

## **Analysis of Wind Speed at Sular -A Bimodal Weibull and Weibull Distribution**

Dr. C. V.Seshaiah<sup>1</sup>, D. Indhumathy<sup>2</sup>

<sup>1</sup>(Department of Mathematics, Sri Ramakrishna Engineering College, India)

<sup>2</sup>(Department of Mathematics, Sri Ramakrishna Engineering College, India)

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**Abstract:** The great demand for the energy supply coupled with inadequate energy resources creates an emergency to find a new solution for the energy shortage. Due to developing environmental concern use of renewable energy sources is very essential. If the wind speed distribution is determined, then the wind power density distribution can easily be obtained accordingly. At a particular wind farm the available energy generated by a wind power generator system depends on the characteristics of wind such as mean wind speed and standard deviation. Since it is hard to predict the variation on annual mean wind speed, wind speed fluctuations during a year can be well characterized in terms of the probability density function. Due to this reason, the proper specification of the wind speed distribution is of immense importance in the assessment of wind energy potential. In this work, we estimate Weibull parameters and spatially explicit Bimodal Weibull & Weibull parameters at a specific location and investigate the reliability of these estimates, spatial and temporal trends and calculation of average wind speed. The maximum likelihood, least square method, Energy Pattern factor method, Empirical Method are used to estimate the parameters. Chi-square test,  $R^2$ , Root Mean Square Error test are the statistical tests used to test the goodness of fit of the wind speed frequency distribution.

**Keywords:** Weibull distribution, Bimodal Weibull & Weibull distribution, Mean wind speed, Maximum likelihood method, Least Square method, etc.

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### **1. INTRODUCTION**

The As an energy source, it seems winds were harnessed even in the ancient times. Perhaps, the earliest use of wind energy was in the form of sails to steer ships. Egyptian civilization, even as far back as 3000 BC, was using boats and ships with sails. There is a mention of ambitious plans of Mesopotamian ruler Hammurabi to use windmills for irrigation around 1790 BC.

Over the last 50-60 years, electricity generation has grown in most parts of the world. The steepest growth has been in developing countries and therefore, balancing energy access, economic development and environmental sustainability is going to be a major challenge not only for the nations concerned but also for the global community. Across the world, greater dependence on imported fossil fuels adds to national energy insecurity and uncertainty. Most countries do not have adequate indigenous fossil fuel reserves and hence have to import oil, coal and gas. Fluctuations in price and supply uncertainty result in serious long-term concerns for national economies. Because of this, both developed and developing countries are becoming increasingly more interested in using pollution free, cost effective and renewable sources of energy. Energy and environment are the twin major crises in the world. Those regions of the world that face energy resources constraints to meet their electricity demand are also the regions where wind energy has been utilized to the largest extent.

For the proper assessment, the variability of the wind over time can be divided into three distinct time scales. Firstly, the large time scale variability describes the variations of the amount of wind from one year to another, or even over periods of decades or more. Secondly, the medium time scale covers periods up to a year. These seasonal variations of the wind are much more predictable. Finally, the short term time scale variability covers time scales of minutes to seconds, also well known by the term "turbulence" and which is of critical interest in the wind turbine design process. Therefore, accurate knowledge about the wind characteristics is needed for planning, design and operation of wind turbines.

Of the total installed renewable power across the country, over 55% is wind power. Even though wind leads India's renewable power sector, it has huge growth potential. Nevertheless, they are often affected by the distortion in the wind flow due to platform structures. Hence, analysis from this data is challenging and relies heavily on the proper wind flow modeling technique. Wind measurements around 10 m above sea level can also be obtained from buoy-mounted anemometers.

At a specific wind farm the available electricity generated by a wind power generator system depends on mean wind speed, standard deviation. Since variation on annual mean wind speed is hard to predict, wind speed variations during a year can be well characterized in terms of the probability density function(pdf). In literature, the pdf is defined as a mathematical function describing the relative likelihood for this RV to occur at

a given point in the observation space. The wind speed probability distribution for a certain location is crucial in determining the performance of energy conversion systems.

Several PDF has been used for this purpose but the commonly used PDF by most researchers to study the wind profile at wind sites is the Weibull functions[1]. For more than half a century the Weibull distribution has attracted the attention of statisticians working on theory and methods as well as various fields of statistics. Hundreds of papers have been written on this distribution; however the research is still ongoing. Together with the normal, exponential distributions, the Weibull distribution is the most popular model in statistics. It is of utmost interest to theory orientated statisticians because of its great number of special features.

There are methods which have so far predominantly used for fitting the measured wind speed probability distribution in a given location over a period of time, typically monthly or yearly. In the literature it is common to fit these functions to compare which one fits the measured distribution best in particular location. During this comparison process, parameter on which the suitability of the fit is judged are required. The bias in the estimation of wind power varies greatly in space because of the large spatial heterogeneity of the Weibull shape parameter. The suitability of the method may vary with the sample data size, sample data distribution, sample data format and goodness of fit [2].

Wind speed variability can be usefully summarized in most areas by the parameters of a Weibull distribution. The Weibull distribution is widely used in life testing and reliability studies. Weibull distribution that is the most widely used and accepted in the specialized literature on wind energy and other renewable energy sources. Research is ongoing worldwide on the Weibull distribution to find the most reliable methods for wind energy estimation.

The two-parameter Weibull distribution is not necessary more biased in more economically viable wind areas (large speed) compared to low wind speed areas in terms of wind power density. The change of shape parameter of the Weibull distribution will have different impacts on the wind energy potential in areas with different mean wind speed. Power generation for the same mean wind speed can be very different for different Weibull shape parameters.

Several authors have indicated that W-pdf should not be used in a generalized way, as it is unable to represent some wind regimes, such as those which describe wind speed frequency histograms which present bimodality. Alternatives for such regions include a bimodal probability distribution, proposed for La Ventosa, Mexico [3] or the general probability distribution presented by [4]. To overcome such situations, in this paper, the Bimodal Weibull and Weibull distribution is used to model single-site hourly average wind speeds.

A bimodal Weibull and Weibull distribution is even more useful because it is additionally able to represent heterogeneous wind regimes in which there is evidence of bimodality. The observed wind data describe wind speed frequency histogram which present bimodality.

In this work, we estimate spatially explicit Weibull & Weibull parameters over global land areas and investigate the reliability of these estimates, spatial and temporal trends, and implications for wind power estimates by including a mixing parameter, say  $w$ , which represents the proportion of mixing associated to the two component models. The mixture distribution produced from the combination of two Weibull distributions has a number of parameters which include shape parameters, scale parameters in addition to the mixing parameter.

First chapter includes the introduction about two parameter Weibull and bimodal Weibull & Weibull distribution. In the second chapter site specification is discussed. In the third chapter the estimation of parameters of Two parameter Weibull distribution using various methods are discussed. In the fourth chapter definition of Bimodal Weibull & Weibull distribution and estimation of its parameters using several methods are discussed. In the last chapter we have the statistical tests to validate the estimation of wind speed and identify the good fit [5].

## **2. SITE SPECIFICATION-WIND SPEED DATA**

Wind characteristics has been studied based on a typically measured and observed two year data source at 10 meter height for every 10 minutes interval at Sulur site. Sulur is an urban town located at Coimbatore in Tamil Nadu. The latitude and longitude are the geo coordinates of Sulur. Sulur is located in the UTC 5.30 live zone and it follows Indian standard time. Wind measurements above the sea level around 10 m can be collected from anemometers. Mean wind speed data of Sulur were collected between the time period 2012-2014. Our study includes the mean wind speed data for 24 months observed at Sulur at a mass height of 10 m. Measured wind speed data are commonly available in time series format, in which each data point represents either an instantaneous sample wind speed or an average wind speed over some time period [6].

An example of such data (giving hourly averages over a 24hr period is given) in Table -1. Sometimes wind speed data may be available in monthly distribution format [7] Table -2. The methods described in

following sections can be used to estimate the Weibull parameters, and the bimodal Weibull and Weibull parameters given, wind speed in either time-series or frequency distribution format

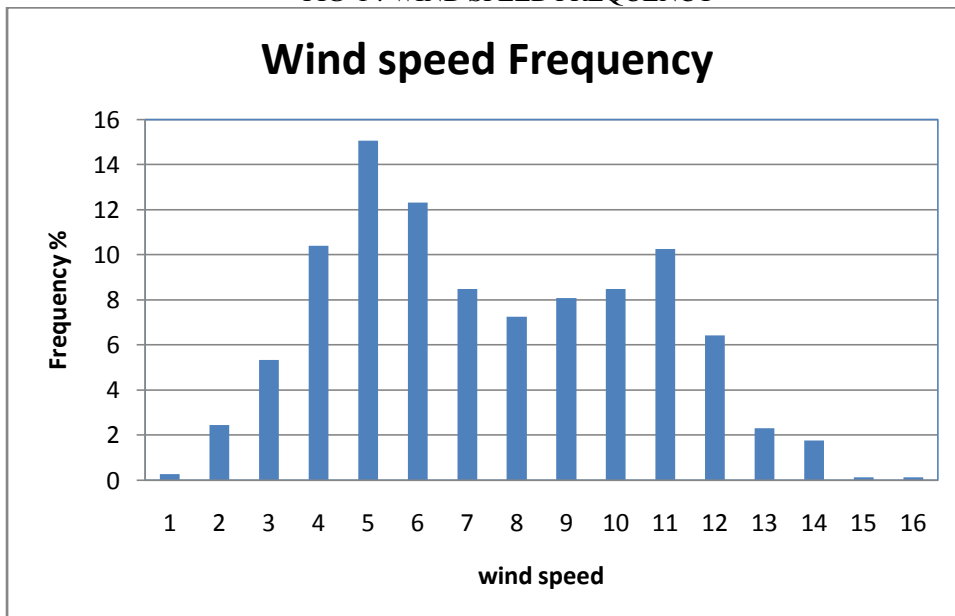
TABLE 1 : WIND SPEED DATA IN 24 HOUR TIME-SERIES FORMAT

Hour	Wind Speed (km/hr)	Hour	Wind Speed (km/hr)	Hour	Wind Speed (km/hr)	Hour	Wind Speed (km/hr)
1	NNE 3.7	7	EAST 11.1	13	EAST 22.2	19	NE 7.4
2	NNW 5.6	8	ENE 9.3	14	ENE 16.7	20	NE 7.4
3	NNE 5.6	9	ENE 9.3	15	ENE 11.1	21	VAR 5.6
4	NE 7.4	10	EAST 11.1	16	EAST 14.8	22	NNE 5.6
5	NE 7.4	11	NE 18.5	17	EAST 14.8	23	NNE 7.4
6	ENE 9.3	12	EAST 22.2	18	EAST 9.3	24	NE 5.6

TABLE 2 : WIND SPEED DATA MONTHLY IN TIME-SERIES FORMAT

Month	Wind Speed (km/hr)	Month	Wind Speed (km/hr)
Jan	8	July	18
Feb	10	Aug	17
Mar	10	Sep	16
Apr	11	Oct	9
May	15	Nov	9
June	19	Dec	8

FIG 1 : WIND SPEED FREQUENCY



### 3. TWO PARAMETER WEIBULL DISTRIBUTION

#### 3.1 DEFINITION:

Let  $(\Omega, F, P)$  be a probability space. A real valued random variable  $X: \Omega \rightarrow R$  is said to have a two parameter Weibull distribution if it has a probability density function (pdf).

$$f(v : \alpha, \beta) = \begin{cases} \frac{\alpha}{\beta} \left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v}{\beta}\right)^\alpha\right], & v \geq 0 \\ 0, & v < 0 \end{cases} \quad (1)$$

where  $\alpha > 0$  is the dimensionless shape parameter and  $\beta > 0$  is the scale parameter.

And the cumulative distribution function is given by

$$F(v : \alpha, \beta) = \begin{cases} 0 & , v < 0 \\ 1 - \exp \left[ - \left( \frac{v}{\beta} \right)^\alpha \right] & , v \geq 0 \end{cases} \quad (2)$$

### 3.2 DETERMINATION OF WEIBULL PARAMETERS

For estimating the parameters of the Weibull wind speed distribution four methods are presented [8].

#### 3.2.1 LEAST SQUARE METHOD

Least Square method requires the wind data to be in cumulative frequency distribution format. Time series data must therefore be first sorted into bins [9]. In this distribution method, the wind speed data are interpolated by a straight line, using the concept of least squares.

The cumulative distribution function is converted to logarithmic form and the resultant equation is

$$\ln[-\ln(1 - F(v))] = \alpha \ln v - \alpha \ln \beta \quad (3)$$

The shape parameter  $\alpha$  is the slope of the straight line and scale parameter  $\beta$  is the value of the term  $-\alpha \ln \beta$ .

#### 3.2.2 MOMENT METHOD

The mean wind speed and the variance are calculated from the observed wind data and used to estimate the parameters

$$\alpha = \left( \frac{\sigma}{\bar{v}} \right)^{-1.086} \quad (4)$$

$$\beta = \frac{\bar{v}}{\Gamma\left(1 + \frac{1}{\alpha}\right)} \quad (5)$$

The relation between the parameters  $\alpha$  and  $\beta$  is given by

$$\frac{\bar{v}}{\beta} = \Gamma\left(1 + \frac{1}{\alpha}\right) \text{ and } \frac{\sigma}{\bar{v}} = \frac{\sqrt{\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right)}}{\Gamma\left(1 + \frac{1}{\alpha}\right)} \quad (6)$$

Where  $\bar{v}$  is the average wind speed,  $\sigma$  is the standard deviation of the wind speed and  $\Gamma$  is the gamma function.

$\Gamma(t)$  appears in the definition of the student's 't' distribution and is the continuous function

$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$  defined for positive real numbers 't'.

#### 3.2.3 THE MAXIMUM LIKELIHOOD METHOD

The Weibull distribution can be fitted to time series wind data using the maximum likelihood method as suggested by Costa Rocha et al and Stevens et al .The shape parameter  $\alpha$  and the scale parameter  $\beta$  are estimated using the following two equations:

$$\alpha = \left( \frac{\sum_{i=1}^n v_i^\alpha \ln(v_i)}{\sum_{i=1}^n v_i^\alpha} - \frac{\sum_{i=1}^n \ln(v_i)}{n} \right)^{-1} \quad (7)$$

$$\beta = \left( \frac{1}{n} \sum_{i=1}^n v_i^\alpha \right)^{1/\alpha} \quad (8)$$

Where  $v_i$  is the wind speed in time step i and n is the number of nonzero wind speed data points.

Eq.(7) must be solved using an iterative procedure (k=2 is a suitable initial guess), after which Eq.(8) can be solved explicitly. Care must be taken to apply Eq.(7) only to the nonzero wind speed data points.

#### 3.2.4 ENERGY PATTERN FACTOR METHOD

This is a new method suggested by Akdag Ali [10]. It is related to the averaged data of wind speed. This method has simpler formulation, easier implementation and also requires less computation. The method is defined by the following equations :

$$E_{pf} = \frac{\bar{v}^3}{(v^3)^2} \quad (9)$$

$$\alpha = 1 + \frac{3.69}{(E_{pf})^2} \quad (10)$$

$$\beta = \frac{\bar{v}}{\Gamma\left(1 + \frac{1}{\alpha}\right)} \quad (11)$$

Eqn.(9) is known as the energy pattern factor method which can be solved numerically or approximately by power density technique. Once  $\alpha$  is determined,  $\beta$  can be estimated using Eqn.(11).

TABLE -3 : ESTIMATION OF TWO PARAMETER WEIBULL

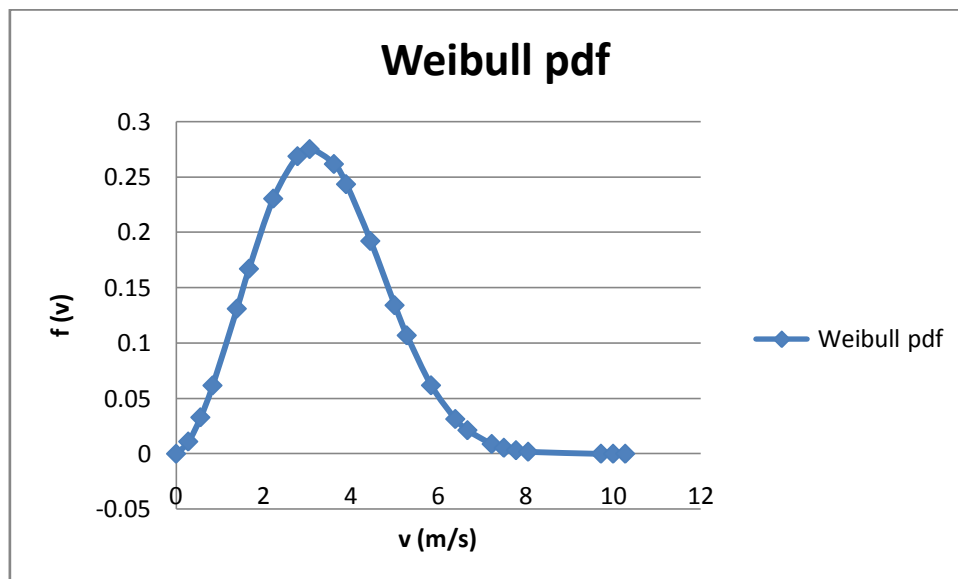


FIG 2: TWO PARAMETER WEIBULL PDF

TEST S	$\alpha$	$\beta$
LS	2.584320	3.763621
MLE	2.431344	3.785243
EPF	2.384077	3.779660
EMPIRICAL	2.439150	3.778000

**4. BIMODAL WEIBULL AND WEIBULL DISTRIBUTION**

In this section, we will see about the Weibull and Weibull distribution, denoted by WW ( $v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega$ ) and prove some of its basic properties. Here  $v[m/s]$  is the wind speed,  $\alpha_1$  and  $\alpha_2$  are the scale parameters established by the left and right Weibull distribution respectively,  $\beta_1$  and  $\beta_2$  are dimensionless shape parameters established by the left and right Weibull distribution respectively; and  $\omega$  is the weight component of the left Weibull distribution ( $0 < \omega < 1$ ). The weight component  $\omega$  can be obtained using the following formulas

$$\bar{v} = \omega \bar{v}_1 + (1 - \omega) \bar{v}_2 \tag{13}$$

$$\text{And } \sigma^2 = \omega (\sigma_1^2 - (\omega - 1)(\bar{v}_1 - \bar{v}_2)^2) - (\omega - 1)\sigma_2^2 \tag{14}$$

$\bar{v}[m/s]$  is the wind speed,  $\bar{v}_1$  and  $\bar{v}_2$  are the average wind speed of the left and right Weibull distribution respectively,  $\sigma_1^2$  and  $\sigma_2^2$  are the variance of the left and right Weibull distribution respectively [11].

A random variable  $V$  that is distributed as  $V_i$  with mixing parameters  $\omega_i$  (such that  $\omega_1 + \omega_2 = 1$ ) is said to have a two-component mixture Weibull and Weibull Distribution.

The density function of  $V$ , which depends on the parameters ( $\alpha_1, \beta_1, \alpha_2, \beta_2$ ) is given by

$$f(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega) = \omega f(v; \alpha_1, \beta_1) + (1 - \omega) f(v; \alpha_2, \beta_2) \\ = \omega \left\{ \frac{\alpha_1}{\beta_1} \left( \frac{v}{\beta_1} \right)^{\alpha_1 - 1} \exp \left[ - \left( \frac{v}{\beta_1} \right)^{\alpha_1} \right] \right\} + (1 - \omega) \left\{ \frac{\alpha_2}{\beta_2} \left( \frac{v}{\beta_2} \right)^{\alpha_2 - 1} \exp \left[ - \left( \frac{v}{\beta_2} \right)^{\alpha_2} \right] \right\} \tag{15}$$

The Weibull and Weibull distribution with parameters  $\alpha_1, \beta_1, \alpha_2, \beta_2 > 0$  has cumulative distribution function (cdf) given by [12].

$$FF(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega) = P(V \leq v) = \omega F(v; \alpha_1, \beta_1) + (1 - \omega) F(v; \alpha_2, \beta_2), \\ = \begin{cases} \omega \left\{ 1 - \exp \left[ - \left( \frac{v}{\beta_1} \right)^{\alpha_1} \right] \right\} + (1 - \omega) \left\{ 1 - \exp \left[ - \left( \frac{v}{\beta_2} \right)^{\alpha_2} \right] \right\}, & v \geq 0 \\ 0, & v < 0 \end{cases} \tag{16}$$

**3.1 DETERMINATION OF WEIBULL PARAMETERS**

For estimating the parameters of the Weibull wind speed distribution four methods are presented [13].

#### 4.1.1 LEAST SQUARE METHOD

In the least square method, the parameters are estimated using

$$\text{Min } S = \sum_{i=1}^n \left\{ F(v_i) - \omega \left\{ 1 - \exp \left[ - \left( \frac{v_i}{\beta_1} \right)^{\alpha_1} \right] \right\} - (1 - \omega) \left\{ 1 - \exp \left[ - \left( \frac{v_i}{\beta_2} \right)^{\alpha_2} \right] \right\} \right\}^2 \quad (17)$$

Here the constraints  $\alpha_1, \alpha_2, \beta_1, \beta_2 > 0$  and  $0 \leq \omega \leq 1$ .

The observed wind speed values  $v_i$  are sorted in ascending order and the occurrence of relative frequencies is assigned to each and the cumulative frequency is then calculated.

The cumulative distribution function is converted to logarithmic form and the resultant equation is

$$\ln \ln \{1 - F(v)\}^{-1} = \alpha_i \ln v_i - \alpha_i \ln \beta_i \quad (18)$$

$$Y = a + bX \text{ where } Y = \ln \ln \{1 - F(v)\}^{-1} \text{ and } X = \ln v_i \quad (19)$$

$$a = -\alpha_i \ln \beta_i \text{ and } b = \alpha_i. \quad (20)$$

$$a = \frac{\sum_{i=1}^n X_i^2 \sum_{i=1}^n Y_i - \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n \sum_{i=1}^n X_i^2 - (\sum_{i=1}^n X_i)^2}, \quad b = \frac{n \sum_{i=1}^n X_i Y_i - \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n \sum_{i=1}^n X_i^2 - (\sum_{i=1}^n X_i)^2} \quad (21)$$

The shape parameter  $\alpha$  is the slope of the straight line and scale parameter  $\beta$  is the value of the term  $-\alpha \ln \beta$ .

#### 4.1.2 MOMENT METHOD

The parameter  $\alpha_1, \alpha_2, \beta_1, \beta_2, \omega$  are estimated using the method of moments given by

$$\bar{v}_r = \omega \beta_1^r \Gamma \left( 1 + \frac{r}{\alpha_1} \right) + (1 - \omega) \beta_2^r \Gamma \left( 1 + \frac{r}{\alpha_2} \right) \text{ where } \alpha_1, \alpha_2, \beta_1, \beta_2 > 0 \text{ and } 0 \leq \omega \leq 1. \quad (22)$$

Where  $v_r$  denotes the set of statistical moments with respect to the origin of the sample. A good initial value will often make the iterative process to converge faster. For this purpose we have used the following expression to determine the initial values of the parameter in the estimation of WW pdf.

$$\alpha_1 = \left( \frac{\sigma_1}{\bar{v}_1} \right)^{-1.086} \quad (23)$$

$$\beta_1 = \bar{v}_1 \left[ \Gamma \left( 1 + \frac{1}{\alpha_1} \right) \right]^{-1} \quad (24)$$

$$\alpha_2 = \left( \frac{\sigma_2}{\bar{v}_2} \right)^{-1.086} \quad (25)$$

$$\beta_2 = \bar{v}_2 \left[ \Gamma \left( 1 + \frac{1}{\alpha_2} \right) \right]^{-1} \quad (26)$$

Where  $v_1, v_2, \sigma_1, \sigma_2$  are mean and standard deviation of the left and right peak.

#### 4.1.3 THE MAXIMUM LIKELIHOOD METHOD

The Maximum likelihood method to find the value of the parameters  $\alpha_1, \alpha_2, \beta_1, \beta_2$  which maximizes the function of log likelihood is given by

Maximize  $\ln L (v_i: \alpha_1, \alpha_2, \beta_1, \beta_2, \omega)$

$$\begin{aligned} LL &= \sum_{i=1}^n \ln \{ \omega f(v_1, \alpha_1, \beta_1) + (1 - \omega) f(v_2, \alpha_2, \beta_2) \} \\ &= \ln \prod_{i=1}^n \omega \left\{ \frac{\alpha_1}{\beta_1} \left( \frac{v_i}{\beta_1} \right)^{\alpha_1 - 1} \exp \left[ - \left( \frac{v_i}{\beta_1} \right)^{\alpha_1} \right] \right\} + (1 - \omega) \left\{ \frac{\alpha_2}{\beta_2} \left( \frac{v_i}{\beta_2} \right)^{\alpha_2 - 1} \exp \left[ - \left( \frac{v_i}{\beta_2} \right)^{\alpha_2} \right] \right\} \\ &= \sum_{i=1}^n \ln \omega \left\{ \frac{\alpha_1}{\beta_1} \left( \frac{v_i}{\beta_1} \right)^{\alpha_1 - 1} \exp \left[ - \left( \frac{v_i}{\beta_1} \right)^{\alpha_1} \right] \right\} + (1 - \omega) \left\{ \frac{\alpha_2}{\beta_2} \left( \frac{v_i}{\beta_2} \right)^{\alpha_2 - 1} \exp \left[ - \left( \frac{v_i}{\beta_2} \right)^{\alpha_2} \right] \right\} \end{aligned} \quad (27)$$

Several Numerical methods were used to solve this equation, we have used Newton Raphson iterative method to solve this equation. The shape factors  $\alpha_1, \alpha_2$  are estimated by

$$\alpha_i = \left( \frac{\sum_{i=1}^n v_i^{\alpha_i} \ln(v_i)}{\sum_{i=1}^n v_i^{\alpha_i}} - \frac{\sum_{i=1}^n \ln(v_i)}{n} \right)^{-1}, \quad (28)$$

$$\beta_i = \left( \frac{1}{n} \sum_{i=1}^n v_i^{\alpha_i} \right)^{1/\alpha_i}, \quad (29)$$

Where  $v_i$  is the wind speed in time step  $i$  and  $n$  is the number of nonzero wind speed data points.

#### 4.1.4 ENERGY PATTERN FACTOR METHOD

It is directly related to the mean wind speed data. This method has simple formulation, easy implementation and also requires less computation. Definition of the energy pattern factor  $E_{pf}$  is given by [14]

$$E_{pf} = \frac{\text{Amount of total power available in the wind}}{\text{Total power calculated by cubing the mean wind speed}} \quad (30)$$

$$E_{pf} = \frac{\frac{1}{n} \sum_{i=1}^n \bar{v}_i^3}{\left( \frac{1}{n} \sum_{i=1}^n \bar{v}_i \right)^3} \quad (31)$$

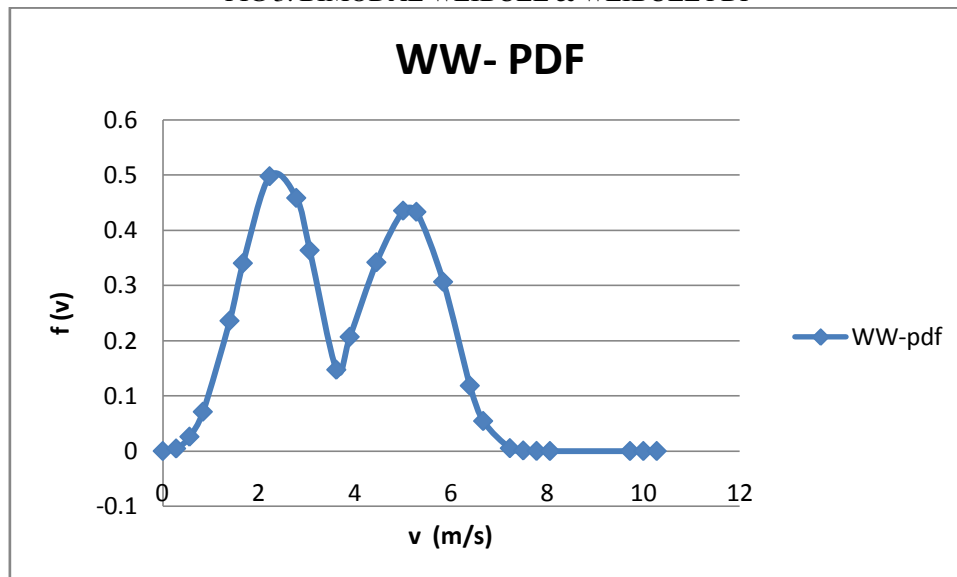
$$\alpha_i = 1 + \frac{3.69}{(E_{pf})^2} \quad (31)$$

$$\beta_i = \frac{\bar{v}_i}{\Gamma\left(1+\frac{1}{\alpha_i}\right)} \tag{32}$$

The parameters are estimated using equation (31) and (32).

TABLE -4 : BIMODAL WEIBULL & WEIBULL PARAMETERS

FIG 3: BIMODAL WEIBULL & WEIBULL PDF



### 5. STATISTICAL ANALYSIS

To evaluate the performance of the Weibull distribution [15] three statistical methods of accuracy were used. RMSE test,  $R^2$  and *Chi – Square* tests defined by

#### 5.1 $R^2$ TEST

$R^2$  is widely used for goodness of fit comparisons and hypothesis testing as it quantifies the correlation between the observed cumulative probabilities and the predicted cumulative probability of a wind speed distribution. The larger the values of  $R^2$  indicates a better fit of the model cumulative probabilities  $F_W(v)$  to the observed cumulative probabilities  $F_{obs}(v)$ .

$R^2$  is defined as

$$R^2 = \frac{\sum_{i=1}^n (F_W(v) - \overline{F_W(v)})^2}{\sum_{i=1}^n (F_W(v) - \overline{F_W(v)})^2 + \sum_{i=1}^n (F_{obs}(v) - F_W(v))^2} \tag{32}$$

#### 5.2 ROOT MEAN SQUARE ERROR

The root mean squared error (RMSE) provides a term by term comparison of the actual deviation between observed probabilities and the modeled Weibull probabilities. A lower value of RMSE indicates a better distribution function model. RMSE should be close to zero for a better fit.

$$RMSE = \left[ \frac{\sum_{i=1}^n (F_{obs}(v) - F_W(v))^2}{N} \right]^{\frac{1}{2}} \tag{33}$$

TESTS	$\alpha_1$	$\alpha_2$	$\beta_1$	$\beta_2$
LS	<b>3.38192</b>	6.29637	2.66000	5.41492
MLE	3.25216	6.38860	<b>2.37929</b>	5.26293
EPF	3.85618	6.40486	<b>2.63343</b>	5.26294
MM	<b>3.51636</b>	6.17992	2.64675	5.27318

#### 5.3 CHI SQUARE

The Chi square test returns the mean square value of the deviations between the experimental and the observed values for the distributions and it is expressed as

$$\chi^2 = \frac{\sum_{i=1}^N (F_{obs}(v) - F_W(v))^2}{F_W(v)} \tag{34}$$

TABLE-5: GOODNESS OF FIT FOR TWO PARAMETER WEIBULL

TESTS	RMSE	CHI SQUARE	R <sup>2</sup>
LS	<b>0.044231</b>	<b>0.001789</b>	<b>0.999763</b>
MLE	<b>0.039334</b>	<b>0.001703</b>	<b>0.999812</b>
EPF	<b>0.043777</b>	<b>0.001691</b>	<b>0.999824</b>
EMPIRICAL	<b>0.039250</b>	<b>0.001715</b>	<b>0.999811</b>

FIG 4: WEIBULL PDF USING DIFFERENT METHODS

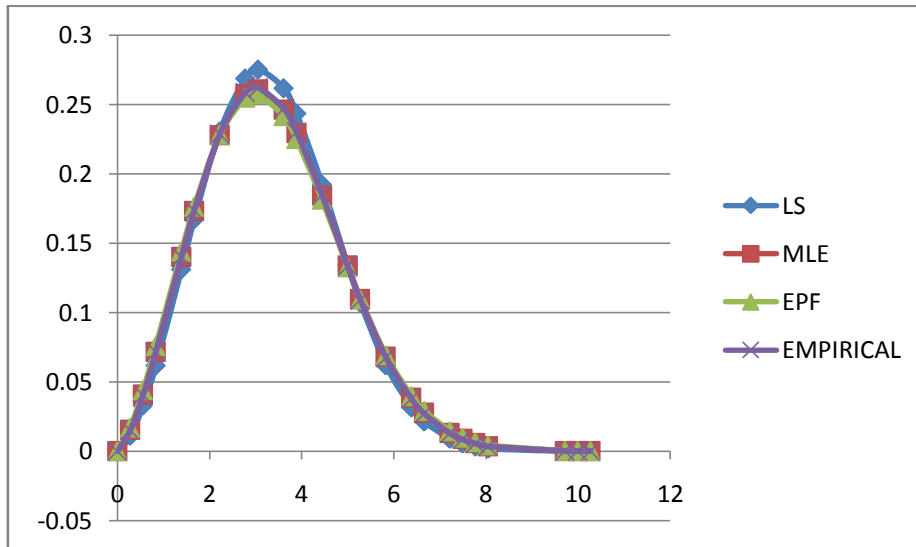


FIG 5: COMPARISON OF W-PDF & WW- PDF

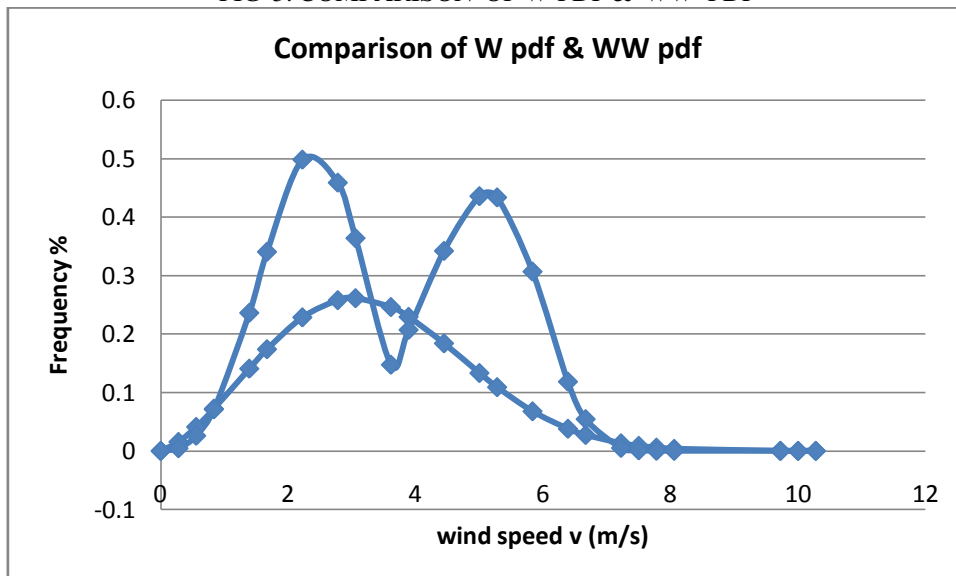




TABLE-6 : GOODNESS OF FIT FOR BIMODAL WEIBULL & WEIBULL

TESTS	RMSE	CHI SQUARE	R <sup>2</sup>
MLE	<b>0.029648</b>	<b>0.001021</b>	<b>0.990943</b>
LS	<b>0.029796</b>	<b>0.001144</b>	<b>0.990852</b>
EPF	<b>0.039656</b>	<b>0.001146</b>	<b>0.990938</b>
MM	<b>0.037753</b>	<b>0.001131</b>	<b>0.990879</b>

From Fig 1 & 5, it is clear that bimodal Weibull & Weibull PDF fits the exact frequency distribution with two peaks. While two parameter Weibull distribution shows only one peak which is entirely different from the frequency peaks.

## 6. CONCLUSION

The Sular site has an evidence of Bimodality which is accurately represented by the Bimodal Weibull & Weibull PDF. The two parameter Weibull distribution does not fit for the wind speed data at this particular site. It is evident that Maximum Likelihood method gives a better approximation in estimation of parameters for bimodal Weibull & Weibull PDF. Our study reveals that only Bimodal Weibull & Weibull distribution suits the locations where wind speed frequency represent two peaks. Our result may help the investors to identify the accurate mean wind speed at a particular site.

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