 31.9775, 35.6292, 39.4784 are the positive & real roots of the transcendental equation $Ce_{2n}(a,q_{2n,m})$. The foregoing analysis are performed by setting the radiation coefficients constants, $k_i=0.86 \ (i=1,2)$ so as to obtain considerable mathematical simplicities. Numerical calculations are depicted in the following figures with the help of MATHEMATICA software.

From Fig. 1. (a) and Fig. 1.(b), it is clear that the temperature falls as the time proceeds along radial direction and is greatest in a steady & initial state, it can be seen that the temperature change on the heated surface decreases when the radius of plate increases for different thickness.

From Fig 2. (a), it can be depicts that deflection is maximum initially and decreases with time, same nature of thermal deflection can be observed on radial direction.

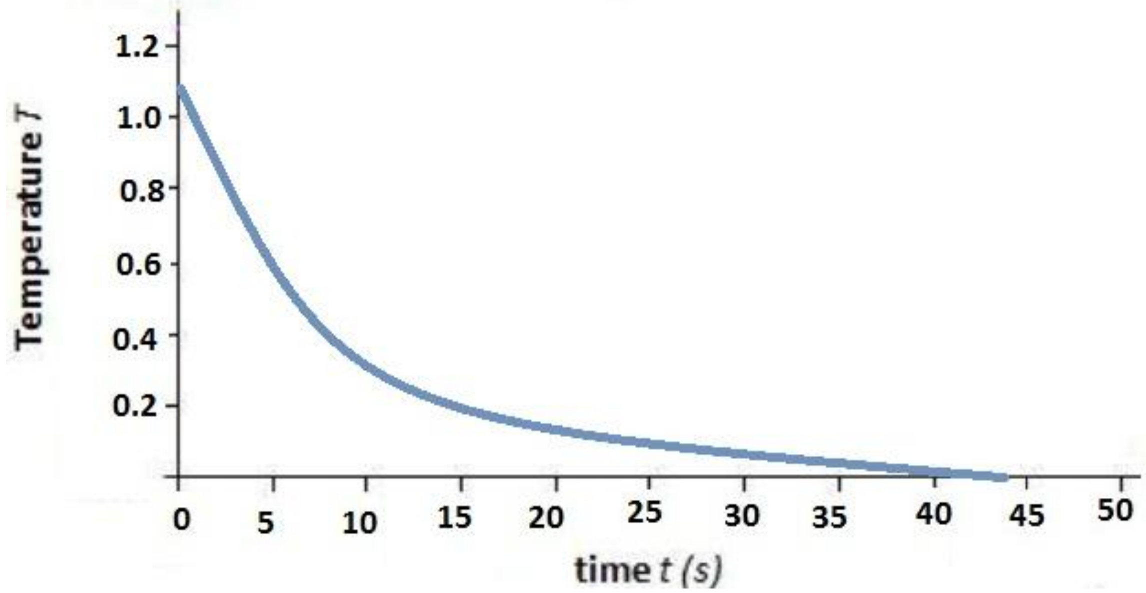


Fig. 1 (a)Temperature distribution versus ξ at z= 3.7,η= 90.

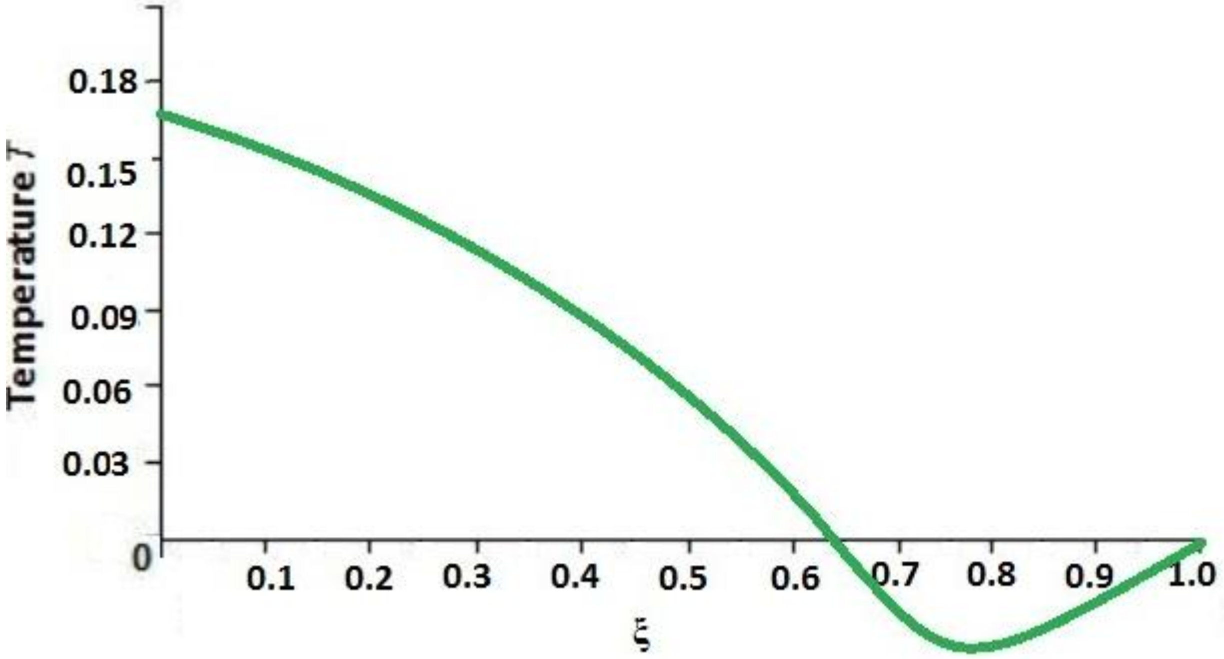


Fig. 1 (b)Temperature distribution versus ξ at z=0,η= 90.

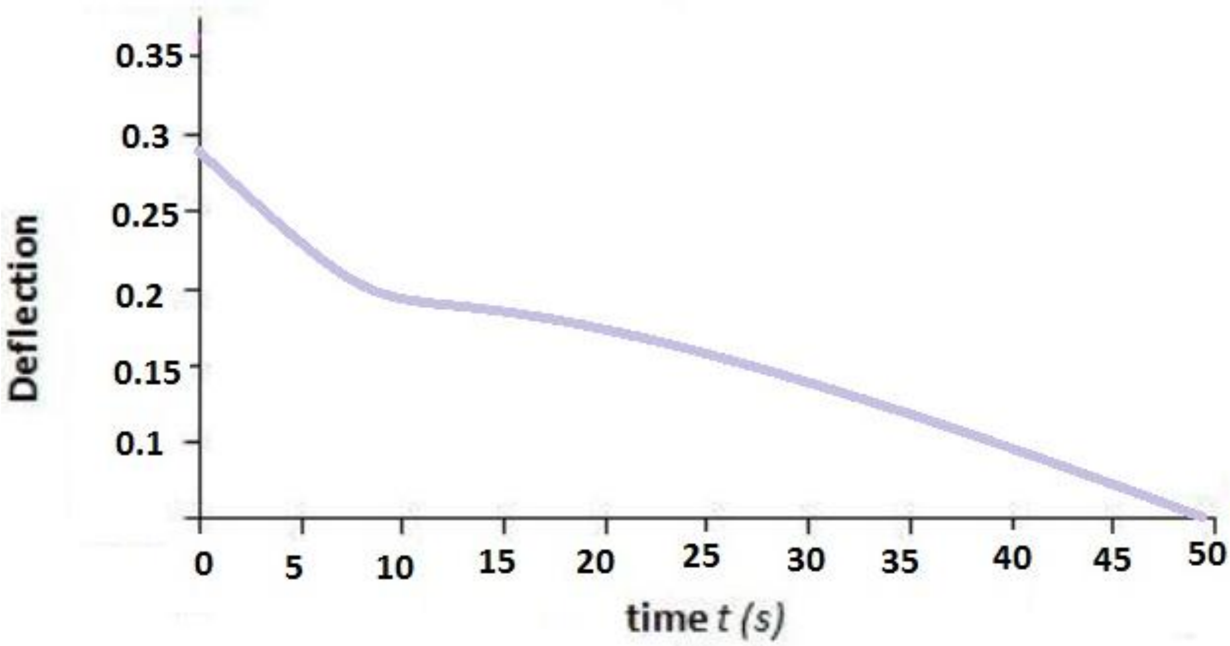


Fig. 2 (a)Temperature distribution versus t at fix $\xi$ .

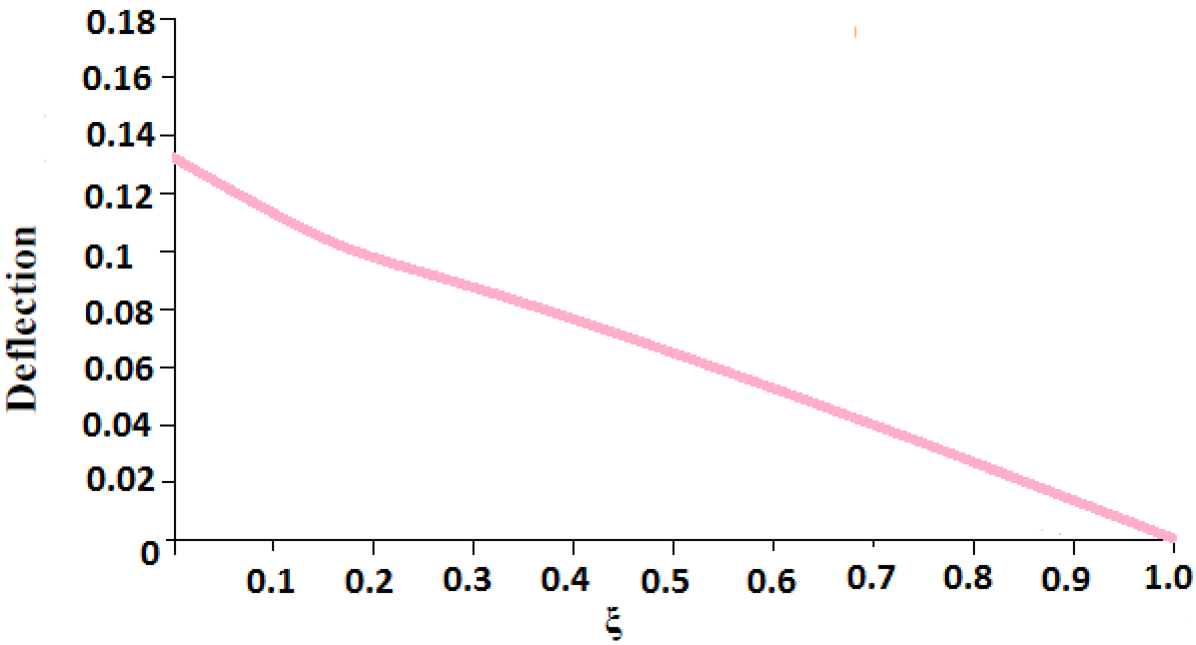


Fig. 2 (b) Deflection versus $\xi$ at z=0,$\eta$= 90.

## V. CONCLUSION

The analysis of non-stationary two-dimensional heat conduction equation is investigated with the integral transformation. With proposed integral transformation method, it is possible to apply widely to analysis stationary as well as non- stationary temperatures. Further the study is extended to find the thermal deflection using thermal bending moment using elliptical coordinate system.

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