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Degree Strong Star Dominating Sets in a Graph

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Abstract: In administrative set up in bygone days, a head of section in an institution is vested with enough powers and the workers in that section are kept isolates without communication between them. A graph model for this is a star with at least three vertices. The centre has degree greater than the degree of the pendant vertices and the pendant vertices are independent. The concept of strong, weak domination was introduced by E. Sampathkumar and L. Pushpa Latha.[6] In the case of a star, the pendant vertices form an independent weak set dominated strongly by the center. This idea leads to the definition of degree strong star sets. In these sets, every element behaves like a pendant of a star. Also the concept of degree strong star dominating set arises from degree strong star sets. In this paper, degree strong star sets and upper strong star number are defined. A detailed study is made of degree strong star sets and degree strong star domination.

Keywords: degree strong star sets, degree strong star number, degree strong star domination and irredundance.
AMS Subject Classification: 05C69.

I. Introduction

Let $G=(V,E)$ be a simple graph. Let $u,v \in V(G)$. u strongly (weakly) dominates v if u and v are adjacent and $\deg(u) \geq \deg(v)$ $(\deg(u) \leq \deg(v))$.[6] In this case of $K_{1,n}$, the center strongly dominates all the pendant vertices and the pendant vertices are independent. This leads to the definition of degree strong star set. A subset S of $V(G)$ is a degree strong star set if every u in S is a pendant vertex of a maximum independent set which is strongly dominated by a vertex of the graph. For example, the pendant vertices of a star form a degree strong star set. If D is a subset of $V(G)$ such that every vertex in the complement of D is a member of a maximum independent weak set dominated by a vertex of D, then D is called a degree strong star dominating set of G. Properties of dss sets, independent dss sets and this domination are studied. Further degree strong star irredundance is also defined and studied.

1.1 Definition

Let G be a simple graph. Let S be a subset of $V(G)$. S is called a degree strong star set (dss set) of G if for every u in S there exists v in $V(G)$ such that u is a member of a maximum independent weak set dominated by v.

1.2 Remark

Let $d_{SS}(G) = \min\{|S|: \text{S is a maximal strong star set of G}\}$. $d_{ss}(G)$ is called the strong star number of G.

1.3 Remark

Let $D_{SS}(G) = \max\{|S|: \text{S is a maximal strong star set of G}\}$. $D_{ss}(G)$ is called the upper strong star number of G.

1.4 Example

(i) Let $G = K_n$. Any set of k vertices of $K_n (1 \leq k \leq n)$ is a degree strong star set(dss set).

(ii) Let $G = K_{1,n}$. Any K_2 in $K_{1,n}$ is a dss set.

(iii) Let $G = K_{m,n}$. If $m = n$, then the two partite sets are dss sets. If $m < n$, then the partite set with n elements is a dss set.

(iv) Let $G = P_n$. Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices. Then $S = \{u_2, u_3, \ldots u_{n-1}\}$ is a dss set.

(v) Let $G = C_n$. Let $u_1, u_2, u_3, \ldots\ldots, u_n$ be the vertices. Then $S = \{u_1, u_2\}$ is a dss set.

(vi) Let $G = W_n$. Let $u_1, u_2, u_3, \ldots\ldots, u_{n-1}$ be the vertices in the rim and u_n be the central vertex . Then $S = \{u_n\}$ is a dss set

1.5 Remark

The property of being a degree strong star set is hereditary.

1.6 Example

(i) $d_{ss}(K_n) = n, \quad D_{ss}(K_n) = n$.

(ii) $d_{ss}(K_{1,n}) = n, \quad D_{ss}(K_{1,n}) = n$

(iii) $d_{ss}(P_n) = V(P_n)$

(iv) $d_{ss}(C_n) = V(C_n)$

(v) $d_{ss}(W_n) = n-1, \quad (n \geq 5)$

(vi) $d_{ss}(D_{r,s}) = V(D_{r,s}), \; if \quad r = s, \; and \; V(D_{r,s}) - 1, \; if \quad r \neq s$

(vii) $d_{ss}(P) = V(P)$, where P is the Petersen graph.

1.7 Observation

(i) Let G be a graph with a full degree vertex. Then $dss(G) = \beta_0(G)$.

(ii) Let G be a graph. A dss set S of G is maximal if and only if every u in $V - S$ does not belong to a maximal independent weak set dominated by a vertex of G.

1.8 Example

(i) Let $G = K_n$. Let $u_1, u_2, u_3, \ldots\ldots, u_n$ be the vertices of K_n. Then $S = \{u_1, u_2, \ldots. u_n\}$ is a maximal dss set of G .

(ii) Let $G = K_{1,n}, \; (n \geq 3)$. Let v be the center and $u_1, u_2, u_3, \ldots\ldots, u_n$ be the pendant vertices. Then $S = \{u_1, u_2, \ldots. u_n\}$ is a maximal dss set.

1.9 Definition

A dss set S is said to be an independent dss set if S is independent.

1.10 Example

(i) In K_n, any single vertex forms an independent dss set.

(ii) In $K_{1,n}$, the set of all pendant vertices is an independent dss set.

1.11 Remark

The property of independent dss set is hereditary.

1.12 Definition

The maximum cardinality of an independent dss set is called the independence dss number of G and is denoted by $Id_{ss}(G)$. The minimum cardinality of a maximal independent dss set of G is denoted by $id_{ss}(G)$

1.13 Example

(i) $Id_{ss}(K_n) = 1$, $\quad id_{ss}(K_n) = 1$

(ii) $Id_{ss}(K_{1,n}) = n$, $\quad id_{ss}(K_{1,n}) = n$

(iii) $Id_{ss}(K_{m,n}) = \max\{m, n\}$, $\quad id_{ss}(K_{m,n}) = \max\{m, n\}$

(iv) $Id_{ss}(P_n) = \left\lceil \frac{n}{2} \right\rceil$, $\quad id_{ss}(P_n) = \begin{cases} \frac{n}{2}, & if\ n\ is\ even \\ \left\lfloor \frac{n}{2} \right\rfloor, & if\ n\ is\ odd \end{cases}$

(v) $Id_{ss}(C_n) = \left\lceil \frac{n}{2} \right\rceil$, $\quad id_{ss}(C_n) = \begin{cases} \frac{n}{2}, & if\ n\ is\ even \\ \left\lfloor \frac{n}{2} \right\rfloor, & if\ n\ is\ odd \end{cases}$

(vi) $Id_{ss}(D_{r,s}) = r + s$, $\quad id_{ss}(D_{r,s}) = \max\{r, s\} + 1$

(vii) $Id_{ss}(P) = 4$, $\quad id_{ss}(P) = 3$, where P is the Petersen graph.

1.14 Observation

(i) $1 \le dss(G) \le n$

(ii) $d_{ss}(G) = 1$, *if only if* $G = K_1$.

Proof:

If $|V(G)| \ge 2$, then $d_{ss}(G) \ge 2$. Hence $d_{ss}(G) = 1$, implies, $G = K_1$.

(iii) $d_{ss}(G) = n-1$, *if only if* $G = W_n\ (n \ge 5)$ *or* $G = K_{1,n}$ *or* $G = W_n \cup H$ *or* $K_{1,n} \cup H$,

where $d_{ss}(H) = |V(G)|$.

II. Degree Strong Star Dominating Set

2.1 Definition

Let $G = (V, E)$ be a simple graph. Let D be a subset of V(G). D is called a degree strong star dominating set of G, if for any v in $V - D$ there exists u in D such that v is a member of a maximum independent weak set dominated by u. The minimum cardinality of a degree strong star dominating set of G (hereafter abbreviated as $dssd$ set) is called the degree strong star domination number of G and is denoted by $\gamma^{ds}(G)$. The existence of $dssd$ set is guaranteed in any graph G, since $V(G)$ is a $dssd$ set.

2.2 $\gamma^{ds}(G)$ for standard graphs:

(i) $\gamma^{ds}(K_n) = 1$

(ii) $\gamma^{ds}(K_{1,n}) = 1$

(iii) $\gamma^{ds}(P_n) = \left\lceil \frac{n}{3} \right\rceil$

(iv) $\gamma^{ds}(C_n) = \left\lceil \frac{n}{3} \right\rceil$

(v) $\gamma^{ds}(W_n) = 1$

(vi) $\gamma^{ds}(K_{m,n}) = \begin{cases} 2, & if\ \mathrm{m = n} \\ m, & if\ \mathrm{m} < n \end{cases}$

(vii) $\gamma^{ds}(D_{r,s}) = 2$

(viii) $\gamma^{ds}(P) = 3$, where P is the Petersen graph.

2.3 Remark

Degree strong star domination property is super hereditary. Hence a $dssd$ set is minimal if and only if it is 1-minimal.

2.4 Definition

Let D be a $dssd$ set of a graph G. Then D is minimal if and only if for any u in D is one of the following holds.

(i) u is a strong isolate of D

(ii) there exists $v \in V - D$ such that u is strongly dominated only by $u \in D$

(iii) there exists $v \in V - D$ such that v is a member of a minimum independent weak set

2.5 Definition

A subset S of $V(G)$ is called dss irredundant if S satisfies the above conditions of a minimal dss set

2.6 Remark

The property of dss irredundance is hereditary.

2.7 Definition

The minimum cardinality of a maximal dss irredundant set of G is called the dss irredundance number of G and is denoted by $ir^{ds}(G)$. The maximum cardinality of a maximal dss irredundant set of G is called the upper dss irredundance number of G and is denoted by $IR^{ds}(G)$

2.8 Theorem

Any minimal $dssd$ set is a maximal dss irredundant set

Proof: Routine.

2.9 Corollary

$$ir^{ds}(G) \le \gamma^{ds}(G) \le \Gamma^{ds}(G) \le IR^{ds}(G)$$

2.10 Remark

In the following graph G, $ir^{ds}(G) < \gamma^{ds}(G)$

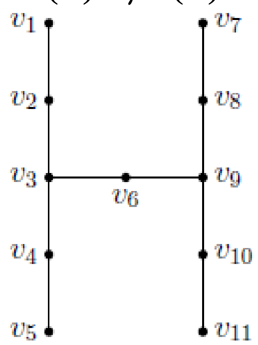

Fig: 1

$\{v_2, v_3, v_4, v_8, v_9, v_{10}\}$ is a γ^{ds} - set of G .Hence $\gamma^{ds}(G) = 6.$ $\{v_2, v_3, v_8, v_9\}$ is a dss irredundant set of G. Therefore, $ir^{ds}(G) \le 4.$ Therefore, $ir^{ds}(G) < \gamma^{ds}(G)$

2.11 Theorem

For any graph G, $\dfrac{\gamma^{ds}(G)}{2} < ir^{ds}(G) \le \gamma^{ds}(G) \le 2ir^{ds}(G) - 1$

Proof: Routine

2.12 Definition

A subset D of V(G) is called an independent $dssd$ set if D is an independent and D is a $dssd$ set.

2.13 Example

(i) In $K_{1,n}$, the central vertex is a $dssd$ set and it is independent

(ii) In C_4 , the diagonal points constitute an independent $dssd$ set.

2.14 Problem

Find conditions for existence of an independent $dssd$ set.

Let G be a graph which admits independent $dssd$ sets. The minimum cardinality of a maximal independent $dssd$ set of G is called the independence $dssd$ number of G and is denoted by $i^{ds}(G)$. The maximum cardinality of a maximal independent $dssd$ set of G is called the upper $dssd$ number of G and is denoted by $I^{ds}(G)$.

2.15 Remark

$\gamma^{ds}(G) \le i^{ds}(G) \le I^{ds}(G) \le \beta_0(G) \le \Gamma^{ds}(G)$

There are graphs in which $\gamma^{ds}(G) < i^{ds}(G)$.

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