International Journal of Latest Engineering and Management Research (IJLEMR)
ISSN: 2455-4847
www.ijlemr.com || Volume 02 - Issue 10 || October 2017 || PP. 58-69

Analysis and Design of LDPC coded Multiuser Discrete Wavelet based MIMO-OFDM Systems

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Abstract: In this paper, the performance analysis and design optimization of low-density parity check (LDPC) coded Multiuser Discrete Wavelet based multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems for high data rate wireless transmission is considered. The tools of density evolution with mixture Gaussian approximations are used to optimize irregular LDPC codes and to compute minimum operational signal-to-noise ratios (SNRs) for ergodic MIMO OFDM channels. In particular, the optimization is done for various MIMO OFDM system configurations, which include a couple of antennas, and Quadrature Phase shift keying (QPSK) demodulation scheme; It is shown that along with the optimized irregular LDPC codes, a turbo iterative receiver that consists of a soft maximum a posteriori (MAP) demodulator and a belief-propagation LDPC decoder can perform within the ergodic capacity of the MIMO OFDM systems under consideration. It is also shown that compared with the optimal MAP demodulator-based receivers, the receivers employing a low-complexity linear minimum mean-square-error soft-interference-cancellation (LMMSE-SIC) demodulator have a small performance loss in spatially correlated MIMO channels. The optimized performance is compared with the corresponding LDPC coded FFT based MIMO OFDM system. It is also shown that compared with the existing FFT based receivers, the DWT based receivers employing a low-complexity linear minimum mean-square-error soft-interference-cancellation (LMMSE-SIC) demodulator have a small performance loss in spatially correlated MIMO channels.

Keywords: LDPC, LMMSE-SIC, MIMO, mixture Gaussian, MAP

1. Introduction

One of the ambitious design goals of fourth–generation (4G) wireless cellular systems is to reliably provide very high data rate transmission, for example, around 100 Mb/s peak rate for downlink and around 30 Mb/s sum rate for uplink transmission. Due to its higher rate requirement, the downlink transmission is especially considered to be a bottleneck in system design. In this paper, the feasibility of downlink transmission in 4G wireless systems through the physical layer (PHY) design and optimization of low-density parity check (LDPC) coded Discrete wavelet based multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) communications is demonstrated. In the considered systems, different users access the downlink channels in a time-division multiple accessing (TDMA) manner, dynamic with a certain scheduling scheme [1]. Compared with other alternative solutions, the MIMO-OFDM downlink transmission proposed here attempts to balance between high rate transmission and low receiver complexity of mobile devices, where the former primarily counts on the LDPC-coded MIMO techniques [2].

In this paper, the schemes that require no CSI at transmitter is considered and aim to achieve very high data rate. In particular, an LDPC-coded MIMO OFDM scheme proposed in [2] is focused. For a fixed target data rate (e.g., 100 Mb/s), we optimize and compare the performance of the LDPC-coded DWT based MIMO OFDM system with LDPC-coded FFT based MIMO OFDM systems with Quadrature Phase shift keying (QPSK) demodulation scheme. For a fair comparison, the quantity $SNR_{min,op}(dB) - C^{-1}(R)(dB)$ as the performance measure is adopted, which reflects how many decibels the minimum operational $SNR_{min,op}$ is above the SNR required by the information theoretic channel capacity $C(\cdot)$ to support a target information rate . The notion of data rate (in the unit of bits per second) will be differentiated from information rate (in the unit of bits per second per Hertz) when the bandwidth (in the unit of Hertz) is not specified or fixed. Considered the suboptimal linear minimum mean-square-error based soft interference cancellation (LMMSE-SIC) demodulator with a complexity at $\mathcal{O}(|\Omega|^3)$ instead of maximum a posteriori (MAP) demodulator with a complexity at $\mathcal{O}(|\Omega|^N)$.where $|\Omega|$ is the constellation size of modulator, and N is the number of transmitter antennas. Also considered in this paper, the spatially correlated MIMO channels where the channel capacity can be substantially reduced from the spatially uncorrelated MIMO channel model [4].

The work in [3] studied the LDPC code design in the MIMO systems under the framework of turbo iterative signal processing and decoding via the tools of EXIT charts. In this work, the techniques of density evolution with mixture Gaussian approximations [5], [7] to design and optimize the irregular LDPC codes, as well as to compute the $SNR_{min,op}$ for ergodic MIMO-OFDM channels are employed. Furthermore, from the LDPC profiles that are optimized for the ergodic channels, heuristically constructed small block-size irregular LDPC codes for outage MIMO OFDM channels. In the end, quantitative results from computer simulations give rise to a number of useful observations and conclusions in the design and optimization of the LDPC coded DWT based MIMO OFDM systems.

The paper is organized as follows: In Section II, an LDPC-coded DWT based MIMO OFDM system with a brief summary of the system model, background on the LDPC codes and channel capacity of MIMO OFDM modulation is described. In Section III, a turbo iterative receiver is introduced, with a brief review of the different demodulation and the decoding schemes. In Section IV, the procedure of analyzing and optimizing the LDPC codes for MIMO OFDM systems is discussed. In Section V, the performance comparison and optimization results for LDPC-coded DWT and FFT based MIMO OFDM systems with different system configurations are demonstrated and discussed. Section VI contains the conclusions.

2. System Description of LDPC coded DWT based MIMO OFDM

Consider a LDPC-coded MIMO OFDM system with *K* subcarriers, *M* transmitter antennas, and *N* receiver antennas, signaling through frequency-selective fading channels. The transmitter structure is illustrated in Fig.1. A block of *k* bits of information data is encoded by a rate $r = k/n$ LDPC code. The output n coded bits are interleaved, modulated by Quadrature Phase shift Keying (QPSK) constellation into a block of $n\,/\log_2|\Omega|$ QPSK symbols. During each OFDM slot, NK out of the total $n\,/\log_2|\Omega|$ QPSK symbols are transmitted from *K* OFDM subcarriers and *N* transmitter antennas simultaneously. Due to the inherent random structure of LDPC codes, the *NK* symbols can be mapped to *K* subcarriers and *N* transmitter antennas in any order. Without loss of generality, assume $(n\,/\log_2|\Omega|)/(NK) = \tilde{n}$, i.e., the total block of QPSK symbols is transmitted in $\tilde{n}$ OFDM slots.

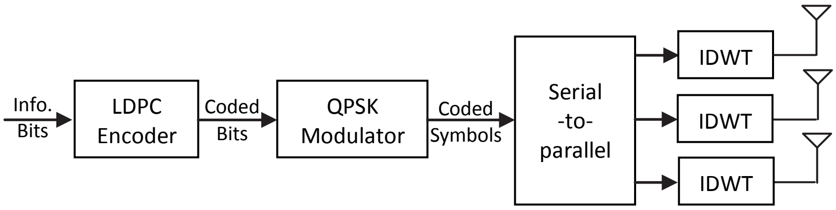

Fig.1 Transmitter Structure of an LDPC coded DWT based MIMO OFDM system

Note that in Fig.1, LDPC could also be replaced by other error-control codes such as Turbo codes; however, the relatively low and scalable decoding complexity and the freedom for code optimization make LDPC codes a more favorable candidate [2].

2.1 MIMO OFDM Modulation

Consider a quasi static block fading model for the studied MIMO OFDM modulation, and assume that the fading channels remain static during each OFDM slot but vary independently from one OFDM slot to another. Furthermore, for practical MIMO-OFDM systems with spatial (antenna) correlations, the frequency domain channel response matrix at the *k*th (*k* = 0,...., *K*−1) subcarrier and the *p*th (*p* = 0,...., $\tilde{n}$−1) OFDM slot is given by [8]:

$$\boldsymbol{H}[p,k] = \sum_{l=0}^{L-1} \boldsymbol{R}_l^{1/2}\, H_l[p] \boldsymbol{S}_l^{1/2} exp\left(\frac{-j2\pi lk}{K}\right) \tag{1}$$

where $\boldsymbol{R}_l = \boldsymbol{R}_l^{1/2}\boldsymbol{R}_l^{1/2}$, and $\boldsymbol{S}_l = \boldsymbol{S}_l^{1/2}\boldsymbol{S}_l^{1/2}$ represent the receive and transmit spatial-correlation matrices, which are determined by the spacing and the angle spread of MIMO antennas. *L* is the number of resolvable paths of the frequency-selective fading channels; $H_l[p]$ is the matrix with entries being independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian, distributed as $\sim \mathcal{N}_c(0, \beta_l^2)$, and is assumed to be independent for different *l* and different *p*; in addition, the power of $H_l[p]$, for all *l* is normalized by letting $\sum_{l=0}^{L-1} \beta_l^2 \equiv 1$.

Assume proper cyclic insertion and sampling, the MIMO-OFDM system with *K* subcarriers decouples frequency-selective channels into *K* correlated flat-fading channels with the following input–output relation:

$$\boldsymbol{y}[p,k] = \sqrt{\frac{\text{SNR}}{N}}\boldsymbol{H}[p,k]\,\boldsymbol{x}[p,k] + \boldsymbol{z}[p,k] \tag{2}$$
$$k = 0, 1, \ldots, K-1, p = 0,1,\ldots,\tilde{n}-1$$

Where $\boldsymbol{H}[p,k] \in C^{M\times N}$ is the matrix of complex channel frequency responses defined in (1); $\boldsymbol{x}[p,k] \in \Omega^N$ and $\boldsymbol{y}[p,k] \in C^M$ are, respectively, the transmitted signals and the received signals at the *k*th subcarrier and the *p*th slot; $\boldsymbol{z}[p,k] \in C^M$ is the additive noise with i.i.d. entries $\boldsymbol{z}[p,k] \sim \mathcal{N}_c(0,\boldsymbol{I})$; and SNR denotes the average signal-to-noise ratio at each receiver antenna. Note that, only the fixed/deterministic signal constellation Ω is considered and its averaged power is normalized to be one.

Assuming Gaussian signaling (i.e., $\Omega \to C$) for MIMO OFDM channels with infinite fading channel observations (i.e, $\tilde{n}\to\infty$), the *ergodic capacity* is given by (3), as shown below:

$$C_{erg}(\text{SNR}) \triangleq E\left\{\underbrace{\frac{1}{K\tilde{n}}\sum_{k=0}^{K-1}\sum_{p=0}^{\tilde{n}-1}\left[\log_2 det\left(\boldsymbol{I}_M + \frac{\text{SNR}}{N}\boldsymbol{H}[p,k]\boldsymbol{H}^H[p,k]\right)\right]}_{\mathcal{J}_{|\mathcal{H}}(\text{SNR})}\right\} \tag{3}$$

where ^{H} denotes the Hermitian transpose; the expectation is taken over random channel states $\mathcal{H}$, with $\mathcal{H} \triangleq \{\boldsymbol{H}[p,k]\}_{p,k}$; and $\mathcal{J}_{|\mathcal{H}}(\text{SNR})$ is the instantaneous mutual information conditioned on $\mathcal{H}$. For MIMO OFDM channels with finite fading channel observations (i.e, $\tilde{n} \ll \infty$), the *outage capacity/probability* is a more sensible measure. For a target information rate R, the outage probability is given by:

$$P_{out}(R, \text{SNR}) = P(\mathcal{J}_{|\mathcal{H}}(\text{SNR}) < R) \tag{4}$$

In practice, the transmitted signals usually take values from constraint constellation, i.e., $\boldsymbol{x} \in \Omega^N$. In this case, the mutual information is computed instead, shown below, where the expectation is taken over random noise vector $\boldsymbol{z} \sim \mathcal{N}_c(0,\boldsymbol{I})$:

$$\mathcal{J}_{|\mathcal{H}}(\text{SNR}) \triangleq N log_2|\Omega| - \frac{1}{K\tilde{n}|\Omega|^N} \times \sum_{k=0}^{K}\sum_{p=0}^{\tilde{n}-1}\sum_{j=0}^{|\Omega|^N-1} E\left\{\log_2 \sum_{i=0}^{|\Omega|^N-1} exp\left[-\left\|\sqrt{\frac{\text{SNR}}{N}}\,\boldsymbol{H}[p,k](\boldsymbol{x}^i - \boldsymbol{x}^j) + \right.\right.\right.$$
$$z2+z2 \tag{5}$$

2.2 Low-Density Parity Check (LDPC) Codes

A low density parity check (LDPC) code is a linear block code specified by a very sparse parity check matrix. The parity check matrix $\boldsymbol{P}$ of a *regular (n,k,s,t)* LDPC code of rate $r = k/n$ is a $(n-k)\times n$ matrix, which has s ones in each column and $t > s$ ones in each row, where $s \ll n$, and the ones are typically placed at random in the parity check matrix. When the number of ones in every column is not the same, the code is known as an *irregular* LDPC code. Although deterministic construction of LDPC codes is possible, in this paper only pseudo-random constructions are considered.

The code with parity check matrix $\boldsymbol{P}$ can be represented by a bipartite graph that consists of two types of nodes—variable nodes and check codes. Each code bit is a variable node, whereas each parity check or each row of the parity check matrix represents a check node. An edge in the graph is placed between variable node *i* and check node *j* if $P_{j,i} = 1$. That is, each check node is connected to code bits whose sum modulo-2 should be zero. Irregular LDPC codes are specified by two polynomials $\lambda(x) = \sum_{i=1}^{d_{l_{max}}} \lambda_i\, x^{i-1}$ and $\rho(x) = \sum_{i=1}^{d_{r_{max}}} \rho_i\, x^{i-1}$, where λ_i is the fraction of edges in the bipartite graph that are connected to variable nodes of degree *i*, and ρ_i is the fraction of edges that are connected to check nodes of degree *i*. Equivalently, the degree profiles can also be specified from the node perspective, i.e., two polynomials $\tilde{\lambda}(x) = \sum_{i=1}^{d_{l_{max}}} \tilde{\lambda}_i\, x^{i-1}$ and $\tilde{\rho}(x) = \sum_{i=1}^{d_{r_{max}}} \tilde{\rho}_i\, x^{i-1}$, where $\tilde{\lambda}_i$ is the fraction of variable nodes of degree *i*, and $\tilde{\rho}_i$ is the fraction of check nodes of degree *i*.

3. Iterative Receiver Structure

A serial concatenated turbo iterative receiver is employed as shown in Fig.2 to approach the maximum likelihood (ML) receiver performance of joint MIMO OFDM demodulation and LDPC decoding.

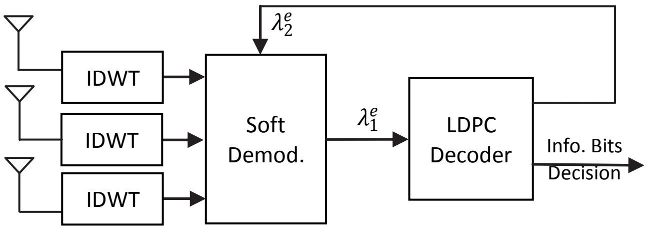

Fig.2 Turbo receiver structure for an LDPC-coded DWT based MIMO OFDM system

The extrinsic information of the LDPC-coded bits is iteratively passed between a soft demodulator and a soft belief-propagation LDPC decoder. In each demodulator decoder, a number of inner iterations are performed within the soft LDPC decoder, during which time; extrinsic information is passed along the edges in the bipartite graph.

Here, all extrinsic information (message) is in log-likelihood (LLR) form, and the variable $\boldsymbol{L}$ is used to refer to extrinsic information. The variable f is used to denote the probability density function of the extrinsic information, and m is used to denote the mean of $\boldsymbol{L}$. Superscript *(p,q)* is used to denote quantities during the *p*th round of inner decoding within the LDPC decoder and the *q*th stage of outer iteration between the LDPC decoder and the MIMO OFDM demodulator. For the quantities passed between the soft MIMO OFDM demodulator and the soft LDPC decoder, only one superscript *q*, the iteration number of turbo iterative receiver, is used. A subscript $D \rightarrow L$ denotes quantities passed from the demodulator to the LDPC decoder, and vice versa, $D \leftarrow L$.

3.1 Demodulation of MIMO OFDM

Assuming the perfect CSI at the receiver, it is clear from (1) that the demodulation of the received signals at a particular sub-carrier and a particular slot can be carried out independently. As illustrated in Fig. 3, at the qth turbo iteration, the soft MIMO OFDM demodulator computes extrinsic information of the LDPC code bit b_i as:

$$L_{D\rightarrow L}^{q}(b_i) = g\left(\boldsymbol{y}, \{L_{D\rightarrow L}^{q-1}(b_i)\}_{\boldsymbol{j}}\right) \tag{6}$$

where $\boldsymbol{y}$ is the received data; $\{L_{D\leftarrow L}^{q-1}(b_i)\}_j$ is the extrinsic information computed by LDPC decoder in the previous turbo iteration at the first turbo iteration $L_{D\leftarrow L}^{q-1}(b_i) \equiv 0,\ \forall_j$; and $g(\cdot)$ denotes the demodulation function, which is described below.

At a given subcarrier and time slot, $N \log_2|\Omega|$ LDPC code bits are transmitted from N transmitter antennas. In a maximum *a posterior* (MAP) MIMO-OFDM demodulator, $L_{D\rightarrow L}^{q}(b_i)$ $(i = 1, \ldots, N \log_2|\Omega|)$ is computed as follows:

$$L_{D\rightarrow L}^{q}(b_{i,j}) = \log \frac{\sum_{\boldsymbol{x}^+ \in C_i^+} \exp\left(-\left\|\boldsymbol{y} - \sqrt{\frac{\text{SNR}}{N}}\boldsymbol{H}\boldsymbol{x}^+\right\|^2 + \sum_{j=1}^{N \log 2|\Omega|} \{\boldsymbol{x}^+\}_j \frac{L_{D\leftarrow L}^{q-1}(b_{k,j})}{2}\right)}{\sum_{\boldsymbol{x}^+ \in C_i^-} \exp\left(-\left\|\boldsymbol{y} - \sqrt{\frac{\text{SNR}}{N}}\boldsymbol{H}\boldsymbol{x}^-\right\|^2 + \sum_{j=1}^{N \log 2|\Omega|} \{\boldsymbol{x}^-\}_j \frac{L_{D\leftarrow L}^{q-1}(b_{k,j})}{2}\right)} - L_{D\leftarrow L}^{q-1}(b_{i,j}) \tag{7}$$

Where C_i^+ is the set of $\boldsymbol{x}$ for which the *i*th LDPC-coded bit is +1, and C_i^- is similarly defined; $\{\boldsymbol{x}^+\}_j$ denotes the corresponding *j*th binary bit in symbol $\boldsymbol{x}^+$, and similarly, so does $\{\boldsymbol{x}^-\}_j$. The soft MAP demodulator in (7) has a complexity at $\mathcal{O}(|\Omega|^N)$ and can only be used in practice for small constellation size and small number of transmit antennas.

Hence describe a suboptimal soft demodulator, which is based on the linear minimum-mean-square-error soft-interference-cancellation (LMMSE-SIC) techniques [9] and has a relatively low complexity at $\mathcal{O}(|\Omega|^3)$. Based on the *a priori* LLR of the code bits provided by the LDPC decoder $\{L_{D\leftarrow L}^{q-1}(b_i)\}_i$, form soft estimates of the symbol transmitted from the *j*th $(j = 1, 2, \ldots, N)$ antenna as:

$$\tilde{x}_j \triangleq \sum_{\hat{x}\in\Omega} \hat{x}\, P(x_j = \hat{x}) = \sum_{\hat{x}\in\Omega} \hat{x} \prod_{l=1}^{\log_2|\Omega|} \left[1 + \exp\left(-\{\hat{x}\}_l \cdot L_{D\leftarrow L}^{q-1}(b_{l,j})\right)\right]^{-1} \tag{8}$$

Where $b_{l,j}$ denotes the corresponding lth binary bit in symbol x_j.
Denote

$$\tilde{\boldsymbol{x}}_j \triangleq \left[\tilde{x}_1, \dots, \tilde{x}_{j-1}, 0, \tilde{x}_{j+1}, \dots, \tilde{x}_{N-1}\right]^T \tag{9}$$

And performing a soft interference cancellation $\boldsymbol{y}_j$ to obtain:

$$\tilde{\boldsymbol{y}}_j \triangleq \boldsymbol{y} - \boldsymbol{H}\tilde{\boldsymbol{x}}_j = \boldsymbol{H}\left(\boldsymbol{x} - \tilde{\boldsymbol{x}}_j\right) + n \tag{10}$$

Next, an Instantaneous Linear MMSE filter is applied to $\tilde{\boldsymbol{y}}_j$ to obtain,

$$z_j = \boldsymbol{w}_j^H \tilde{\boldsymbol{y}}_j \tag{11}$$

where the filter $\boldsymbol{w}_j \in C^M$ is chosen to minimize the mean square error between symbol x_j and the filter output z_j, i.e.,

$$\begin{aligned} \boldsymbol{w}_j &= \arg\min_{\boldsymbol{w} \in C^M} E\left\{\left|x_j - \boldsymbol{w}^H \tilde{\boldsymbol{y}}_j\right|^2\right\} \\ &= \sqrt{\frac{N}{\mathrm{SNR}}}\left(\boldsymbol{H}\Delta_j \boldsymbol{H}^H + \frac{N}{\mathrm{SNR}} I\right)^{-1} \boldsymbol{He} \end{aligned} \tag{12}$$

$$\begin{aligned} \text{Where,}\quad \Delta_j &= \mathrm{cov}\left\{x_j - \tilde{x}_j\right\} \\ &= \mathrm{diag}\left\{1 - |\tilde{x}_1|^2, \dots, 1 - \left|\tilde{x}_{j-1}\right|^2, 1, 1 - \left|\tilde{x}_{j+1}\right|^2, \dots, 1 - |\tilde{x}_N|^2\right\} \end{aligned} \tag{13}$$

and $\boldsymbol{e}$ denotes an M-sized vector with all zero entries, except for the jth entry being 1. The detailed derivation of (12) is further referred to [9].
As in [9], we approximate the soft instantaneous MMSE filter output in (11) as Gaussian distributed i.e.,

$$p\left(z_j \middle| x_j\right) \sim \mathcal{N}_c(\mu_j x_j, \eta_j^2) \tag{14}$$

Conditioned on x_j, the mean and variance of are given, respectively, by:

$$\mu_j = E\left\{z_j x_j^*\right\} = \boldsymbol{e}^T \boldsymbol{H}^H \left(\boldsymbol{H}\Delta_j \boldsymbol{H}^H + \frac{N}{\mathrm{SNR}} I\right)^{-1} \boldsymbol{He} \tag{15}$$

$$\eta_j^2 = \mathrm{var}\left\{z_j\right\} = E\left\{\left|z_j\right|^2\right\} - \mu_j^2 = \mu_j - \mu_j^2 \tag{16}$$

The extrinsic information $L_{D\to L}^q\left(b_{i,j}\right)$ of the corresponding ith binary bit in symbol x_j, delivered by the LMMSE-SIC demodulator is calculated as:

$$L_{D\to L}^q(b_i) = \log \frac{\sum_{\boldsymbol{x}^+ \in S_{i,j}^+} \exp\left(-\frac{-\left\|z_j - \mu_j \boldsymbol{x}^+\right\|^2}{\eta_j^2} + \sum_{k=1}^{\log_2|\Omega|} \left\{x_j^+\right\}_k \cdot \frac{L_{D\leftarrow L}^{q-1}(b_k)}{2}\right)}{\sum_{\boldsymbol{x}^+ \in S_{i,j}^-} \exp\left(-\frac{-\left\|z_j - \mu_j \boldsymbol{x}^-\right\|^2}{\eta_j^2} + \sum_{k=1}^{\log_2|\Omega|} \left\{x_j^-\right\}_k \cdot \frac{L_{D\leftarrow L}^{q-1}(b_k)}{2}\right)} - L_{D\leftarrow L}^{q-1}(b_i) \tag{17}$$

Where $S_{i,j}^+$ is the set of all possible values of x_i for which the ith LDPC-coded bit is +1, and $S_{i,j}^-$ is similarly defined; $\left\{x_j^+\right\}_k$ denotes the corresponding kth binary bit in symbol x_j^+, and similarly, so does $\left\{x_j^-\right\}_k$. Note that the LMMSE-SIC demodulator extracts the extrinsic LLR of code bit b_i from z_j, which is the scalar output of the LMMSE filter in (11), whereas the MAP demodulator collects the extrinsic LLR from $\mathbf{y}$, which is the M-size vector of the received signals. The complexity of soft LMMSE-SIC demodulator is significantly lower than that of the soft MAP demodulator, especially when N and $|\Omega|$ are large.

3.2 Decoding of LDPC Codes

The message-passing decoding algorithm is used to decode the LDPC codes [6]. To describe the message-passing decoding algorithm, the following notations are first introduced. In the bipartite graph of the LDPC codes, the variable (bit) nodes are numbered from 1 to n, the check nodes from 1 to $n-k$. The degree of the ith variable node is denoted by v_i, and the degree of the ith check node is denoted by Δ_i.

Denote by $\{e_{i,1}^b, e_{i,2}^b, \ldots, e_{i,v_i}^b\}$ the set of edges connected to the ith variable node and by $\{e_{i,1}^c, e_{i,2}^c, \ldots, e_{i,\Delta_i}^c\}$ the set of edges connected to the ith check node. That is, $e_{i,k}^b$ denotes the kth edge connected to the ith variable node, and $e_{i,k}^c$ denotes the kth edge connected to the ith check node. The particular edge or bit associated with an extrinsic message is denoted as the argument of L. A subscript $b \to c$ denotes quantities passed from the variable nodes to the check nodes of the LDPC code, and vice versa. For example, $L_{b\to c}^{p,q}(e_{i,k}^b)$ denotes the extrinsic LLR passed from a variable node to a check node along the jth edge connected to the ith variable node, during the pth iteration within the LDPC decoder and the qth iteration between the LDPC decoder and the demodulator. For the pdf's f and means m, no argument is used since they do not depend on the particular edge, variable node, or check node.

The message-passing decoding algorithm of LDPC codes is summarized as follows.

- *Iterate between variable node update and check node update*: For $p = 1, 2, \ldots, P$

a) *Variable node update*: For each of the variable nodes $i = 1, 2, \ldots, n$, for every edge connected to the variable node, compute the extrinsic message passed from the variable node to the check node along the edge, given by:

$$L_{b\to c}^{p,q}(e_{i,j}^b) = L_{D\to L}^{q}(b_i) + \sum_{k=1,k\neq j}^{v_i} L_{b\leftarrow c}^{p-1,q}(e_{i,k}^b) \tag{18}$$

b) *Check node update*: For each of the check nodes $i = 1, 2, \ldots, n-k$, for all edges that are connected to the check node, compute the extrinsic message passed from the check node to the variable node, given by:

$$L_{b\leftarrow c}^{p,q}(e_{i,j}^c) = 2\tanh^{-1}\left[\prod_{k=1,k\neq j}^{\Delta_i} \tanh\left(\frac{L_{b\to c}^{p,q}(e_{i,k}^c)}{2}\right)\right] \tag{19}$$

- *Compute extrinsic messages passed back to the demodulator:*

$$L_{D\leftarrow L}^{q}(b_i) = \sum_{k=1}^{v_i} L_{b\leftarrow c}^{P,q}(e_{i,k}^b) \tag{20}$$

- *Store check node to variable node messages*: For all edges, set:

$$L_{b\leftarrow c}^{0,q+1}(e_{i,k}^b) = L_{b\leftarrow c}^{P,q}(e_{i,k}^b) \tag{21}$$

After sufficient Q times turbo receiver iterations, final hard decisions on information and parity bits are made as:

$$\tilde{b}_i = sign\left[L_{D\to L}^{Q}(b_i) + L_{D\leftarrow L}^{Q}(b_i)\right] \tag{22}$$

4. Analysis and Optimization of LDPC-coded MIMO-OFDM

In this section, how to analyze and optimize the LDPC-coded MIMO OFDM systems via the techniques of density evolution with mixture Gaussian approximations [7] is described. The principal idea of density evolution [5], [6] is to treat the extrinsic information that is passed in the iterative process as random variables. Then, by estimating the pdf of the random variables as a function of SNR and iteration number, the probability of error at every iteration can be computed. When the length of the code words $n \to \infty$, the extrinsic information passed along the edges connected to every check node and variable node can be assumed to be independent variables. This makes it possible to compute the pdfs relatively easily. The minimum SNR for which the probability of error tends to zero is called the minimum operational SNR, which is denoted by $SNR_{min,op}$.

4.1 Mixture Gaussian Approximation to the Distribution of Extrinsic Messages

It is known that the extrinsic information passed from soft LDPC decoder to soft MIMO OFDM demodulator $f_{D\leftarrow L}^{q}$ can be modeled as mixture symmetric Gaussian distributed [10]. A mathematical expression of $f_{D\leftarrow L}^{q}$ can be referred to (43). a mixture Gaussian model is due to the code structure and the message-passing decoding algorithm of LDPC codes is remarked.

On the other hand, the pdf of the extrinsic information passed from soft MIMO OFDM demodulator to soft LDPC decoder $f_{D\to L}^{q}$ in general has no closed-form expression. Next, the pdf of the soft demodulator output LLR is modeled as:

$$f_{D\to L}^{q} \cong \sum_{j=1}^{J} \pi_j \, \mathcal{N}(m_j, 2m_j) \tag{23}$$

In particular, the interest is in approximating the exact pdf in (23) with finite J terms. For a fixed number of mixtures J, based on the observations $\Xi \triangleq \{\xi_i, i = 1, \dots, n\}$, the parameters $\theta \triangleq \{\pi_j, m_j, j = 1, \dots, J\}$ can be estimated using the expectation-maximization (EM) algorithm, which is explained next.

Denote $\phi(x: \mu, \sigma^2)$ as the pdf of a $\mathcal{N}(\mu, \sigma^2)$ random variable. Then, the maximum likelihood (ML) estimate of the parameters θ is given by:

$$\begin{aligned}\hat{\theta} &= \arg \max_{\theta: \sum_{j=1}^{J} \pi_j = 1} \log p_\theta(\Xi) \\ &= \arg\max_{\theta: \sum_{j=1}^{J} \pi_j = 1} \sum_{i=1}^{n} \log \sum_{j=1}^{J} \pi_j \, \phi(\xi_i: m_j, 2m_j)\end{aligned} \tag{24}$$

Direct solution to the above maximization problem is very difficult. The EM algorithm [11] is an iterative procedure for solving this ML estimation problem.

In the EM algorithm, the observation Ξ is termed as *incomplete* data. Starting from some initial estimate $\theta^{(0)}$, the EM algorithm solves the ML estimation problem (24) by the following iterative procedure:

- E-step: Compute

$$Q\left(\theta \middle| \theta^{(i)}\right) = E_{\theta^{(i)}}\{\log p_\theta(\mathrm{X}) | \Xi\} \tag{25}$$

- M-step: Solve

$$\theta^{(i+1)} = \arg\max_\theta Q\left(\theta \middle| \theta^{(i)}\right) \tag{26}$$

Define the following hidden data $Z = \{z_i, i = 1, \dots, n\}$, where z_i is the *J*-dimensional indicator vector such that

$$z_{i,j} = \begin{cases} 1, & if \ \xi_i \sim \mathcal{N}_c(m_j, 2m_j) \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

The complete data is then $X = (\Xi, Z)$. Having,

$$p_\theta(\Xi, Z) = \prod_{i=1}^{n} \prod_{j=1}^{J} \left[\pi_j \, \phi(\xi_i: m_j, 2m_j)\right]^{z_{i,j}} \tag{28}$$

The log-likelihood function of the complete data is given by

$$\log p_\theta(\Xi, Z) = \sum_{i=1}^{n} \sum_{j=1}^{J} z_{i,j} \log \pi_j + \sum_{i=1}^{n} \sum_{j=1}^{J} z_{i,j} \left[-\frac{1}{2} \log 2m_j - \frac{(\xi_i - m_j)^2}{4m_j}\right] + C \tag{29}$$

where C is a constant. The E-step can then be calculated as follows:

$$\begin{aligned} Q(\theta, \theta') &= E_\theta'\{\log p_\theta(\Xi, Z) | \Xi\} \\ &= \sum_{i=1}^{n} \sum_{j=1}^{J} \hat{z}_{i,j} \left[\log \pi_j - \frac{1}{2} \log 2m_j - \frac{(\xi_i - m_j)^2}{4m_j}\right] + C \end{aligned} \tag{30}$$

Where,

$$\hat{z}_{i,j} = \frac{\phi(\xi_i: m_j', 2m_j') \pi_j'}{\sum_{l=1}^{J} \phi(\xi_i: m_l', 2m_l') \pi_l'} \tag{31}$$

In addition, the M-step is calculated as follows: To obtain $\{\pi_j\}$, we have

$$\frac{\partial Q(\theta, \theta')}{\partial \pi_j} = 0 \Rightarrow \pi_j = \frac{1}{n} \sum_{i=1}^{n} \hat{z}_{i,j}, j = 1, \dots, J \tag{32}$$

To obtain $\{m_j\}$, we have

$$\frac{\partial Q(\theta, \theta')}{\partial m_j} = 0 \Rightarrow m_j = -1 + \sqrt{1 + \frac{\sum_{i=1}^{n} \hat{z}_{i,j} \xi_i^2}{\sum_{i=1}^{n} \hat{z}_{i,j}}}, j = 1, \dots, J \tag{33}$$

Finally the EM algorithm for calculating the Gaussian mixture parameters for the extrinsic messages passed from the demodulator is summarized as follows:

- *Given the demodulator extrinsic messages* $\{\xi_i\}$, *the number of mixture components* J, *and the total number of EM iterations I, starting from the initial parameters* $\theta^{(0)}$;

a) For $i = 1, \dots, I$, Let $\theta' = \theta^{(i-1)}$ and calculate $\{\hat{z}_{i,j}, i = 1, \dots, n; j = 1, \dots, J\}$ according to (31).

b) Calculate $\{\pi_j, j = 1, \dots, J\}$ according to (32) and calculate $\{m_j, j = 1, \dots, J\}$ according to (33). Set $\theta^{(i)} = \theta$.

In the above EM algorithm, the number of mixture components J is fixed. Note that when J increases, $\log p_\theta(\Xi)$ increases, or $-\log p_\theta(\Xi)$ decreases. The minimum description length (MDL) principle can be used to determine J [12],

$$\hat{J}_{MDL} = \arg\min_J \left\{-\log p_{\theta,J}(\Xi) + \frac{J}{2}\log n\right\} \tag{34}$$

Where, in the MDL criterion, a penalty term $(J/2)\log n$ is introduced. Hence, first set an upper bound of the number of mixture components, J_{max}. In addition, for each $J \le J_{max}$, the above EM algorithm will be implemented and the corresponding MDL value is calculated. Finally, the optimal J with the minimum MDL is chosen.

4.2 Density Evolution with Gaussian Approximation

In [5], it was assumed that the extrinsic message at the output of each variable or check node is Gaussian and symmetric (i.e., the variance is twice the mean). Therefore, the pdf of the extrinsic messages at the output of each variable or check node is entirely characterized by its mean. Only the AWGN channel was considered in [5], and since the pdf at the channel output is Gaussian for the AWGN channel, this characterization was accurate. Here, treated the more general case where the pdf at the demodulator output is a mixture of symmetric Gaussians and derive the steps involved in computing the pdfs of the extrinsic LLRs at each iteration. It is shown that due to the assumption that the demodulator output is a mixture of symmetric Gaussian pdfs, and can easily track the pdfs of the extrinsic LLRs within the LDPC code without having to numerically convolve or evaluate pdfs. This significantly reduces the complexity in code design without sacrificing much performance. Similar to [5], assume that $f_{b\leftarrow c}^{p,q}$ is Gaussian at each check node. Due to the irregularity of the LDPC codes, this assumption means that the pdfs of the extrinsic LLRs are all mixtures of symmetric Gaussian pdfs, and only the means of the component Gaussian pdfs have to be evaluated.

Specifying the procedure for computing the pdfs of the extrinsic messages passed around in the turbo iterative receiver algorithm described in Section III. Denoting $\psi(x) = E\{\tanh[1/2\mathcal{N}_c(x, 2x)]\}$, we get the following:

a) Initialization: Set $f_{b\leftarrow c}^{0,0} = \delta(x)$, and $f_{D\leftarrow L}^{0} = \delta(x)$

b) Turbo iterative iterations: For $q = 1,2, \dots, Q$, do the following.

i. Compute the pdf of extrinsic messages passing from the demodulator: $f_{D\rightarrow L}^{q}$ is computed as a function of SNR and $f_{D\leftarrow L}^{q-1}$ using the method in Section IV-A to obtain,

$$f_{D\rightarrow L}^{q} = \sum_{j=1}^{J} \pi_j\, \mathcal{N}(\mu_j, 2\mu_j) \tag{35}$$

ii. Compute the pdf of the LDPC extrinsic messages:

- *Iterate between variable node update and check node update*: For $p = 1,2, \dots, P$, do the following.

a) At a variable node of degree: From (18), we can see that the pdf of the extrinsic LLR that is passed along an edge connected to a variable node of degree i, which is denoted by $f_{b\leftarrow c,i}^{p,q}$, is the convolution of $f_{D\rightarrow L}^{q}$ with $(i-1)$ convolutions of the pdf $f_{b\leftarrow c}^{p-1,q}$, with itself. This can be simplified by making the assumption that the output extrinsic from the variable node of degree excluding the contribution from the channel is Gaussian. The same assumption has been made in [5]. That is:

$$\begin{aligned} f_{b\leftarrow c,i}^{p,q} &= f_{D\rightarrow L}^{q} \otimes \mathcal{N}\left((i-1)m_{b\leftarrow c}^{p-1,q}, 2(i-1)m_{b\leftarrow c}^{p-1,q}\right) \\ &= \sum_{j=1}^{J} \pi_j\, \mathcal{N}(\mu_j, 2\mu_j) \otimes \mathcal{N}\left((i-1)m_{b\leftarrow c}^{p-1,q}, 2(i-1)m_{b\leftarrow c}^{p-1,q}\right) \\ &= \sum_{j=1}^{J} \pi_j\, \mathcal{N}\left(\mu_j + (i-1)m_{b\leftarrow c}^{p-1,q}, 2\left(\mu_j + (i-1)m_{b\leftarrow c}^{p-1,q}\right)\right) \end{aligned} \tag{36}$$

Since λ_i fractions of the edges are connected to variable nodes of degree i, the pdf of the extrinsic message passed from the variable nodes to the check nodes along an edge is:

$$f_{b\to c}^{p,q} = \sum_{j=2}^{d_{l,max}} \lambda_i f_{b\to c,i}^{p,q} = \sum_{j=1}^{J} \sum_{i=2}^{d_{l,max}} \pi_j \lambda_i \mathcal{N}\left(\mu_j + (i-1)m_{b\leftarrow c}^{p-1,q}, 2\left(\mu_j + (i-1)m_{b\leftarrow c}^{p-1,q}\right)\right) \quad (37)$$

b) *At check node of degree j*: Assume that the *i*th check node is of degree *j* and that the extrinsic LLR at the output of this check node is Gaussian with mean $m_{b\leftarrow c,j}^{p,q}$. To compute this mean, taking the expectation on both sides of (19) and get:

$$E\left\{\tanh\left(\frac{L_{b\leftarrow c}^{p,q}(e_{i,r}^{c})}{2}\right)\right\} = E\left\{\left[\prod_{k=1,k\neq j}^{j} \tanh\left(\frac{L_{b\to c}^{p,q}(e_{i,k}^{c})}{2}\right)\right]\right\} = \left[E\left\{\tanh\left(\frac{L_{b\to c}^{p,q}(e_{i,k}^{c})}{2}\right)\right\}\right]^{j-1} \quad (38)$$

where (38) follows from the fact that $L_{b\to c}^{p,q}(e_{i,k}^{c})$ and $L_{b\to c}^{p,q}(e_{i,s}^{c})$ are identically distributed and are independent for $k \neq s$. Since the distribution of $L_{b\leftarrow c}^{p,q}(e_{i,r}^{c})$ will be same for all r i.e., the message passed along all the edges connected to a check node have the same distribution. Using the distribution of $L_{b\to c}^{p,q}(e_{i,k}^{c})$ given in (37) and the definition of the function $\psi(\cdot)$, we get:

$$\psi\left(m_{b\leftarrow c,j}^{p,q}\right) = \left(\sum_{j=1}^{J} \sum_{i=2}^{d_{l,max}} \pi_j \lambda_i \psi\left(m_{b\to c,i}^{p,q}\right)\right)^{j-1} \quad (39)$$

Where $m_{b\leftarrow c,j}^{p,q}$ is the mean of the LLR passed along the edge of a check node of degree *j*. Therefore:

$$m_{b\leftarrow c,j}^{p,q} = \psi^{-1}\left[\left(\sum_{j=1}^{J} \sum_{i=2}^{d_{l,max}} \pi_j \lambda_i \psi\left(m_{b\to c,i}^{p,q}\right)\right)^{j-1}\right] \quad (40)$$

Since ρ_j fractions of the edges are connected to checks of degree *j*, the pdf of the extrinsic message passed from the check node to variable node is:

$$f_{b\leftarrow c}^{p,q} = \sum_{j=2}^{d_{l,max}} \rho_j \mathcal{N}\left(m_{b\leftarrow c,j}^{p,q}, 2m_{b\leftarrow c,j}^{p,q}\right) \quad (41)$$

c) *Message passed back to the demodulator*: At variable node of degree *i*, by taking expectation on both sides of (20), we get:

$$m_{D\leftarrow L}^{q}(i) = i m_{b\leftarrow c}^{p-1,q} \quad (42)$$

Since a fraction $\tilde{\lambda}_i$ of the *nodes* have degree *i*:

$$f_{D\leftarrow L}^{q} = \sum_{i=2}^{d_{l,max}} \tilde{\lambda}_i \mathcal{N}\left(m_{D\leftarrow L}^{p}(i), 2m_{D\leftarrow L}^{p}(i)\right) \quad (43)$$

c) The $SNR_{min,op}$ can be computed as the minimum SNR for which the mean $m_{D\leftarrow L}^{Q}$ tends to ∞.

The procedure for computing the $SNR_{min,op}$ for a given degree profile $(\lambda(x), \rho(x))$ can be used in conjunction with an optimization procedure to design optimal LDPC-coded MIMO OFDM systems. The idea is to find optimal $\lambda(x)$ and $\rho(x)$ such that the $SNR_{min,op}$ is minimized and the rate of the LDPC code is $r = 1 - \int_0^1 \rho(x)dx / \int_0^1 \lambda(x)dx$ is noted. A nonlinear optimization procedure called differential evolution [13] is employed here to solve the above optimization problem.

5. Simulation Results

A complete simulation results have been obtained for the two demodulator cases via., MAP and LMMSE-SIC for a 2×2 MIMO OFDM under the AWGN fading conditions. Fig.3 shows the performance of the soft LDPC coded FFT based MIMO OFDM channel without any spatial correlation. Here it can be shown clearly that the uncoded MIMO OFDM starts at a better Bit Error Rate (BER) at low E_b/N_o (SNR) values. But as the SNR value increased, the LDPC coded MIMO OFDM converged rapidly compared to the uncoded one showing a better result (reduced) in BER for the LMMSE-SIC demodulator at an SNR value of 15dB.

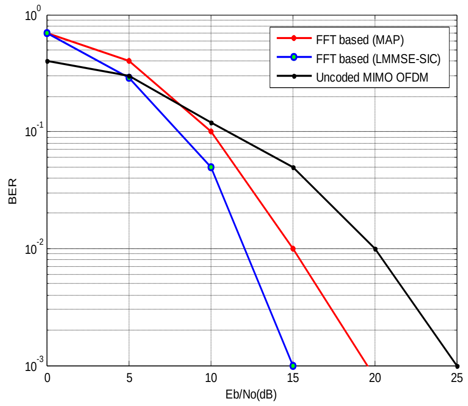

Fig.3 Performance for outage 2×2 LDPC-coded FFT based MIMO OFDM system with no spatial correlation

Fig.4 depicts the performance of the soft LDPC coded DWT based MIMO OFDM channel without any spatial correlation. Here also it can be shown that the uncoded MIMO OFDM starts at a better BER at low E_b/N_o values. But as the SNR value increased, the LDPC coded MIMO OFDM converged rapidly compared to the uncoded one showing reduction in BER for the LMMSE-SIC demodulator at an SNR value of 12dB. But compared to its counterpart in Fig 4 the LMMSE-SIC demodulator for DWT based system is close enough to the uncoded one even at the low SNR values.

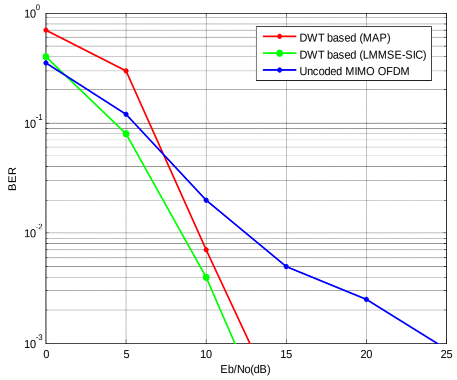

Fig.4 Performance for outage 2×2 LDPC-coded DWT based MIMO OFDM system with no spatial correlation

The performance comparison in terms of BER versus SNR of DWT and FFT based LDPC coded MIMO OFDM system for a MAP demodulator is exhibited in Fig.5. It was uncovered from the figure that the DWT based system outperformed the FFT based one. Here the DWT based spatially correlated MAP demodulator is close enough to the error rate of uncorrelated MAP than that of the FFT based spatially correlated MAP.

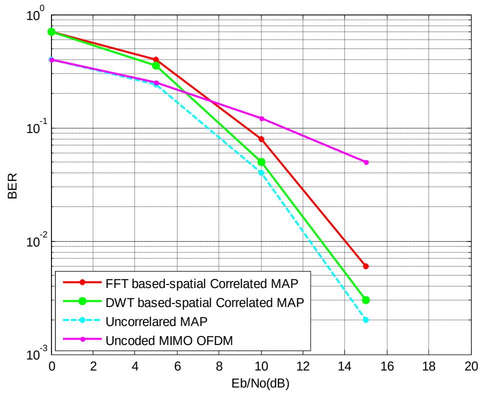

Fig.5 BER vs E_b/N_o comparison of 2×2 DWT and FFT based LDPC-coded MIMO OFDM system for MAP Demodulator

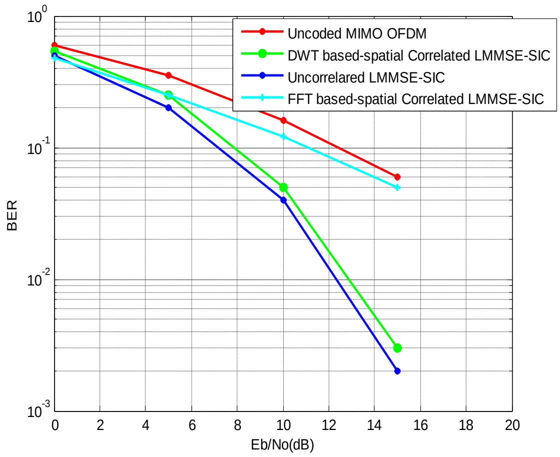

Fig.6 BER vs E_b/N_o comparison of 2×2 DWT and FFT basedLDPC-coded MIMO OFDM system for LMMSE-SIC Demodulator

Fig.6 shows the performance comparison in terms of BER versus SNR of DWT and FFT based LDPC coded MIMO OFDM system for a LMMSE-SIC demodulator. The above graph clearly states that the DWT based system outperformed the FFT based one and the DWT based spatially correlated LMMSE-SIC demodulator is close enough to the error rate of uncorrelated LMMSE-SIC than that of the FFT based spatially correlated LMMSE-SIC.

6. Conclusions

Discrete wavelet based MIMO OFDM system is a blossoming technique which has been proven to be the best alternate from the conventional FFT based system. The results obtained had shown it clearly the performance analysis and design optimization of low-density parity check (LDPC) coded Multiuser Discrete Wavelet based multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems for high data rate wireless transmission. In particular, the optimization is done for various MIMO OFDM system configurations, which include a couple of antennas, and Quadrature Phase shift keying (QPSK) demodulation scheme. It is shown that along with the optimized irregular LDPC codes, a turbo iterative receiver that consists of a soft maximum *a posteriori* (MAP) demodulator and a belief-propagation LDPC decoder can perform within the ergodic capacity of the MIMO OFDM systems under consideration. It is also shown that compared with the optimal MAP demodulator-based receivers, the receivers employing a low-complexity linear minimum mean-square-error soft-interference-cancellation (LMMSE-SIC) demodulator have a small performance loss in spatially correlated DWT based MIMO channels. The optimized performance is compared with the corresponding LDPC coded FFT based MIMO OFDM system and shown that, the DWT based receivers employing a low-complexity LMMSE-SIC demodulator have a small performance loss in spatially correlated MIMO channels.

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