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Isolate Vertex Resolving Partition in a Graph

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Abstract: Let G = (V, E) be a simple connected graph. A partition π = {V_1, V_2, V_3….,V_k } is called a resolving partition of G if for any u ϵ V(G) , the code of u with respect to π (denoted by $c_π$(u)) namely (d(u,V_1),d(u, V_2),….,d(u, V_k)) is distinct for different u ϵ V(G) where d(u, V_i)= min{d(u, x) / x ϵ V_i }. The minimum cardinality of a resolving partition of a graph G is called the partition dimension of G and is denoted by pd (G) Several types of resolving partition have been considered like connected resolving partition , metric chromatic number of a graph (that is, independent resolving partition) , equivalence resolving partition etc. In this paper, a new type of resolving partition is considered and in this partition, each element of the partition contains an isolate vertex in the subgraph induced by that element. This is a generalization of an independent resolving partition. This resolving partition is called an isolate vertex resolving partition in a graph. The minimum cardinality of an isolate vertex resolving partition is denoted by pd_{is}(G). This parameter for some well known graphs is found. Graphs for which pd_{is}(G) = 2 or pd_{is}(G) = n are characterized.
Keywords: Central vertex, Isolate vertex partition dimension, Isolate Vertex resolving partition, Partition Dimension, Resolving partition.

1. Introduction

In this paper, G is a simple, finite, connected and undirected graph.

Definition 1.1. [1] Let G = (V, E) be a simple, finite, connected and undirected graph. A partition π ={V_1,V_2, V_3….,V_k } of V (G) is called a resolving partition of G if the code $c_π$ (u) = (d(u,V_1),d(u,V_2),….,d(u,V_k)) is distinct for different u ϵ V(G) where d(u, V_i)= min{d(u,x) / x ϵ V_i }. The minimum cardinality of a resolving partition of a graph G is called the partition dimension of G and is denoted by pd (G).

Definition 1.2. Let G = (V,E) be a simple connected graph. Let π = {V_1, V_2, V_3,…..,V_k } be a partition of V(G). If each < V_i > contains an isolate and if π is a resolving partition, then π is called an isolate vertex resolving partition. The trivial partition namely π = {{u_1}, {u_2},…., {u_n}} where V(G) = {u_1, u_2 ,….., u_n} is an isolate vertex resolving partition.
The minimum cardinality of an isolate vertex resolving partition is called the isolate vertex partition dimension of G and is denoted by pd_{is}(G).

Remark 1.3. Every independent resolving partition is an isolate vertex resolving partition. Therefore
$pd_{is}(G) \leq ipd(G) \leq pd(G)$.

2. pd_{is} (G) FOR WELL KNOWN GRAPHS

1. pd_{is} (K_n) = n
2. pd_{is} ($K_{1,n}$) = n + 1
3. pd_{is} ($K_{m,n}$) = m + n
4. pd_{is} ($K_{a1,a2,\ldots a n}$) = a_1+ a_2 + ………+ a_n
5. pd_{is} (K_m (a_1, a_2 , …., a_m)) = max { a_1, a_2,……., a_m}

Theorem 2.1. pd_{is} (W_n) ≥ 5 if n ≥ 14.
Proof: Suppose n ≥ 14.
Let π = {{u}, V_1, V_2, V_3} be an isolate vertex partition, where u is the central vertex. Every vertex V_i (1 ≤ i ≤ 3) has code 1 with respect to {u} and 0 with respect to V_i. Since there are only two more partitions and since the codes with respect to these two partitions can be either (1,1) or (1,2) or (2,1) or (2,2), | V_2| ≤ 4 for all i. That is

$|V_1 \cup V_2 \cup V_3| \leq 12$. Since there are 13 vertices in $V_1 \cup V_2 \cup V_3$, we get a contradiction. Therefore $pd_{is}(W_n) \geq 5$ if $n \geq 14$.

Remark 2.2.

1. $pd_{is}(W_n) \geq 6$ if $n \geq 34$.
2. $pd_{is}(W_n) \geq 7$ if $n \geq 82$.
3. $pd_{is}(W_n) \geq k + 1$ if $n \geq (2^{k-2} + 1) + (k-2)\, 2^{k-2} + 1$.

Theorem 2.3. Let G be a connected graph of order n. Then $pd_{is}(G) = 2$ if and only if $G = K_2$.

Proof: Let $\pi = \{V_1, V_2\}$ be a minimum isolate vertex resolving partition.
When n = 2, V_1 and V_2 are singletons and $G = K_2$. When n = 3, π is not an isolate vertex resolving partition. Suppose n = 4. Then $G = K_4, C_4, K_{1,3}, K_4 - e, P_4, K_{1,3} + e$ and pdis(G) = 4. Let $n \geq 5$. Let $\Pi = \{V_1, V_2\}$ be a minimum isolate vertex resolving partition. V_1 contains an isolate vertex say z. Then $d(z,V_2) = 1$. Suppose V_1 contains three vertices say z, x and y. $c_\Pi(z) = (0, 1)$. Therefore $c_\Pi(x) = (0, t)$, where $t \neq 1$ and $c_\Pi(y) = (0, t')$, where $t' \neq 1$, $t' \neq t$. Therefore one of t, $t' \geq 3$. Let $t' \geq 3$.Then there exists a shortest path
$y, y_1, y_2, \ldots, y_{t'} = v \in V_2$. Therefore $y, y_1, y_2, \ldots, y_{t'-1} \in V_1$. Also $d(y_{t'-1}, V_2) = 1$, a contradiction.
(Since $d(z,V_2) = 1$). Suppose V_1 contains two vertices z, x. $d(z, V_2) = 1$. Let $d(x,V_2) = t$. Then $t \geq 2$. There exists a shortest path $x, x_1, x_2, \ldots, x_t = v \in V_2$. Since $t \geq 2$, $x_{t-1} \in V_1$ and $d(x_{t-1},V_2) = 1$, a contradiction. If $V_1 = \{z\}$ and if $\langle V_2 \rangle$ has an isolate say u, then u and z are adjacent and any path from any other vertex of $\langle V_2 \rangle$ to z must contain u. Hence u is not an isolate of $\langle V_2 \rangle$, a contradiction. Therefore $pd_{is}(G) \geq 3$. The converse is obvious.

Theorem 2.4. $pd_{is}(G) = n$ if and only if V(G) can be partitioned into subsets V_1 and V_2 such that $G = \langle V_1 \rangle + \langle V_2 \rangle$. If any of $\langle V_1 \rangle$ and $\langle V_2 \rangle$ is connected, then its diameter less than or equal to 2 and if one of them is disconnected, then it is totally disconnected.

Proof: Suppose V(G) can be partitioned into subsets V_1 and V_2 such that $G = \langle V_1 \rangle + \langle V_2 \rangle$.
Case(i): Let one of $\langle V_1 \rangle$, $\langle V_2 \rangle$ be independent and other is connected with diameter less than or equal 2. Let $diam(\langle V_2 \rangle) = 2$. Let $x, y \in V_1$. Let $\pi = \{\{x,y\},\{x_i\}\}$ where x_i runs over all the vertices of $V_1 - \{x, y\}$ and V_2. $d(x,x_i) = 2 = d(y,x_i)$ for all $x_i \in V_1 - \{x, y\}$ $d(x,x_i) = 1 = d(y,x_i)$ for all $x_i \in V_2$. Therefore π is not a resolving partition. Suppose $x, y \in V_2$. Then $d(x,x_i) = 1 = d(y,x_i)$ for all $x_i \in V_1$. Let $z \in V_2 - \{x, y\}$. If z is adjacent with y, then there exists a path x,z,…..,u,y of length greater than or equal to 3, a contradiction. (Since $diam(\langle V_2 \rangle) = 2$). Therefore z is adjacent with both x and y. (If z is not adjacent with both x and y, then $diam(\langle V_2 \rangle) = 2$). Therefore d(x,z) = d(y,z). Therefore π is not a resolving partition. Therefore $pd_{is}(G) = n$. Let $diam(\langle V_2 \rangle) = 1$. Then $\langle V_2 \rangle$ is complete. Therefore no two points of V_2 can be independent. Therefore $pd_{is}(G) = n$.
Case(ii): Let both $\langle V_1 \rangle$ and $\langle V_2 \rangle$ be independent. Then $pd_{is}(G) = n$.
Case(iii): Suppose $\langle V_1 \rangle$ is independent and $\langle V_2 \rangle$ is disconnected but not totally disconnected.
Let $V_1 = \{u_1, u_2, \ldots, u_k\}$. Let $V_2 = \{\{u_{k+1}, u_{k+2}, \ldots, u_n\}$. Let H_1 and H_2 be the components of $\langle V_2 \rangle$. Since $\langle V_2 \rangle$ is not totally disconnected, at least one of H_1, H_2 contains at least two vertices. Let H_1 contain at least two vertices. Since H_1 is connected, there exists two adjacent vertices in H_1 say x, z. Let $y \in V(H_2)$. Let $\pi = \{\{x,y\},\{x_i\}\}$ where x_i runs over all the vertices of V_1, $H_1 - \{x\}$, $H_2 - \{y\}$. Since $z \in V(H_1)$, $\{z\} \in \pi$. d(x,z) = 1, d(y,z) = 2. Therefore π is an isolate vertex resolving partition of G, a contradiction, since $|\pi| = n - 1$.
Case(iv): Suppose $\langle V_1 \rangle$ and $\langle V_2 \rangle$ are connected and $diam(\langle V_i \rangle) \leq 2$, $1 \leq i \leq 2$. If $diam(\langle V_i \rangle) = 1$ for i = 1, 2 then G is complete and hence $pd_{is}(G) = n$. If $diam(\langle V_1 \rangle) = 2$ or $diam(\langle V_2 \rangle) = 2$ then proceeding as in Case(i), we get that $pd_{is}(G) = n$.
Case(v): Suppose $\langle V_1 \rangle$ is connected and $\langle V_2 \rangle$ is disconnected but not totally disconnected. Then proceeding as in Case (iii), we get a contradiction.
Conversely, suppose $pd_{is}(G) = n$.
Suppose V (G) cannot be partitioned into subsets V_1 and V_2 such that $G = \langle V_1 \rangle + \langle V_2 \rangle$.
Suppose G is complete. Then G can be partitioned into subsets V_1 and V_2 such that $G = \langle V_1 \rangle + \langle V_2 \rangle$.
Suppose G is not complete. Let $\{u_1, u_2, \ldots, u_k\}$ be a maximum independent set of G. Then $k \geq 2$.
Let $V_1 = \langle \{u_1, u_2, \ldots, u_k\} \rangle$ and $V_2 = \langle V - V_1 \rangle$. Since $G \neq \langle V_1 \rangle + \langle V_2 \rangle$, there exists $u_i \in V_1$ and $y \in V_2$ such that u_i and y are not adjacent. Since u_i is not an isolate of G and since u_i is an isolate of $\langle V_1 \rangle$, u_i is adjacent with some vertex say $z \in V_2$. Then $\pi = \{\{y,z\}, \{x_i\}\}$ where $x_i \in V_1$ or $x_i \in V_2 - \{y, z\}$, is an isolate vertex resolving partition. Since y, z are resolved by u_i. Therefore $pd_{is}(G) \leq n - 1$, a contradiction. Therefore G can be partitioned into subsets V_1 and V_2 such that $G = \langle V_1 \rangle + \langle V_2 \rangle$.
Suppose $\langle V_1 \rangle$ and $\langle V_2 \rangle$ are connected. Suppose $diam(\langle V_1 \rangle) \geq 3$. (similar proof if $diam(\langle V_2 \rangle) \geq 3$). Then there exists a path $u = u_0, u_1, \ldots, u_k = v$ in $\langle V_1 \rangle$ where $k = diam(\langle V_1 \rangle) \geq 3$. Let $\pi = \{\{u_0,u_k\}, \{x_i\}\}$where $x_i \in V_1$

– $\{u_o, u_k\}$, $x_i \in V_2$. u_o, u_k are resolved by u_i. Therefore $pd_{is}(G) \leq n - 1$, a contradiction. Therefore diam($< V_1 >$) $\leq$ 2.

Suppose V_1 is independent and $< V_2 >$ is connected. Suppose diam($< V_2 >$) ≥ 3. Then proceeding as above $pd_{is}(G) \leq n - 1$, a contradiction. Therefore diam($< V_2 >$) ≤ 2. Therefore $G = < V_1 > + < V_2 >$ and if any of $< V_1 >$ and $< V_2 >$ is connected, then its diameter less than or equal to 2 and if one of them is disconnected, then it is totally disconnected.

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