

# IMPROVEMENT OF CLOSED LOOP PERFORMANCE BY DESIGNING THE SUPERVISORY BASED SWITCHING QUANTITATIVE FEEDBACK THEORY CONTROLLER

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**Abstract:** In this paper, the robust controller design for the control of highly uncertain plants by using Quantitative Feedback Theory and its comparison with different control techniques. Before going to designing QFT controller by using Genetic Algorithm optimization technique, we design the GA based PID controller and then design the QFT controller for uncertain plant. The main problem in this paper is supervisory based switching QFT control is proposed for the control of highly uncertain plants. According to this strategy, the uncertainty region is divided into smaller regions with a nominal model. The proposed control architecture is made up by these local controllers, which commute among themselves in accordance with the decision of a high level decision maker called the supervisor. The supervisor makes the decision by comparing the candidate local model behavior with the one of the plant and selects the controller corresponding to the best fitted model. A hysteresis switching logic is used to slow down switching for stability reasons. It is shown that this strategy leads to improved closed loop performance, and can also handle the uncertainty sets that cannot be tackled by a single QFT robust controller. Simulation results are proposed to show the effectiveness of the proposed methodology.

**Keywords:** Switching Supervisory Adaptive Control, Robust Control, QFT,GA.

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## 1. INTRODUCTION

For a highly uncertain process, in which parametric uncertainties are very large, finding a single controller that can deal with the entire range of parameter variations may be impossible. To design a satisfactory control system in the presence of large modeling uncertainties, noise, and disturbances, a hierarchical control structure can be used. The control structure consists of a bank of candidate controllers supervised by a logic-based switching [1].

In each fixed, predetermined region of uncertainty, the local controller can achieve the desired performance. Switching is made between the local controllers to support all range of uncertainties. The overall control architecture consists of a bank of controllers (multi-controllers), and a supervisor. The supervisor is also composed of a bank of models (multi-estimators), a monitoring signal generator, and a switching logic. At each time instant, a high level decision maker, the so called supervisor, determines which controller should be placed in the feedback loop. In other words, when estimates of the plant is changed, a new controller may be selected, which is similar to the idea of adaptive control. But unlike the traditional adaptive control strategies, this adaptation takes place in a discrete fashion. As a result, the overall closed loop system can be viewed as a hybrid system.

Before designing QFT controller, we use Genetic Algorithm optimization technique. A basic genetic algorithm comprises three genetic operators.

- selection
- crossover, and
- mutation

One of the main advantages of the supervisory control is its modularity [1]. Designing multi-estimators, multi-controllers, and switching logic can be done mutually independent. This feature enables the designer, to use “off-the-shelf” robust control laws. Based on this idea, in [2], a multi-model adaptive PID controller is

developed and evaluated in a simulation study for a nonlinear pH neutralization process. A methodology that blends robust non-adaptive mixed m-synthesis designs and stochastic hypothesis-testing concepts is introduced in [3].

In [4], linear limitations of linear robust controllers overcome by combining switching and Quantitative Feedback Theory (QFT). QFT is a powerful design methodology that provides a transparent tradeoff between different often conflicting design specifications. It suggests a controller with minimum cost of feedback that satisfies the set of performance specifications in spite of the plant uncertainty [5], [6]. Combining robust designs and switching, the designed controller optimizes the time response of the system by fast adaptation of the controller parameters during the transient response based on the error amplitude [4].

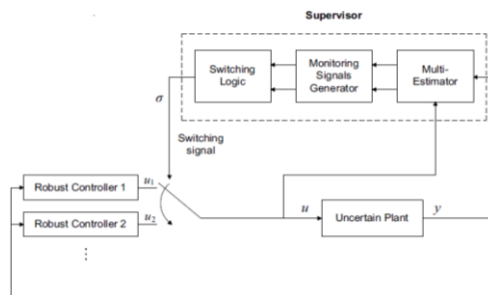
In this paper, the problem of robust adaptive control via combining QFT and switching supervisory control is introduced; the entire region of uncertainty is partitioned into smaller regions. For each region, a QFT controller is employed to attain robust stability and performance despite uncertainties and disturbances. A supervisory architecture orchestrates controller selection. This selection is based on the values of monitoring signals.

This paper is organized as follows: In Section II, switching supervisory control is reviewed. Controller design by using GA based PID controller can be explained in Section III. Fundamentals of QFT and a comparison of the present work with that of [4] are given in Section IV. Our proposed design structure is described in Section V. Simulation results and conclusions are made in Sections VI, and VII, respectively.

## II. Switching Supervisory Control

In supervisory control, the basic idea is to discretize the uncertainty set into a finite number of nominal values and then employ a family of controllers, one for each nominal value. Switching among the controllers is orchestrated by a supervisor in such a way that closed-loop stability is guaranteed. The benefits gained by this approach include (i) simplicity and modularity in design: controller design amounts to controller design for known linear time-invariant systems for which various computationally efficient controller design tools are available; (ii) ability to handle larger classes of systems than is possible with other approaches (see [1] for more discussion).

We quickly review here the supervisory control framework for adaptively controlling plants with large modeling uncertainty (see Figure 1); for details, see e.g. ([7], Chapter 6) or [1] and the references therein.



The supervisory control framework

Consider an uncertain linear plant  $M_p$  parameterized by a Parameter  $p$ , and denote by  $p^*$  the true but unknown parameter:

$$M_p: \begin{cases} \dot{x} = A_p x + B_p u \\ y = C_p x \end{cases}$$

Where  $x \in \mathbb{R}^{n-x}$  is the state,  $u \in \mathbb{R}^{n-u}$  is the input, and  $y \in \mathbb{R}^{n-y}$  is the output. The parameter  $p^* \in \mathbb{R}^{n-p}$  belongs to a known finite set  $P: = \{p_1.. p_m\}$ , where  $m$  is the cardinality of  $P$ .

It is assumed that  $(A_p, B_p)$  is stabilizable and  $(A_p, C_p)$  is detectable for every  $p \in P$ .

The supervisor comprises a multi-estimator, monitoring signals, and a switching logic. The estimator-based supervisory control design for the system (1) is briefly outlined below:

**Multi-estimator:** A multi-estimator is a collection of models, one for each fixed parameter  $p \in P$ . The multi-estimator takes in the input  $u$  and produces a bank of outputs  $y_p, p \in P$ . The multi-estimator should have the following matching property: there is  $\hat{p} \in P$  such that

$$|y_{\hat{p}}(t) - y(t)| \leq c_e e^{-\lambda_e(t-t_0)} |y_{\hat{p}}(t_0) - y(t_0)|$$

For all  $t \geq t_0$ , for all  $u$ , and for some  $c_e \geq 0$  and  $\lambda_e > 0$ . One such multi-estimator for (1) is the following dynamics

$$\begin{aligned} \dot{\hat{x}}_p &= A_p \hat{x}_p + B_p u + L_p (y_p - y), \\ y_{\hat{p}} &= C_p \hat{x}_p \end{aligned} \quad (3)$$

For all  $p \in P$ , and the property (2) is satisfied with  $\hat{p} = p^*$ . The matrix  $L_p$  in (3) is such that the eigen values of  $A_p + L_p C_p$  have negative real parts for each  $p \in P$ . (since (3) with  $p = p^*$  and (1) imply that  $(d/dt)(x_{p^*} - \hat{x}) = (A_p + L_p C_p)(x_{p^*} - \hat{x})$  and  $y = C_{p^*} \hat{x}$ ).

**Multi-controller:** A family of candidate controllers  $\{G_p\}$  is designed such that the closed loop system meets the desired robust stability and performance specifications for every  $p \in P$ . Then the family of controllers is  $u_p, p \in P$ .

**Monitoring signals:** Monitoring signals  $\mu_p, p \in P$  are norms of the output estimation errors,  $y_p - y$ . Here, the monitoring signals are generated as

$$\mu_p = \varepsilon_0 + \int_0^t e^{-\lambda(t-s)} \gamma |y_p(s) - y(s)|^2 ds$$

for some  $\gamma, \varepsilon_0, \lambda > 0$ . The numbers  $\gamma, \varepsilon_0$  and  $\lambda$  are design parameters and need to satisfy

$0 < \lambda < \lambda_0$  for some constant  $\lambda_0$  related to the eigen values of the closed loop system (for details on  $\lambda_0$ , see [7]).

**Switching logic:** A switching logic produces a switching signal that nominates the active controller at each time instant. In this paper, we use the scale independent hysteresis switching logic [8]:

$$\sigma(t) := \begin{cases} \arg \min_{q \in \mathcal{P}} \mu_q(t) & \text{if } \exists q \in \mathcal{P} \text{ such that} \\ & (1+h)\mu_q(t) \leq \mu_{\sigma(t^-)}(t) \\ \sigma(t^-) & \text{else,} \end{cases}$$

Where  $h > 0$  is called a hysteresis constant and is a design parameter that prevents excessive switching. Both in theory and in practice, it is important that excessive switching be avoided. The use of a hysteresis term conveniently satisfies this requirement. The control signal applied to the plant is  $u(t) = u_\sigma(t)$ .

### III. GA Based PID Controller

The basic principles of GA were first proposed by Holland. This technique was inspired by the mechanism of natural selection, a biological process in which stronger individual is likely to be the winners in a competing environment. GA uses a direct analogy of such natural evolution to do global optimization in order to solve highly complex problems [5]. The GA architecture is shown in Fig. 4.

A basic genetic algorithm comprises three genetic operators.

- selection
- crossover, and
- mutation

### IV. QUANTITATIVE FEEDBACK THEORY

Quantitative Feedback Theory (QFT) is a powerful tool in the design of robust control systems for uncertain plants. The quantitative approach provides a transparent design methodology which enables the designer to observe clearly the limitations and trade-offs in its design [5].

QFT is devised to design robust controllers for highly uncertain, linear time-invariant (LTI), single-input single-output (SISO) systems, through a two-degrees-of-freedom design structure [6].

A. QFT design Problem

The general QFT feedback structure is shown in Figure 2.

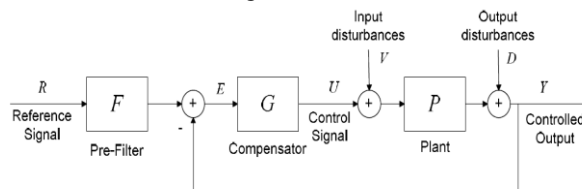


Fig 2. The two-degrees of freedom structure of QFT

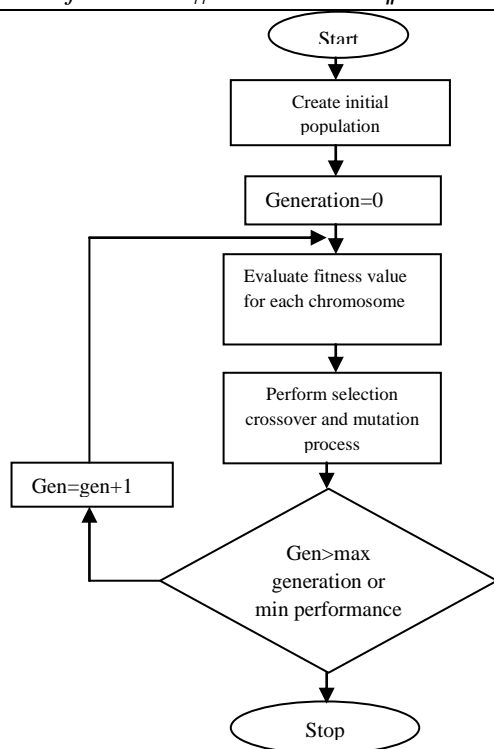


Fig. 4 Genetic algorithm architecture

The blocks to be designed by QFT in Figure 2 are the controller and the pre-filter. The QFT design, performed in the frequency domain, follows very closely classical designs using Bode plots. The model for the open-loop dynamics can either be fixed or include uncertainty. If the problem requires that the specifications be met with the uncertain dynamics, we call it a robust performance problem. That is, the performance specifications must be satisfied for all possible cases admitted by the specific uncertainty model [9].

However, the achieved performance is limited, due to linearity of the controller and its methodology. One approach to overcome this limitation is to use nonlinear controllers [4], [10], [11]). Combining switching and QFT was first introduced in [4] to prioritize some specifications over others according to the state of the system at each time. Switching is used to select the appropriate controller which is determined based on the error amplitude. Two controllers are used: the fast, more stable and imprecise controller is used when the output is too far from the reference, or equivalently when the error amplitude is large. When error is small, a controller is used which reduces the bandwidth to avoid effects of noise, and meanwhile increases the low frequency gain to minimize the jitter and the tracking error.

In this method, both of the controllers are supposed to have the same poles, so that graphical stability criteria can be utilized. This constraint, limits controllers type and therefore Performance of the system. So, a more general approach is introduced to overcome this limitation.

## VI. Switching Supervisory QFT Control

### A. Problem formulation

In the case of highly uncertain plants, two distinct cases can occur:

- A single QFT controller exists for the entire uncertainty range. However, the closed loop performance may not be improved further as desired.
- A single QFT controller does not exist to ensure closed loop robust stability and performance.

In both cases, the QFT strategy needs improvements to fulfill the practical design requirements and meet the challenges of the control of difficult highly uncertain plants.

### B. Class of admissible plants

The goal is to design a control system which can track a predetermined set point in case of plant uncertainty and disturbances. The plant is assumed to be modeled by a stabilizable and detectable SISO linear

system with control input  $u$  and measured output  $y$ , perturbed by a bounded disturbance input  $d$ . It is also assumed that the plant transfer function belongs to a known class of admissible transfer functions of the form:

$$M = \bigcup_{p \in P} M_p$$

Where  $p$  is a parameter taking values in some index set.  $M_p$  is also a family of transfer functions “centered” around some known nominal process model transfer function  $V_p$  [12]. Allowable unmodeled dynamics around the nominal process model transfer functions  $V_p$  could be specified as:

$$M_p := \{v_p(1 + \delta_m) + \delta_a : \|\delta_m\|_{\infty, \lambda} \leq \varepsilon, \|\delta_a\|_{\infty, \lambda} \leq \varepsilon\},$$

Where  $\varepsilon > 0$  and  $\lambda \geq 0$  are two arbitrary numbers. Here,  $\|\cdot\|_{\infty, \lambda}$  denotes  $e^{\lambda t}$ -weighted  $H_\infty$  norm of a transfer function:  $\|v\|_{\infty, \lambda} := \sup_{\text{Re}(s) \geq 0} |v(s - \lambda)|$  (see [12]).

Throughout the paper, we will take  $P$  to be a compact subset of a finite-dimensional normed linear vector space.

By this definition, the entire region of plant uncertainty is partitioned into a set of smaller regions. Each smaller region is presented by a parameter value  $p$ , and  $M_p$  is a model of the plant in that small region.

Considering all permissible uncertainties and disturbances in each smaller region, a controller is designed to perform robust stability and robust set point tracking specifications, via QFT.

### C. Multi-estimator and multi controller

A state-shared multi-estimator of the form

$$\dot{\hat{x}}_E = A_E x_E + L_E y + B_E u, \quad y_p = C_p x_E, \quad e_p = y_p - y,$$

Where  $p \in P$ , and  $A_E$  an asymptotically stable matrix, is utilized here. This type of structure is quite common in adaptive control [13]. Note that even if  $P$  is an infinite set, the above dynamical system is finite-dimensional. In this case the multi-estimator formally has an infinite number of outputs; however they can all be computed from  $x_E$ . Here we use state-sharing not only to generate the estimation errors  $e_p$ , but also in the monitoring signal generator for  $\mu_p$ .

For practical reasons the bank of local controllers can be efficiently implemented (multirealized) by means of a state-shared parameter dependent feedback system. The provided method can implement bumpless transfer, which is an effective way to improve poor transient response of switched systems [13], [14].

## V. SIMULATION RESULTS

In this section, a numerical example is used to illustrate the proposed design method. Consider the following uncertain plant transfer function: ([15])

$$M(s) = \frac{ka}{s(s+a)}$$

To illustrate, the two mentioned distinct cases, for QFT design, two different situations for plant uncertainties are considered.

A. Situation I: Plant uncertainty is:

$$k \in [1, 10], \quad a \in [1, 10].$$

The closed-loop desired specifications are robust stability in terms of a margin specification ( $L_f$  is the loop gain)

$$\left| \frac{L_f}{1 + L_f} \right| \leq 1.2, \quad \omega > 0$$

and a tracking specification enforced to  $\omega_h = 10$  rad/sec.

$$\frac{8400}{(s+3)(s+4)(s+10)(s+70)} \leq T_r \leq \frac{0.6584(s+30)}{s^2 + 4s + 19.75}$$

Where  $T_r = F \frac{PG}{1 + PG}$  is transfer function from reference input to the output.

Similarly, the plant output disturbance control ratio is modeled as

$$T_D(s) = \frac{s(s+70)}{s^2 + 140s + 5225}.$$

In this case, a single QFT design could cover the whole uncertainty range. But to achieve a better performance, switching QFT design is implemented. An identifier-based estimator E for this system may be constructed of the following form as in [13].

$$\dot{x}_E = \begin{bmatrix} A_E & 0 \\ 0 & A_E \end{bmatrix} x + \begin{bmatrix} b_E \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ b_E \end{bmatrix} u, \quad y_p = c_p x_E$$

Where  $(A_E, b_E)$  a stable parameter independent, controllable pair, and

$$\left\{ \begin{bmatrix} A_E & 0 \\ 0 & A_E \end{bmatrix} + \begin{bmatrix} b_E \\ 0 \end{bmatrix} c_p, \begin{bmatrix} 0 \\ b_E \end{bmatrix}, c_p \right\}$$

Is a stabilizable realization of  $M(s)$ . A state-shared implementation of this multi-estimator is then used in the supervisor.

It is possible to construct a multi-estimator  $\Sigma_E$  for this example by picking any two dimensional controllable pair  $(A_E, b_E)$  with  $A_E$  stable, and then defining  $c_p$  so that it represents the plant uncertainty. For simplicity,  $(A_E, b_E)$  is chosen to be in control canonical form and that  $A_E$ 's characteristic polynomial  $\omega_E$  is  $s^2 + \omega_2 s + \omega_1$ . Under these conditions  $c_p$  appear to be the following vector:

$$C_p := [\omega_1 \omega_2 - a \quad k_a \quad 0]$$

The performance signals  $\mu_p, p \in P$  are then constructed using the idea of state-sharing in a similar way as in [7], and [13].

In order to design the multiple-model based switching architecture the region of uncertainty is divided into the following smaller regions:

1.  $1 \leq k \leq 2, 1 \leq a \leq 10$ .
2.  $2 \leq k \leq 5, 1 \leq a \leq 10$ .
3.  $5 \leq k \leq 10, 1 \leq a \leq 10$ .

Now, for each region, a QFT design is employed to achieve robust stability and performance despite uncertainties and disturbances.

The main steps involved in the design of these QFT controllers, such as template generation, loop shaping, prefilter design, manipulation of tolerance bounds within the available freedom, template size considerations and selection of nominal transfer function matrices all require much experience and expertise. Here, as in [16] the various steps of quantitative designs are reformulated and presented in terms of appropriate cost functions and respective algebraic constraints. The resulting nonlinear constrained optimization problem can easily be solved using Genetic Algorithm. Eventually, a supervisory architecture determines the active controller which should be placed in the feedback loop. This selection is based on the values of monitoring signal. A schematic diagram of the overall multiple-model based switching algorithm is depicted in Figure 3.

To demonstrate the robustness of the closed-loop system, in the simulations, the values of the actual parameters  $k$  and  $a$  are not exactly in  $P$ . In the simulations shown in Figure 4, the values of  $k$  and  $a$  are changing with time. Comparing Figs. 4(a) and 4(b), one observes that switching QFT control laws result in control signals about 10 times smaller than those produced by single QFT control: ([15]), without sacrificing the performance.

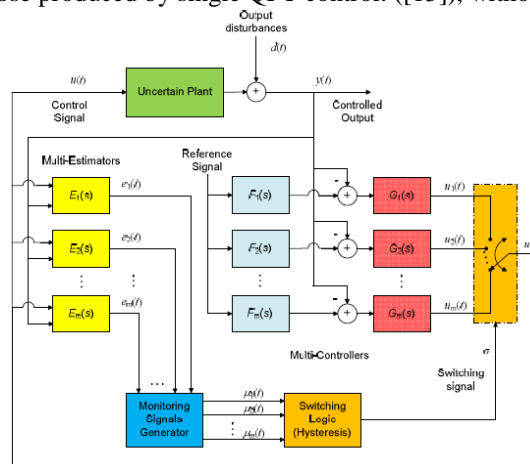


Fig 3. The Supervisory Based Switching QFT control architecture

B. Situation II: Plant uncertainty is:

$$k \in [-10, -1] \cup [1, 10], a \in [1, 10].$$

Robust stability and performance specifications are as in Situation I.

In this case, the sign of the plant gain is unknown. So, design of a single QFT controller is impossible. Switching QFT can overcome this difficulty.

In order to design the supervisory based switching QFT control the region of uncertainty is divided into the following two regions:

1.  $1 \leq k \leq 10, 1 \leq a \leq 10.$
2.  $-10 \leq k \leq -1, 1 \leq a \leq 10.$

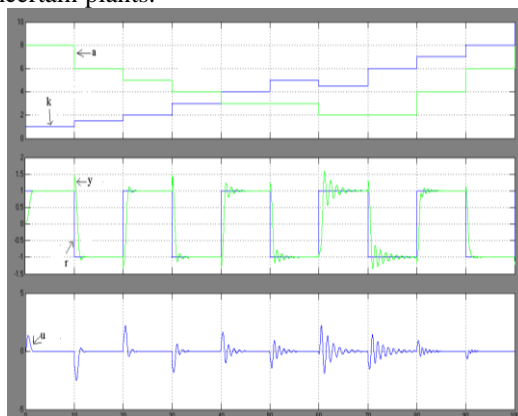
Now, for each region, a QFT design is employed to achieve robust stability and performance despite uncertainties and disturbances.

Here the optimal design of [16] is used. However, it was designed for the case where plant uncertainty is as the first region, it can be used for the second region if it is multiplied by -1. Simulation results are shown in Figure 5.

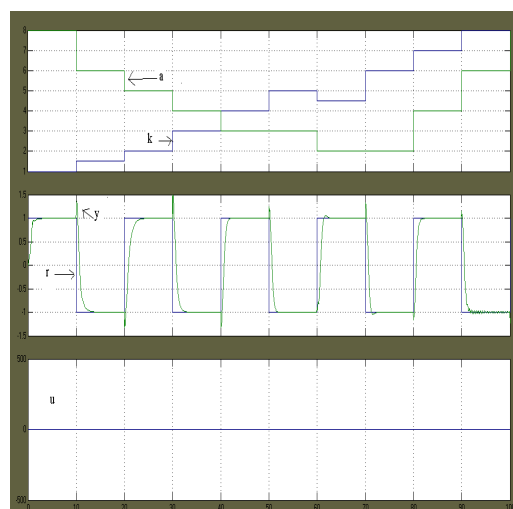
Note that, the changes in the plant parameters could be modeled as output disturbances, which is defined before in the QFT design.

### VI CONCLUSION

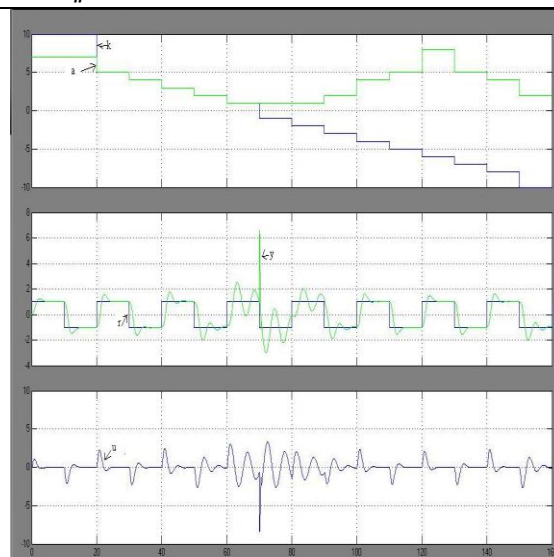
This project presents the application of QFT in switching multiple model based adaptive control. The control structure proposed consists of a bank of candidate state-shared compensators and a supervisor. Each of the candidate controllers is designed in order to achieve the demanded performance in a region of the plant uncertainty. The supervisor consists of a shared-state multi-estimator, a performance signal generator and a hysteresis switching logic scheme. The supervisor chooses the active controller corresponding to the local model which best fits the plant data. Simulation results on a design example are presented showing the performance of this algorithm for controlling uncertain plants.



Plant response for single QFT control



Plant response for multi- QFT control for Example 1



Plant response for multi- QFT control for Example 2

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