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Partially distributed heat supply problem of quasistatic transient thermal stresses in a thick circular plate

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Abstract: The thermoelastic deformation in thin circular plates subjected to partially distributed and axisymmetric heat supply the lower face with upper face at zero temperature while circular surface is thermally insulated. The determination of a quasi-static transient thermal stresses in a thick circular plate subjected to partially distributed heat supply. Initially the plate is kept at zero temperature. The results are obtained in series form in terms of Bessel's functions. The results for displacement and stresses have been computed numerically and illustrated graphically.
Keywords: Unsteady State, Thermoelastic, Thermal Stresses, Partially Distributed Heat Supply.

I. INTRODUCTION

The two-dimensional thermoelastoplastic bending deformation analysis of a plate subject to partially distributed heat supply has done by Taingawa et. al.,. They also analysed the thermal bending moment of a laminated composite rectangular plate due to a partially distributed heat supply. And evaluated temperature change, thermal stress, and thermal deflection of simply supported plates and clamped ones. And they have examined the effect of relaxation on distributions of the thermal stress and thermal deflection for the nonhomogeneous rectangular plate. Masaki et at. considered a circular plate and discuss the transient thermoelastoplastic problems marking use of the stain increment theorem. Introduction the methods of finite Hankel transform and Laplace transform to develop the analysis for the temperature field. And for the theoretical analysis of elastoplastic elastoplastic deformation of the plate, treated the axisymmetric problem, some numerical results are shown and discussed.

Recently Kulkarani and Deshmukh studied the thermoelastic problem a thin circular plate subjected to partially distributed heat supply. Here we consider a thick circular plate and studied thermal stresses due to partially distributed heat supply.

In this paper deals with the determination of a quasi static transient thermal stresses in a thick circular plate subjected to partially distributed heat supply on the lower face with upper face at zero temperature while circular surface is thermally insulated. Initially the plate is kept at zero temperature. The results are obtained in series form in terms of Bessel's functions. The results for displacement and stresses have been computed numerically and illustrated graphically.

II. FORMULATION OF PROBLEM

A thick circular plate with radius a and thickness h defined by $0 \le r \le a, \frac{-h}{2}, \le z \le \frac{h}{2}$ respectively is considerd. Let the plate be subjected to the partially distributed heat supply over the lower surface $\left(z = -\frac{h}{2} \right)$ with upper surface $\left(z = \frac{h}{2} \right)$ at zero temperature and fixed circular edge (r = a) is thermally insulated. Initially the plate is kept at zero temperature. Assume the circular surface of thick plate is traction free under this more realistic prescribed condition the quasi-static thermal stresses are required to be determined.

III. HEAT CONDUCTION EQUATION

The temp of plate at time t satisfy the heat conduction equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{K}\frac{\partial T}{\partial t} \quad (1)$$

With the boundary condition

$$T(r,z,t)=f(r,t),\ z=-\frac{h}{2} \tag{2}$$

$$T(r,z,t)=0,\ z=\frac{h}{2} \tag{3}$$

$$\frac{\partial T(r,z,t)}{\partial r}=0,\ r=a \tag{4}$$

and the initial condition

$$T(r,z,0)=0 \tag{5}$$

whereK is the thermal diffusivity of the material of the plate.

IV. DISPLACEMENT POTENTIAL AND THERMAL STRESSES

The differential equation governing the displacement potential function $\phi(r,z)$

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=K\tau \tag{6}$$

where*K* is the restraint coefficient and temperature change $\tau=T-T_i$, T_i is the initial temperature. Displacement function ϕ is known as. Goodier's thermoelastic displacement potential the displacement function is the cylindrical coordinate system are represented by Michell's function.

$$U_r=\frac{\partial\phi}{\partial r}-\frac{\partial^2 M}{\partial r dz} \tag{7}$$

$$U_z=\frac{\partial\phi}{\partial r}+2(1-\nu)\nabla^2 M-\frac{\partial^2 M}{\partial z^2} \tag{8}$$

The Michells function M must satisfy

$$\nabla^2\nabla^2 M=0 \tag{9}$$

Where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \tag{10}$$

The component of the stresses are represented by thermoelastic displacement potential ∅ and Michells function *M* as

$$\sigma_{rr}=2G\left[\frac{\partial^2\phi}{\partial r^2}-K\tau+\frac{\partial}{\partial z}\left(\nu\nabla^2 M-\frac{\partial^2 M}{\partial r^2}\right)\right] \tag{11}$$

$$\sigma_{\theta\theta}=2G\left[\frac{1}{r}\frac{\partial\phi}{\partial r}-K\tau+\frac{\partial}{\partial z}\left(\nu\nabla^2 M-\frac{1}{r}\frac{\partial M}{\partial r}\right)\right] \tag{12}$$

$$\sigma_{zz}=2G\left[2\frac{\partial\phi}{\partial r^2}-K\tau+\frac{\partial}{\partial z}(2-\nu)\left(\nu\nabla^2 M-\frac{\partial^2 M}{\partial z^2}\right)\right] \tag{13}$$

and

$$\sigma_{rz}=2G\left[\frac{\partial^2\phi}{\partial r\partial z}+\frac{\partial}{\partial r}\left((1-\nu)\nabla^2 M-\frac{\partial^2 M}{\partial z^2}\right)\right] \tag{14}$$

where*G* and ν are shear modulus and poisons ration respectively for the traction free surface stress function

$$\sigma_{rr}=\sigma_{rz}=0\quad at\qquad r=a \tag{15}$$

The equation (1) to (15) constitutesthe mathematical formulation of the problem.

2 SOLUTION

2.1 TEMPERATURE CHANGE

To obtain the expression for temp $T(r,z,t)$

Assume $T(r,z,t)=\sinh\left(z-\frac{h}{2}\right)\sum_{n=1}^{\infty} f_n(t)J_0(\alpha_n r)$ (16)

Where $\alpha_1\alpha_2$ … the root of the transcendental equation $J_1(\alpha_n a)=0$ and $J_n(x)$ is the Bessel's function of the first kind of order *n*.

Using the equation (16) in (1) one obtain

$$f'_n(t)=\left(1-\alpha_n^2\right) k\ t\ f_n(t)$$

On Integrating

$$f_n(t)=A_n e^{\left(1-\alpha_n^2\right)k\,t} \tag{17}$$

A_n is constant.

Using the equation (17) in (16) obtain

$$T(r,z,t)=\sinh\left(z-\frac{h}{2}\right)\sum_{n=1}^{\infty} A_n J_0(\alpha_n r).e^{\left(1-\alpha_n^2\right)k\,t} \tag{18}$$

The constant A_n, can be found from the nature of temperature of lower face using the condition given in the equation in (2), one obtain

$$F(r,t)[H(r-a)-H(r)]=h\,A_n\,J_0(\alpha_n r)e^{\left(1-\alpha_n^2\right)k\,t}$$

Hence by the theory of Bessells function

$$\int_0^a f(r)[H(r-a)-H(r)]r\,J_0(\alpha_n r)dr=h\int_0^a A_n\,r\,J_0^2(\alpha_n r)e^{\left(1-\alpha_n^2\right)k\,t}dr$$

Using

$$\int_0^a r(\alpha_n r)dr=\left(\frac{a^2}{2}\right)J_0^2(\alpha_n a)$$

one obtains,

$$\frac{2}{a^2hJ_0^2(\alpha_n a)}\int_0^a rJ_0(\alpha_n r)f(r,t)[H(r-a)-H(r)]dr=A_n e^{\left(1-\alpha_n^2\right)kt}$$

Now taking Laplace transform

$$\frac{2}{a^2hj_0^2(\alpha_n a)}\int_0^a rJ_0(\alpha_n r)f(r,p)[H(r-a)-H(r)]dr=A_n\frac{1}{P-\left(1-\alpha_n^2\right)kt}$$

where $f(r,p)=L[f(r,t)]$ and p is the Laplace transform parameter

$$A_n=\frac{2\left(P+\alpha_n^2k-k\right)}{a^2J_0^2(\alpha_n a)}\int_0^a rJ_0(\alpha_n r)f(r,p)[H(r-a)-H(r)]dr$$

Finally taking inverse Laplace transform by using convolution theorem

$$A_n=\frac{2}{a^2hJ_0^2(\alpha_n a)}\int_0^t\int_0^a\left\{\frac{1}{u}rJ_0(\alpha_n r)f(r,t-u)[H(r-a)-H(r)]e^{-\alpha_n^2kt\,\delta(u)}\right\}dr\,du$$

Where δ (t) is well known Dirac delta function, One obtain

$$A_n=\frac{2}{a^2hJ_0^2(\alpha_n a)}\int_0^t\int_0^a\left\{rJ_0(\alpha_n r)f(r,t-u)[H(r-a)-H(r)]e^{\left(1-\alpha_n^2\right)kt}\right\}drdu \tag{19}$$

Substituting the equation (19) in the equation (18) one obtain the temperature distribution function as

$$T(r,z,t)=\frac{2}{a^2h}\sinh\left(z-\frac{h}{2}\right)\sum_{n=1}^{\infty}\frac{J_0(\alpha_n r)}{J_0^2(\alpha_n a)}\int_0^t\int_0^a\left\{rJ_0(\alpha_n r)f(r,t-u)[H(r-a)-H(r)]e^{\left(1-\alpha_n^2\right)kt}\right\}drdu \tag{20}$$

Since the initial temperature $T_i = 0$ the temperature charges $\tau = T - T_i \Rightarrow \tau = T$

$$T = \sinh\left(z - \frac{h}{2}\right)\sum_{n=1}^{\infty} A_n J_0(\alpha_n a) exp\left[\left(1-\alpha_n^2\right) k\, t\right] \tag{21}$$

2.2 MICHELLS FUNCTION

Now suitable form of M Satisfying (9)

$$M = \left[H_n J_0(\alpha_n r) + R_n \alpha_n r J_1(\alpha_n r)\right] Cosh\left[\alpha_n\left(z + \frac{h}{2}\right)\right] \tag{22}$$

H_n and R_n are arbitrary function.

Assuming displacement function $\phi(r,z,t)$ which satisfy (6) as

$$\phi(\mathrm{r,z,t}) = \sum_{n=1}^{\infty} \frac{kA_n}{1-\alpha_n^2} J_0(\alpha_n r) \sinh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt} \tag{23}$$

2.3 DISPLACEMENT AND THERMAL STRESSES

Now using the equation (21) (22) and (23) in equation (7) (8) and (11) to (14) one obtain the expression for displacement and stress respectively as

$$U_r = -K\sum_{n=1}^{\infty} \frac{\alpha_n A_n J_1(\alpha_n r)}{\left(1-\alpha_n^2\right)}.\sinh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty} H_n J_1(\alpha_n r) \sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right]$$
$$- \sum_{n=1}^{\infty} \alpha_n^3 R_n\, r J_0(\alpha_n r).\sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right] \tag{24}$$

$$U_z = K\frac{A_n}{(1-\alpha_n^2)} J_0(\alpha_n r).cosh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt} - H_n J_0(\alpha_n r).cosh[_n\left(z + \frac{h}{2}\right)]$$
$$-K\sum_{n=1}^{\infty} R_n[4(1-\nu)J_0(\alpha_n r) - \alpha_n.rJ_1(\alpha_n r)].\mathrm{coshcosh}\left[\alpha_n\left(z + \frac{h}{2}\right)\right] \tag{25}$$

$$\sigma_{rr} = 2G\left\{K\sum_{n=1}^{\infty} \frac{A_n J_1(\alpha_n r)}{\left(1-\alpha_n^2\right)}\left[\sinh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}\right] - K\sum_{n=1}^{\infty} A_n J_0(\alpha_n r).sinh\left(z - \frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt}\right. \tag{26}$$
$$\left.+\sum_{n=1}^{\infty} \alpha_n^2 H_n\left(\alpha_0 J_0(\alpha_n r) - \frac{J_1(\alpha_n r)}{r}\right).\sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right] + \sum_{n=1}^{\infty} R_n\left[(2\nu-1)J_0(\alpha_n r) - r\alpha_n r J_1(\alpha_n r)\right].\sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right]\right\}$$

$$\sigma_{\theta\theta} = 2G\left\{\sum_{n=1}^{\infty}\left[\frac{kA_n \alpha_n J_1(\alpha_n r)}{\left(1-\alpha_n^2\right)} \sinh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}\right] - \sum_{n=1}^{\infty} kA_n J_0(\alpha_n r)\sinh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}\right.$$
$$\left.+\sum_{n=1}^{\infty} \alpha_n^2 H_n\left(\frac{J_1(\alpha_n r)}{r}\right).\sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right] + \sum_{n=1}^{\infty} \alpha_n^3 R_n(2\nu-1)J_0(\alpha_n r)\sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right]\right\} \tag{27}$$

$$\sigma_{zz} = 2G\left\{K\sum_{n=1}^{\infty}\left[\frac{kA_n}{\left(1-\alpha_n^2\right)} J_0(\alpha_n r).\sinh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}\right] - \sum_{n=1}^{\infty} kA_n J_0(\alpha_n r)\sinh\left(z - \frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt}\right.$$
$$\left.+\sum_{n=1}^{\infty} \alpha_n^3 H_n J_0(\alpha_n r).\sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right] + \sum_{n=1}^{\infty} \alpha_n^3 R_n\left[2(2-\nu)J_0(\alpha_n r) - \alpha_n r J_1(\alpha_n r).\sinh\left[\alpha_n\left(z + \frac{h}{2}\right)\right]\right]\right\} \tag{28}$$

$$\sigma_{rz} = 2G\left\{K\sum_{n=1}^{\infty} \frac{\alpha_n kA_n}{\left(1-\alpha_n^2\right)} J_1(\alpha_n r).\cosh\left(z - \frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty} \alpha_n^3 H_n J_1(\alpha_n r).\cosh\left[\alpha_n\left(z + \frac{h}{2}\right)\right]\right.$$
$$\left.+\sum_{n=1}^{\infty} \alpha_n^3 R_n\left[2(1-\nu)J_1(\alpha_n r) + \alpha_n r J_0(\alpha_n r).\cosh\left[\alpha_n\left(z + \frac{h}{2}\right)\right]\right]\right\} \tag{29}$$

Determination of unknown arbitrary function H_n and R_n

On using equation (15) in the equation (26) and (29),one obtain

$$H_n = K\sum_{n=1}^{\infty}\frac{A_n \sinh\left(z-\frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}}{\alpha_n^3 \sinh\left[\alpha_n\left(z-\frac{h}{2}\right)\right]} \tag{30}$$

$$R_n = 0 \tag{31}$$

Substituting the equation (30) and (31) in the equation (24) to (29), the expression for displacement and stresses respectively as

$$U_r = -k\sum_{n=1}^{\infty}\alpha_n A_n J_1(\alpha_n r).\sinh\left(z-\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty}\frac{1}{\alpha_n^3}kA_n J_1(\alpha_n r).\sinh\left(z-\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt} \tag{32}$$

$$U_z = -K\sum_{n=1}^{\infty}\frac{A_n J_0}{1-\alpha_n^2}(\alpha_n r).\sinh\left(z-\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty}\frac{1}{\alpha_n \sinh\left[\alpha_n\left(z-\frac{h}{2}\right)\right]} \times kA_n J_0(\alpha_n r)\cosh\left[\alpha_n\left(z-\frac{h}{2}\right)\right]e^{\left(1-\alpha_n^2\right)kt} \tag{33}$$

$$\sigma_{rr} = 2G\left\{k\sum_{n=1}^{\infty}\frac{A_n J_1(\alpha_n r)}{\left(1-\alpha_n^2\right)}\left[\sinh\left(z-\frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}\right] - k\sum_{n=1}^{\infty}A_n J_0(\alpha_n r)sinh\left(z-\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty}\frac{kA_n}{\alpha_n}\left(\alpha_n J_0(\alpha_n r)-\frac{J_1(\alpha_n r)}{r}\right)\sinh\left(z+\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt}\right\} \tag{34}$$

$$\sigma_{\theta\theta} = 2G\left\{\sum_{n=1}^{\infty}\left[\frac{kA_n\alpha_n J_1(\alpha_n r)}{\left(1-\alpha_n^2\right)}\sinh\left(z-\frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}\right] - \sum_{n=1}^{\infty}k A_n J_0(\alpha_n r)\sinh\left(z-\frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty}\frac{1}{r\alpha_n}k A_n J_1(\alpha_n r)\sinh\left(z+\frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt}\right\} \tag{35}$$

$$\sigma_{zz} = 2G\left\{k\sum_{n=1}^{\infty}\frac{A_n}{1-\alpha_n^2}\sinh\left(z-\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt} - \sum_{n=1}^{\infty}k A_n J_0(\alpha_n r)\sinh\left(z-\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty}kA_n J_0(\alpha_n r)\sinh\left(z+\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt}\right\} \tag{36}$$

$$\sigma_{rz} = 2G\left\{k\sum_{n=1}^{\infty}\frac{k\alpha_n A_n}{\left(1-\alpha_n^2\right)}J_1(\alpha_n r)c\,osh\left(z-\frac{h}{2}\right).e^{\left(1-\alpha_n^2\right)kt} + \sum_{n=1}^{\infty}\frac{1}{\alpha_n^3 \sinh\left[\alpha_n\left(z+\frac{h}{2}\right)\right]}k A_n J_0(\alpha_n r)\cosh\left[\alpha_n\left(z+\frac{h}{2}\right)\right]\sinh\left(z+\frac{h}{2}\right)e^{\left(1-\alpha_n^2\right)kt}\right\} \tag{37}$$

V. UMERICAL CALCULATION

The numerical calculation have been carried out for steel (SN 50C) plate with parameters a = 1m, b = **0.5M, H = 0.4M, THERMAL DIFFUSIVITY K= 15.9 X 10-6(M2 S-1) WITH α1= 3.8317, α2=7.0156, α3 =** 10.1735, α_4= 13.3237, α_5= 16.470, α_6= 19.6159, α_7=22.7601, α_8=25.9037, α_9=29.0468, α_{10}=32.18 are the roots of transcendental equation $J_1(\alpha\,.\,a)=0.$

For convenience setting $A = \frac{-2T_0}{a^2 h 10^5}$, $B = \frac{-2KT_0}{a^2 h 10^5}$, $C = \frac{-4GKT_0}{a^2 h 10^5}$ in the expression (21) and (33) to (37).

In order to examine the influence of partially distributed hear supply on upper surface of the thick circular plate, one performed the numerical calculation r = 0, 0.2, 0.4, 0.6, 0.8, 1 and z = -0.2, -0.1, 0.1, 0.2. Numerical variations in radial direction on the upper surface (z = 0.2) where partially distributed heat supply is given are shown in the following figures with the help of computer programming.

VI. CONCLUSION

In this paper, a thick circular plate is considered and determined the expressions for temperature, displacement and stress function due to partially distributed heat supply on the lower surface. As a special case mathematical model is constructed for $z = -\frac{h}{2}$ i.e. $T = \begin{cases} 0 & \text{otherwise} \\ T_0 (r-a)^2 \left(1 - e^{-kt}\right) & 0 < r \le a \end{cases}$

and performed numerical calculations. The thermoelastic behavior is examined such as temperature change, displacement and stresses with the help of partially distributed heat supply on the lower surface.

From Figure 1: Axial displacement function u_z decreases with the time within circular region $0 \le r \le 0.6$

From Figure 2: Stress function σ_{zz} decreases with the time within circular region $0 \le r \le 0.5$. It develops compressive stresses within circular region $0 \le r \le 0.5$

From Figure 3: The stress function $\sigma_{\theta\theta}$ decreases with the time within circular region $0 \le r \le 0.4$. It develops compressive stresses within circular region $0 \le r \le 0.4$.

From Figure 4: Radial displacement function u_r increases with the time. It shown normal curve within $0 \le r \le 1$.

From Figure 5: The temperature function increases with the time. The graph of temperature function shows partially distribution of heat within $0 \le r \le 1$.

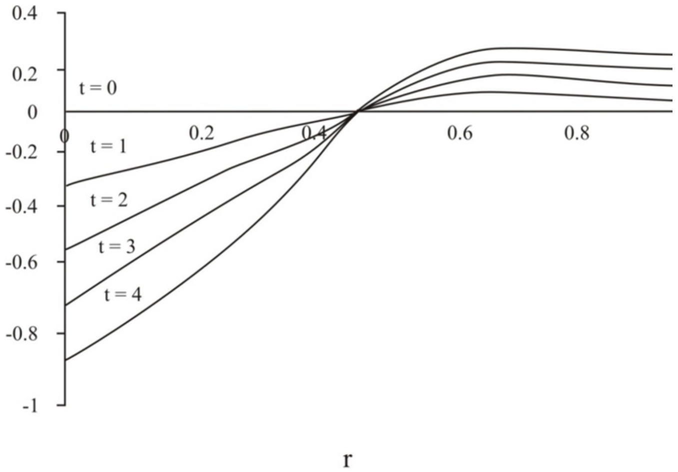

Figure 1: Variation of Axial displacement function $\mathbf{u_z}$ versus **r** for different value of **t**

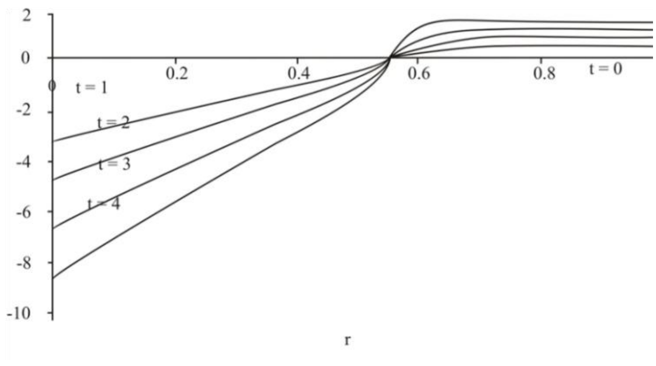

Figure 2: Variation of stress function $\boldsymbol{\sigma_{zz}}$ versus **r** for different value of **t**

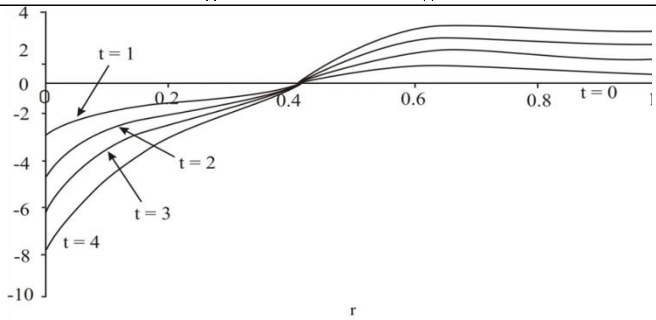

Figure 3: Variation of stress function$\sigma_{\theta\theta}$ versus **r** for different value of **t**

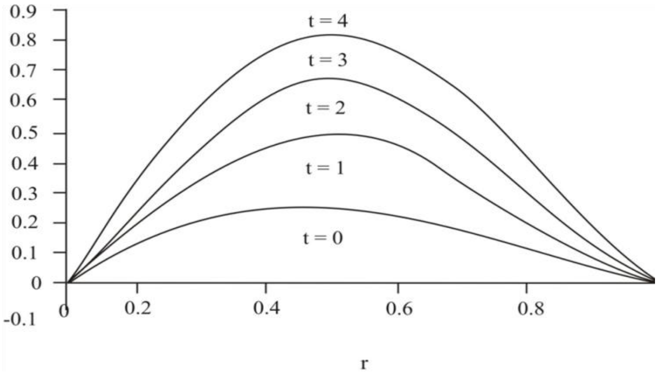

Figure 4: Variation of radial displacement function **u** $_r$ versus **r** for different value of **t**

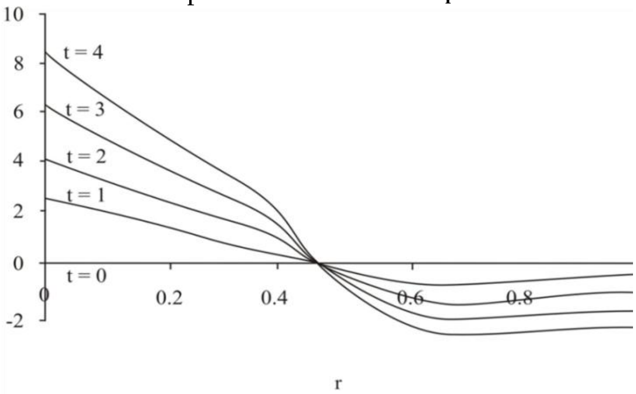

Figure 5: Variation of the temperature function versus **r** for different value of **t**

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