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Application of POD Improved Trajectory Piecewise-Linear Method In Reservoir Simulation

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Abstract: Under the existing calculation conditions, **i**mproving reservoir simulation computing speed is the urgent problem to be solved. At present, the trajectory piecewise-linear (TPWL) reduced order method can be applied to the nonlinear reservoir simulation system. But its disadvantage is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In this paper, we improve the TPWL method from the choice of linear expansion points and the weighting function, and then reducing order of each linear model using proper orthogonal decomposition (POD). It is called POD-ITPWL. We apply the method for a two-phase (oil-water) reservoir model which is solved by full implicit. The example demonstrates that which can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.

Keywords: reservoir simulation; model order reduction; trajectory piecewise-linear; proper orthogonal decomposition

Traditional reservoir simulators solve a set of governing partial differential equations. It entails solving a set of nonlinear algebraic equations by using iteration. As the reservoir models arising from real fields consist of hundreds of thousands or millions of grid blocks, these numerical solutions can be quite time consuming. So, in the case of ensuring the sufficient accuracy of numerical solution, how to greatly accelerate the reservoir simulation speed is the urgent problem to be solved. Model order reduction (MOR) [1-3] techniques have shown promise in alleviating computational demands. For now, TPWL [4-7] reduced order method is be widely used in the nonlinear system. The nonlinear system can be represented as a weighted combined piecewise linear system. The TPWL method is more efficient for the model reduction of nonlinear systems, but the disadvantage of this method is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In this paper, we improve the TPWL method from the choice of linear expansion points and the weighting function, and then reducing order of each linear model using POD method. This method is called POD-ITPWL. We apply the method for a two-phase (oil-water) reservoir model which is solved by full implicit, which can greatly reduce the dimension of reservoir model, so as to improve the operation speed.

1. Reservoir Model

A two dimensional oil-water two phase reservoir model is used. It is assumed that oil and water do not exchange material, the process is isothermal, the fluid is compressible, and the mass conservation equation and Darcy's law can be used to obtain [8]:

$$-\nabla \bullet \left[\frac{k_{ri}\rho_i}{\mu_i} \mathbf{K}(\nabla p_i - \rho_i g \nabla d) \right] + \frac{\partial(\phi S_i \rho_i)}{\partial t} - \rho_i q_i^{'''} = 0 \qquad (1)$$

Where $\mathbf{K}$ is permeability tensor; μ is fluid viscosity; k_r is relative permeability; p is pressure; g is gravity acceleration; d is depth; fluid density; ϕ is porosity; S is fluid saturation; t is time; q''' is a source term expressed as flow rate per unit volume; superscript $i \in \{o, w\}$ is respectively oil phase and water phase. In the equation (1), there are four unknown quantities, p_w and S_o are eliminated by using the auxiliary equation (2) and (3), so that only the state variables p_o, S_w are included in the equation,

$$S_o + S_w = 1 \tag{2}$$

$$p_o - p_w = p_c(S_w) \tag{3}$$

Where $p_c(S_w)$ is oil-water two-phase capillary pressure.

We consider the relatively simple cases and ignore gravity and capillary force. Format to discrete in space by using five point block centered finite difference, we may have the nonlinear first-order differential equation (4), see the specific derivation of literature [9]:

$$\underbrace{\begin{bmatrix} \mathbf{V}_{wp} & \mathbf{V}_{ws} \\ \mathbf{V}_{op} & \mathbf{V}_{os} \end{bmatrix}}_{\mathbf{V}} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{s}} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{T}_w & \mathbf{0} \\ \mathbf{T}_o & \mathbf{0} \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{F}_w(\mathbf{s}) \\ \mathbf{F}_o(\mathbf{s}) \end{bmatrix}}_{\mathbf{F}} \mathbf{q}_{well,t} \tag{4}$$

Where: vector $\mathbf{p}$ and $\mathbf{s}$ is grid center oil pressure p_o and water saturation S_w respectively; $\dot{\mathbf{p}}$ and $\dot{\mathbf{s}}$ is the time t derivative of vector $\mathbf{p}$ and $\mathbf{s}$ respectively; $\mathbf{V}$ is the cumulative matrix; $\mathbf{T}$ is transmission matrix; $\mathbf{F}$ is divided flow matrix; Vector $\mathbf{q}_{well,t}$ is the total flow of oil-water well.

Define the state vector $\mathbf{x}$, input vector $\mathbf{u}$ and output vector $\mathbf{y}$

$$\mathbf{x} \triangleq \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} \qquad \mathbf{u} \triangleq \begin{bmatrix} \breve{\mathbf{q}}_{well,t} \\ \breve{\mathbf{p}}_{well} \end{bmatrix} \qquad \mathbf{y} \triangleq \begin{bmatrix} \bar{\mathbf{p}}_{well} \\ \bar{\mathbf{q}}_{well,w} \\ \bar{\mathbf{q}}_{well,o} \end{bmatrix} \tag{5,6,7}$$

Where vector $\breve{\mathbf{q}}_{well,t}$ and $\breve{\mathbf{p}}_{well}$ represent the well of the constant flow and the bottom hole pressure respectively; The vector $\bar{\mathbf{p}}_{well}$ indicates the output bottom hole flow pressure of the constant flow well; Vector $\bar{\mathbf{q}}_{well,o}$ and $\bar{\mathbf{q}}_{well,w}$ indicate the output oil and water flow of the constant bottom hole pressure respectively.

The equation (4) can be written as the form of state space equation [9]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} \tag{8}$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x},\mathbf{u}) = \mathbf{C}(\mathbf{x})\mathbf{x} + \mathbf{D}(\mathbf{x})\mathbf{u} \tag{9}$$

In the control system, $\mathbf{A}$ is called the system matrix, $\mathbf{B}$ is called the input matrix, $\mathbf{C}$ is called the output matrix, $\mathbf{D}$ is called the direct transfer matrix. Because the elements of the matrix $\mathbf{V}$, $\mathbf{T}$, $\mathbf{F}$, $\mathbf{J}$ are function of the state variables, the system is a nonlinear system.

2. POD-ITPWL Reduced Order Method

In order to construct a POD basis vector, the first we need to run a full order reservoir simulator (training process), and to preserve the state vector $\mathbf{x}$ of each time step (also called the snapshot, including the oil pressure p_o and the water saturation S_w of all grids). Because of pressure and saturation with different physical properties, we use matrix $\mathbf{X}_p$, $\mathbf{X}_S$ to preserve p_o, S_w respectively (hereinafter abbreviated as p and S):

$$\mathbf{X}_p = \left[\mathbf{x}_p^{\ 1}\ \mathbf{x}_p^{\ 2}\ \cdots\ \mathbf{x}_p^{\ m} \right], \qquad \mathbf{X}_S = \left[\mathbf{x}_S^{\ 1}\ \mathbf{x}_S^{\ 2}\ \cdots\ \mathbf{x}_S^{\ m} \right] \tag{10}$$

Assuming that the number of grids in the reservoir model is N, then each vector $\mathbf{x}_p^{\ i}$, $\mathbf{x}_S^{\ i}$ (Superscript i denotes the number of snapshots)in the matrix $\mathbf{X}_p$ and $\mathbf{X}_S$ is N dimension, however, the dimension of the state vector $\mathbf{x}$ of the system is: $n = 2N$. The snapshot needs to be calculated mean value after the snapshot is obtained:

$$\overline{\mathbf{x}}_p = \frac{1}{m}\sum_{i=1}^{m}\mathbf{x}_p^{\ i}, \quad \overline{\mathbf{x}}_S = \frac{1}{m}\sum_{i=1}^{m}\mathbf{x}_S^{\ i} \tag{11}$$

And each snapshot in the data matrix $\mathbf{X}_p$ and $\mathbf{X}_S$ is subtracted from the mean:

$$\begin{aligned} \hat{\mathbf{X}}_p &= \left[\mathbf{x}_p^{\ 1} - \overline{\mathbf{x}}_p,\ \mathbf{x}_p^{\ 2} - \overline{\mathbf{x}}_p,\ \cdots,\ \mathbf{x}_p^{\ m} - \overline{\mathbf{x}}_p \right] \\ \hat{\mathbf{X}}_S &= \left[\mathbf{x}_S^{\ 1} - \overline{\mathbf{x}}_S, \mathbf{x}_S^{\ 2} - \overline{\mathbf{x}}_S,\ \cdots,\ \mathbf{x}_S^{\ m} - \overline{\mathbf{x}}_S \right] \end{aligned} \tag{12}$$

Implementing the above POD reduction process for the matrix $\hat{\mathbf{X}}_p$, the basis vector matrix $\mathbf{\Phi}_{lp}$ and $\mathbf{\Phi}_{ls}$ are obtained, and we combine the two matrices to obtain basis matrix $\mathbf{\Phi}_l$, which includes l columns, and $l = l_p + l_s$.

By using the TPWL method, a set of linearized points is obtained by using a kind of linear expansion point selection algorithm: $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}$.Near the linearization points, a set of linear models are obtained by the linear expansion of the nonlinear term $\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}$:

$$\dot{\mathbf{x}} = \mathbf{G_i}\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G_i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i\mathbf{u}, \quad i = 0, 1, \cdots, (s-1) \tag{13}$$

Where: $\mathbf{G_i}$ is Jacobian matrix of $\mathbf{f(x)}$ at $\hat{\mathbf{x}}_\mathbf{i}$, $\mathbf{B}_i = \mathbf{B}(\hat{\mathbf{x}}_i)$.

By using weighted function, the approximate reduction system of the nonlinear system (8) is obtained by weighted summation of the formula (13)

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i \mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G}_\mathbf{i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i \mathbf{u}), \quad i = 0, 1, \cdots, (s-1) \qquad (14)$$

Setting $\mathbf{x} \approx \mathbf{\Phi}_l \mathbf{z}$, we can get the approximation of nonlinear system (8), (9) for order reduction system

$$\dot{\mathbf{z}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{z})(\mathbf{G}_{ir}\mathbf{z} + \mathbf{V}^T(\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G}_\mathbf{i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_{ir}\mathbf{u}) \qquad (15)$$

$$\mathbf{y} = \mathbf{C}_r \mathbf{z} + \mathbf{D}\mathbf{u} \qquad (16)$$

The disadvantage of TPWL [7] method is that when the state is in the vicinity of the linear points, the TPWL method does not have large distortion; otherwise, if the state is far away from the points, the distortion is obvious. In order to obtain high quality linear expansion, we improve the algorithm in the literature [7], and propose a linear maximum error control based on global expansion point selection algorithm, which is used in reservoir simulation. The specific process of the algorithm is as follows:

1) Give the maximum error control limit α and input vector $\mathbf{u}(t)$;

2) simulate the full order reservoir simulator, save the output state vectors $\{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_M\}$;

3) The initial state $\mathbf{x}_0$ is taken as the first linear expansion point $\hat{\mathbf{x}}_0$, and set $i = 1$;

4) Using the TPWL method to establish a temporary model

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_\mathbf{i}) - \mathbf{G}_\mathbf{i}\hat{\mathbf{x}}_\mathbf{i}) + \mathbf{B}_i\mathbf{u}) \qquad (17)$$

5) The model (17) is simulated and the state vector $\{\tilde{\mathbf{x}}_0, \tilde{\mathbf{x}}_1, \cdots, \tilde{\mathbf{x}}_M\}$ is obtained;

6) $\{\tilde{\mathbf{x}}_0, \tilde{\mathbf{x}}_1, \cdots, \tilde{\mathbf{x}}_M\}$ and $\{\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_M\}$ will be compared to find the maximum error state $\tilde{\mathbf{x}}_k$, and record the maximum error $\eta_{\max}$ and k;

7) If $\eta_{\max} > \alpha$, so select the first $i+1$ linearization point $\hat{\mathbf{x}}_i = \tilde{\mathbf{x}}_k$, and set $i = i+1$, then turn to 4);

If $\eta_{\max} < \alpha$, so the loop ends and the linearization point $\{\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{i-1}\}$ are returned.

Compared with other methods, this method has higher quality, and the reduced order model has smaller dimension, higher accuracy and better scalability [10].

In order to obtain high precision, we improve weight function in the literature [7], as follows:

$$\omega_i(\mathbf{z}) = \left[\frac{d_{\min}}{d_i(\mathbf{z})} e^{-\frac{d_i(\mathbf{z}) - d_{\min}}{D_{\min}}} \right]^p$$

Where: $d_i(\mathbf{z}) = \left|\mathbf{z} - \hat{\mathbf{z}}_i\right|_2^2$, $d_{\min} = \min(d_i(\mathbf{z}))$, $i = 0, 1, \cdots, (s-1)$. $D_{\min}$ is the minimum distance between linearized points $\{\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}\}$.Parameter p is between 1 and 2. At last, all weight functions are standardized and satisfied $\sum_{i=0}^{s-1} \omega_i(\mathbf{z}) = 1$。

3. Example Verification

A two-dimensional oil-water two phase anisotropic reservoir is described. Its grid is divided into 21 * 21, and the distribution of permeability and porosity is shown in Figure 1, 2. The related parameters of reservoir model: thickness h=2m, length and width of grid $\Delta x = \Delta y$ =33m, the viscosity of the crude oil μ_o =5mPa·s, formation water viscosity μ_w =1mPa·s, comprehensive compression coefficient c_t =3.0×10^{-3}MPa^{-1}, the original formation pressure p_i =30MPa, borehole radius r_{well} =0.114m, the end point relative permeability of oil phase k_{ro}^0 =0.9, the end point relative permeability of water phase k_{rw}^0 =0.6, oil phase Corey index n_o =2.0, water phase Corey index n_w =2.0, residual oil saturation S_{or} =0.2, irreducible water saturation S_{wc} =0.2. We use anti five point method well pattern to produce. Center has a water injection well, and four corners have four production wells. We ignore gravity and capillary force.

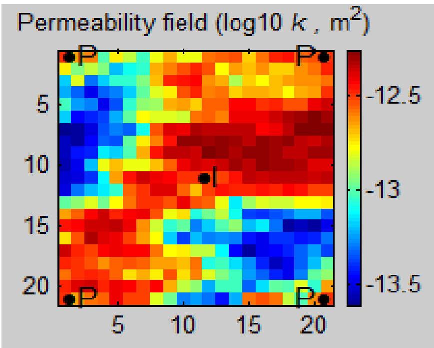

Fig.1 Permeability distribution of reservoir model

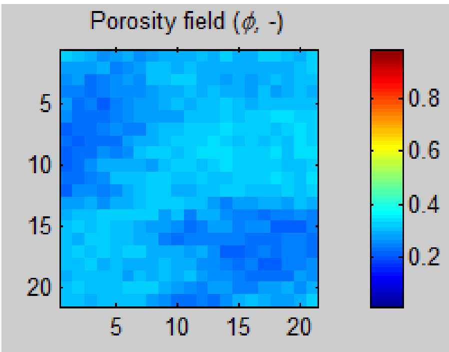

Fig.2 Porosity distribution of reservoir model

We have applied POD-TPWL and POD-ITPWL method respectively to this reservoir. To extract the information needed to reproduce the behavior of system, a full run (referred to as the training simulation) is performed. In the process of training simulation, the BHP of injection well is 35MPa, the BHPs of four production wells are 26MPa. The maximum time step allowed is 20 days. We simulate 1200 days and a total of 62 snapshots for the oil pressure and water saturation states, Jacobian matrices are recorded. The pressure matrix retains 24 singular values, saturation matrix retains 22 singular values, and the dimension of the base matrix Φ_l of POD is $2N \times l$, of which $l = 24 + 22 = 46$. This means that the full order simulator is required to solve $2N = 882$ unknown variable, while the reduced order only needs to solve 46 variables. For POD-TPWL method, we select 10 linearization points. By using the POD-ITPWL method, we obtain 14 linearization points.

We next consider two different scenarios to evaluate the predictive capability of POD-TPWL and POD-ITPWL reduced order model（ROM）.

(1) Schedule 1

We change the bottom-hole pressure of the four production wells, and they are set to 24MPa. The difference is smaller compared with the bottom-hole pressure of training simulation. The injection well BHP is the same as in the training simulation.

Figures 3 through 5 show the oil and water flow rates for production wells, and water injection rates for the injection well using POD-TPWL method. Figures 6 through 8 show the oil and water flow rates for production wells, and water injection rates for the injection well using POD-ITPWL method. The results of POD-TPWL and POD-ITPWL methods demonstrate close agreement with the reference simulation, but the POD-ITPWL method is more accurate compared with POD-TPWL method.

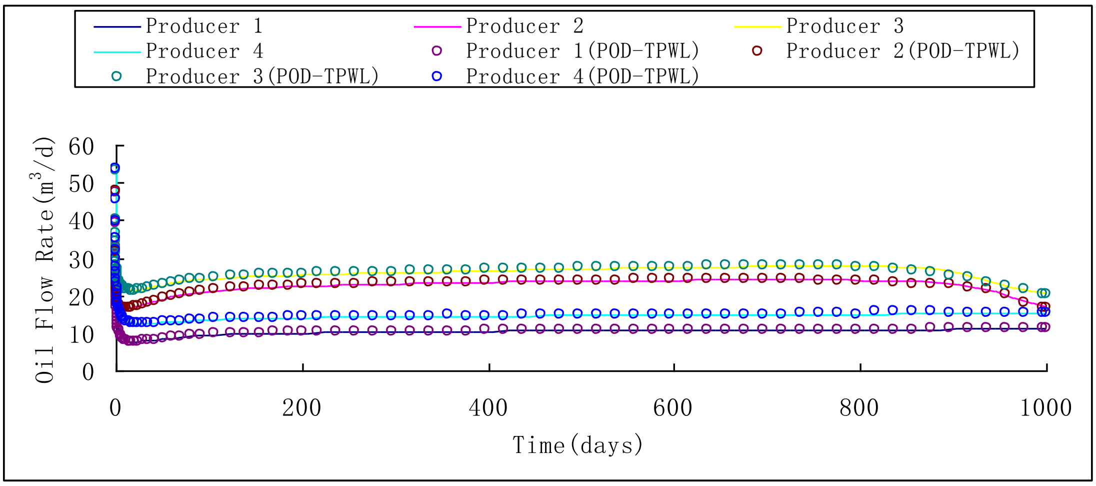

Figure 3 Oil flow rates of four production wells for POD-TPWL (schedule 1)

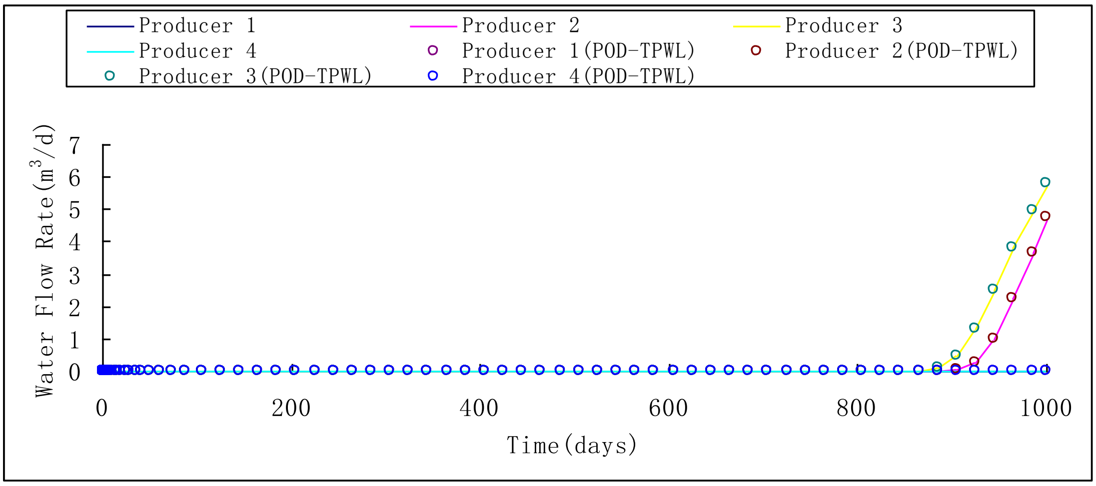

Figure 4 Water flow rates of four production wells for POD-TPWL (schedule 1)

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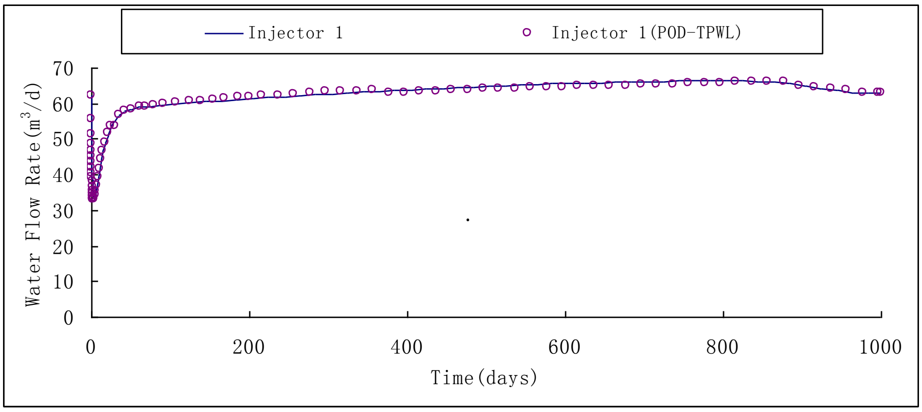

Figure 5 Water flow rate of injection well for POD-TPWL (schedule 1)

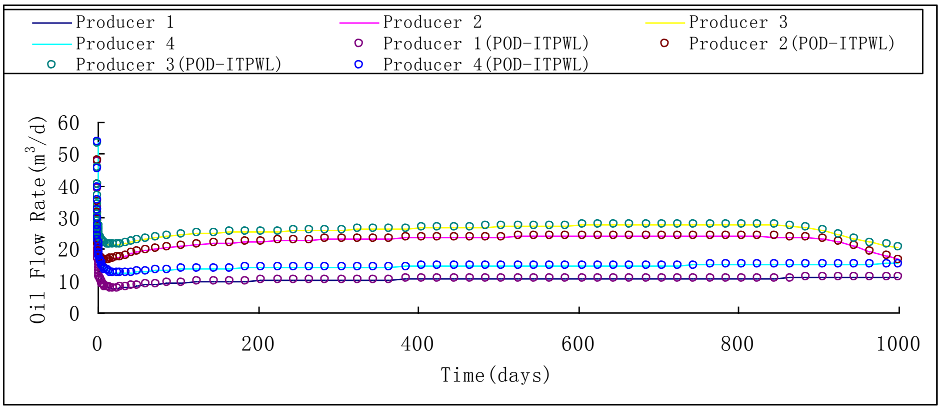

Figure 6 Oil flow rates of four production wells for POD-ITPWL (schedule 1)

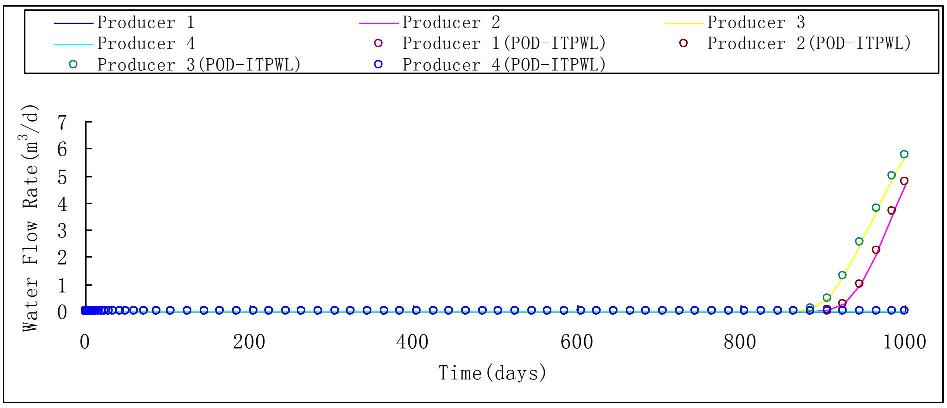

Figure 7 Water flow rates of four production wells for POD-ITPWL (schedule 1)

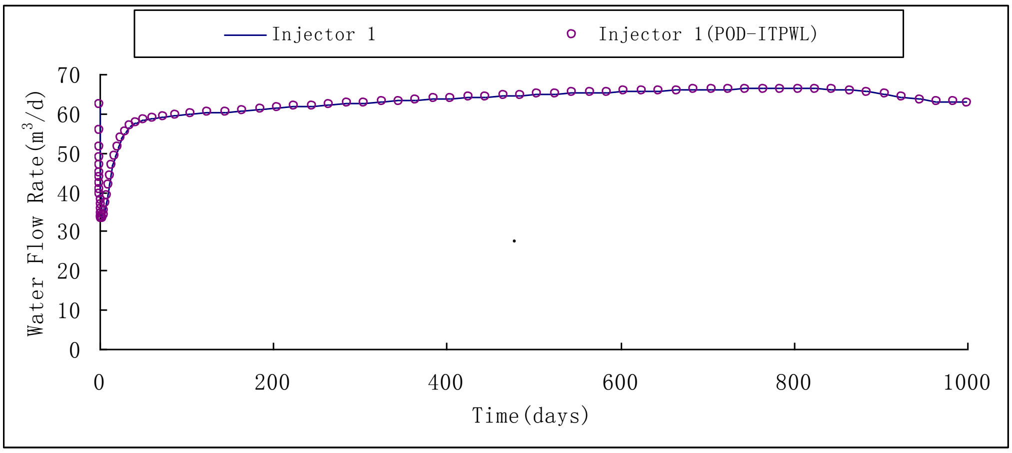

Figure 8 Water flow rate of injection well for POD-ITPWL (schedule 1)

The simulation times for the full-order reservoir simulation, the POD-TPWL reduced-order reservoir simulation, and the POD-ITPWL reduced-order reservoir simulation are given in table 1. The ROM with improved TPWL is able to approximately reduce the simulation time by 5 times compared with time for the full-order reservoir model.

Table 1 Comparison of simulation time (schedule 1)

	full-order	POD-TPWL	POD-ITPWL
Time	95.87s	18.78s	19.82s

(2) Schedule 2

For the schedule II , four production well BHPs are set to 22MPa. The difference is larger compared with the bottom hole pressure of training simulation. The specification for the injection well is the same as in the previous case.

Figures 9 through 11 show the oil and water flow rates for production wells, and water injection rates for the injection well using POD-TPWL method. Figures 12 through 14 show the oil and water flow rates for production wells, and water injection rates for the injection well using POD-ITPWL method. The results demonstrate that when the difference of production well BHPs is larger compared with the bottom hole pressure of training simulation, the accuracy of POD-TPWL method becomes very poor, while the accuracy of POD-ITPWL method is still high.

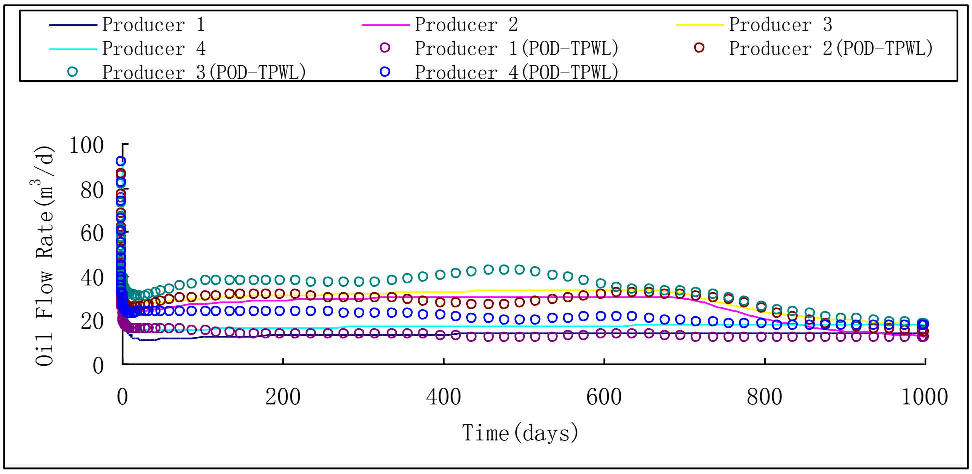

Figure 9 Oil flow rates of four production wells for POD-TPWL (schedule 2)

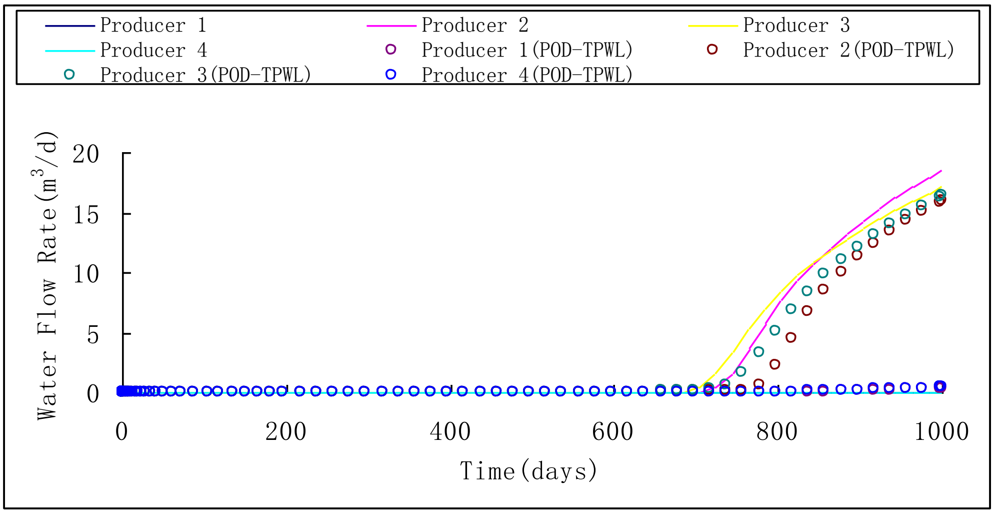

Figure 10 Water flow rates of four production wells for POD-TPWL (schedule 2)

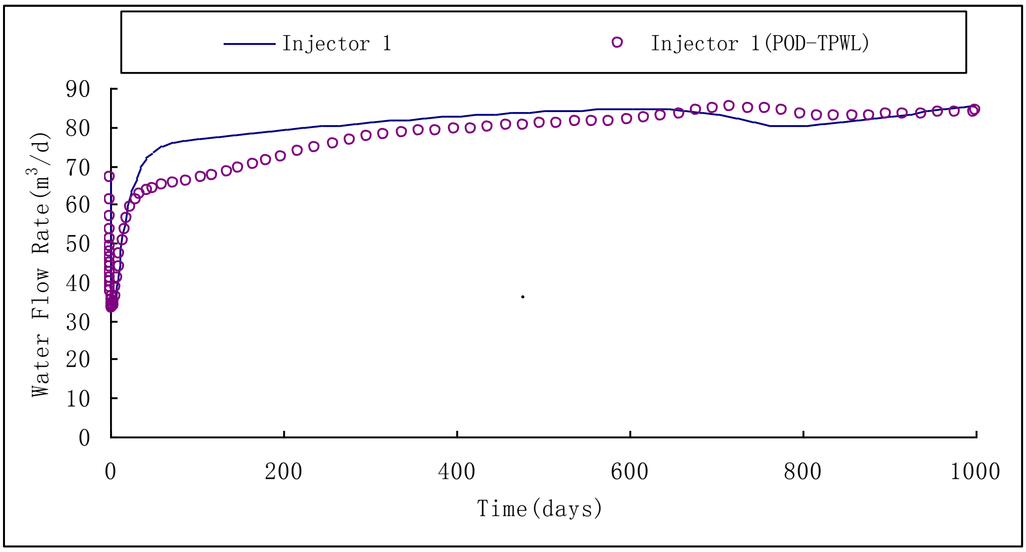

Figure 11 Water flow rate of injection well for POD-TPWL (schedule 2)

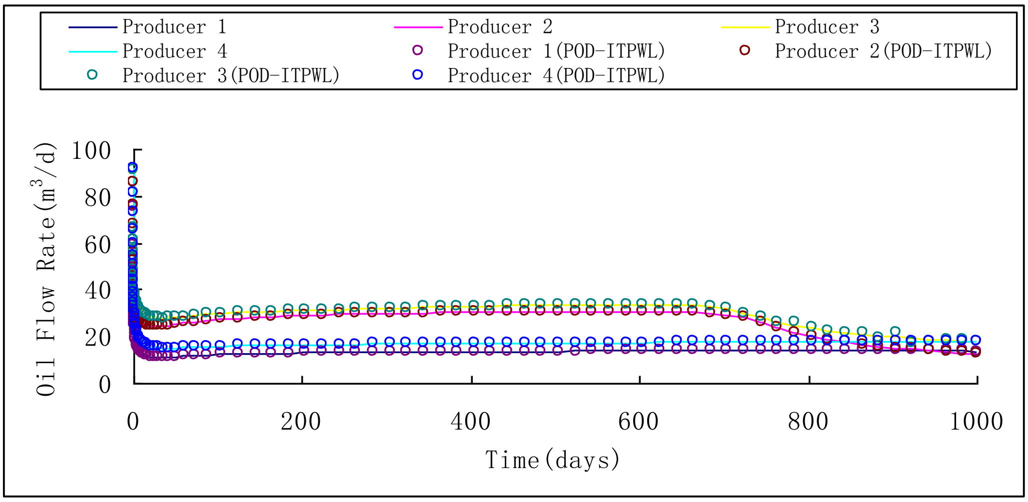

Figure 12 Oil flow rates of four production wells for POD-ITPWL (schedule 2)

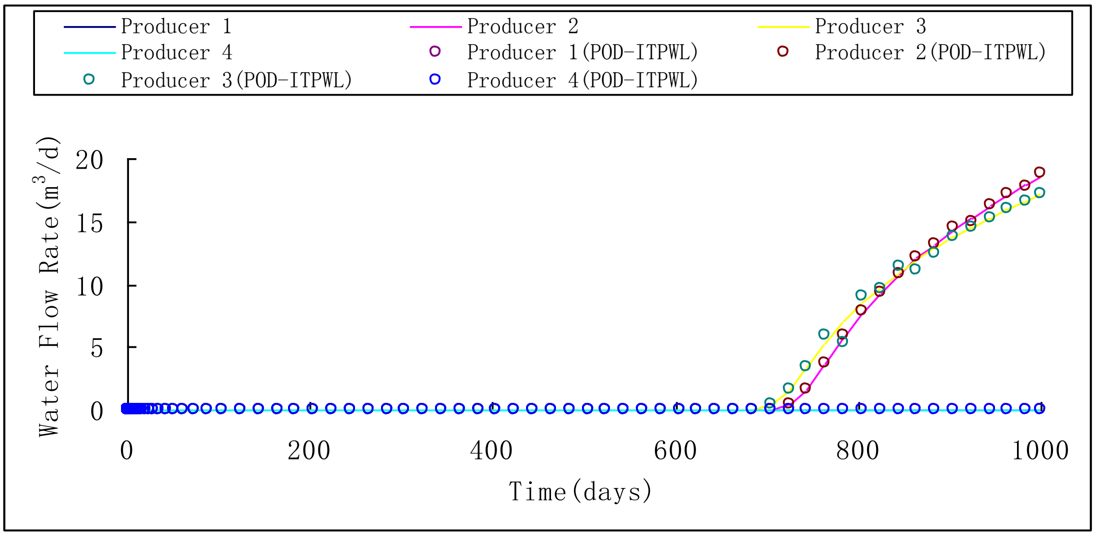

Figure 13 Water flow rates of four production wells for POD-ITPWL (schedule 2)

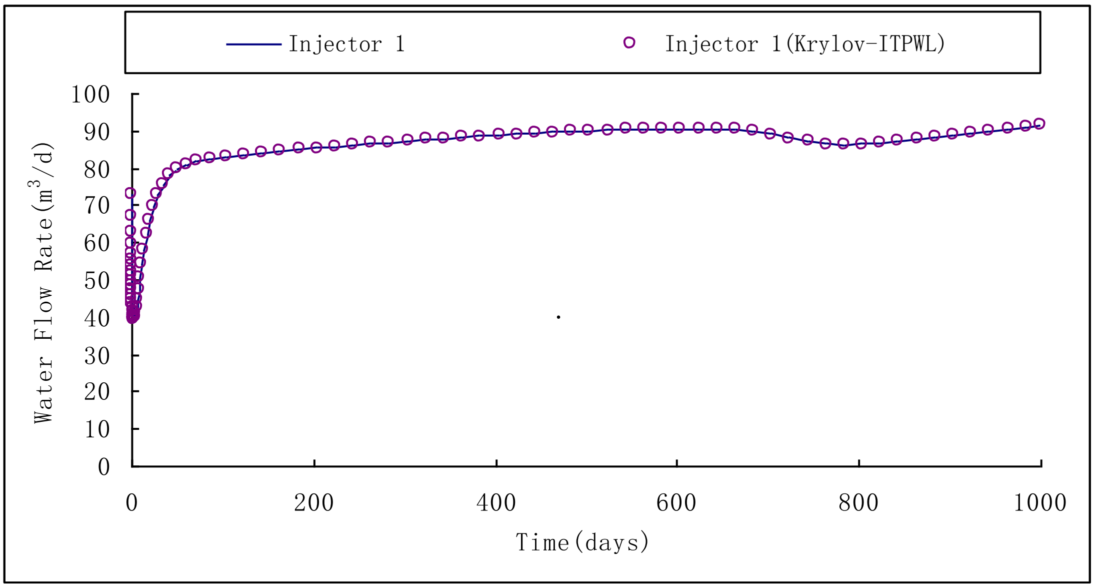

Figure 14 Water flow rate of injection well for POD-ITPWL (schedule 2)

For schedule II, the simulation times are given in table 2. The ROM with improved TPWL is also able to approximately reduce the simulation time by 5 times compared with time for the full-order reservoir model.

Table 2 Comparison of simulation time (schedule 2)

	full-order	POD-TPWL	POD-I TPWL
Time	97.89s	17.67s	18.84s

5. Conclusion

In this work the POD-TPWL and POD-ITPWL methods are applied to a heterogeneous 2D, two-phase (oil-water) model containing 441 grid blocks and five wells. We consider two different scenarios to evaluate the predictive capability of POD-TPWL and POD-ITPWL method. The example demonstrates that if the difference of inputs of testing and training process is smaller, the results of POD-TPWL and POD-ITPWL methods ware close agreement with the full-order simulation. If the difference is larger, the accuracy of POD-TPWL method becomes very poor, while the accuracy of POD-ITPWL method is still high. And POD-ITPWL is able to approximately reduce the simulation time by 4 times compared with time for the full-order reservoir model. Our results show that POD-ITPWL outperforms POD-TPWL in computational accuracy.

This paper demonstrates that the use of reduced-order model based on improved TPWL appears to be a viable approach for reservoir simulation. In future work we plan to test the procedure for larger and more complicated reservoir models.

Reference

[1]. J.S.Han, E.B.Rudnyi, J.G.Korvink. Efficient optimization of transient dynamic problems in MEMS devices using model order reduction, J.Micromech. Microeng. 15(2005)822–832.

[2]. M.Heyouni, K.Jbilou, Matrix Krylov subspace methods for large scale model reduction problems, Appl. Math. Comput. 181(2006)1215–1228.

[3]. M.M.Awais, S.Shamail, N.Ahmed, Dimensionally reduced Krylov subspace model reduction for large scale systems, Appl. Math.Comp. 191(2007) 21–30.

[4]. D.VasiIyev, M. R., and J.White. A TBR-based trajectory piecewise-linear algorithm for generating accurate low-order models for nonlinear analog circuits and MEMS [C]. In:Proceedings of IEEE/ACM Design Automation Conference, 2003:490-495.

[5]. MichaI Rewienski, J. W. A Trajectory Piecewise-Linear Approach to Model Order Reduction and Fast Simulation of Nonlinear Circuits and Micromachined Devices [J]. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 2003, 22: 155-170.

[6]. D. Vasilyev, M. Rewiehski, and J. White. Macromodel generation for BioMEMS components using a stabilized balanced truncation plus trajectory piecewise-linear approach [J]. IEEE Transactions on Computer-aided Design of Integrated Circuits and Systems, 2006, 25(2):285-293.

[7]. M. J. Rewienski. A Trajectory Piecewise-Linear Approach to Model Order Reduction of Nonlinear Dynamical Systems. PhD thesis, Massachusetts Institute of Technology, 2003.

[8]. Khalid Aziz, Antonin Settari. Petroleum Reservoir Simulation [M]. London: Applied Science Publishers, 1979:128-133.

[9]. J. D. Jansen. Systems Description of Flow Through Porous Media [M].Springer: SpringerBriefs in Earth Sciences, 2013:21-36.

[10]. Liu Y, Yuan W Z, Chang H L. A Global Maximum Error Controller-Based Method for Linearization Point Selection in Trajectory Piecewise-Linear Model Order Reduction. IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 2014, 33(7) : 1100－1104.