

# COMPARISON BETWEEN NEWTON RAPHSON AND CONTINUOUS POWER FLOW IN POWER SYSTEM USING MATLAB

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**ABSTRACT:** Now these days load flow is a very important and fundamental tool for the analysis of any power systems and in the operations as well as planning stages. Certain applications, particularly in distribution automation and optimization of a power system, require repeated load flow solutions. In these applications it is very important to solve the load flow problem as efficiently as possible. Since the invention and widespread use of digital computers and many methods for solving the load flow problem have been developed. Most of the methods have “grown up” around transmission systems and, over the years, variations of the Newton method have become the most widely used. Some of the methods based on the general meshed topology of a typical transmission system are also applicable to distribution systems which typically have a radial or tree structure. Specifically, we will compare the standard Newton Method, and the continuous power flow method.

**KEYWORDS**— Load Flow, Newton Raphson, Numerical Analysis, Power System, continuous power flow.

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## I. INTRODUCTION

LOAD flow study also known as power flow study, is an important tool involving numerical analysis applied to a power system. A power-flow study usually uses simplified notation such as a one-line diagram and per-unit system, and focuses on various forms of AC power (i.e.: voltages, voltage angles, real power and reactive power). Load-flow studies are performed to determine the steady-state operation of an electric power system. It calculates the voltage drop on each feeder, the voltage at each bus, and the power flow in all branch and feeder circuits. Determine if system voltages remain within specified limits under various contingency conditions, and whether equipment such as transformers and conductors are overloaded. It is used to identify the need for additional generation, capacitive, or inductive support, or the placement of capacitors and/or reactors to maintain system voltages within specified limits. Losses in each branch and total system power losses are also calculated. It is necessary for planning, economic scheduling, and control of an existing system as well as planning its future expansion.

In recent years, the increase in peak load demand and power transfers between utilities has elevated concerns about system voltage security. Voltage collapse has been deemed responsible for several major disturbances and significant research efforts are under way in an effort to further understand voltage phenomena. A large portion of this research is concentrated on the steady state aspects of voltage stability. Indeed, numerous authors have proposed voltage stability indexes based upon some type of power flow analysis. A particular difficulty being encountered in such research is that the Jacobian of a Newton-Raphson power flow becomes singular at the steady state voltage stability limit. In fact, this stability limit, also called the critical point, is often defined as the point where the power flow Jacobian is singular. As a consequence, attempts at power flow solutions near the critical point are prone to divergence and error. For this reason, double precision computation and anti-divergence algorithms such as the one found in have been used in attempts to overcome the numerical instability.

In continuous power flow, Jacobian can be avoided by slightly reformulating the power flow equations and applying a locally parameterized continuation technique. During the resulting “continuous power flow”, the reformulated set of equations remains well-conditioned so that divergence and error due to a singular Jacobian are not encountered. As a result, single precision computations can be used to obtain power flow solutions at and near the critical point.

The continuous algorithm used in this work is used to find a path of equilibrium solutions of a set of nonlinear equations. One particular application of these algorithms has been in civil engineering where the equilibrium solutions of the equations describing a structure have been studied under a change in a load intensity parameter.

## II. POWER FLOW PROBLEM FORMULATION

The goal of a power flow study is to obtain complete voltage angle and magnitude information for each bus in a power system for specified load and generator real power and voltage conditions. Once this information is known, real and reactive power flow on each branch as well as generator reactive power output can be analytically determined.

The solution to the power flow problem begins with identifying the known and unknown variables in the system. The known and unknown variables are dependent on the type of bus. A bus without any generators connected to it is called a Load Bus. A bus with at least one generator connected to it is called a Generator Bus. The exception is one arbitrarily-selected bus that has a generator. This bus is referred to as the slack bus.

In the power flow problem, if the real power and reactive power at each Load Bus are known. For this reason, Load Buses are also known as PQ Buses. For Generator Buses, it is assumed that the real power generated  $P_G$  and the voltage magnitude  $|V|$  is known. For the Slack Bus, it is assumed that the voltage magnitude  $|V|$  and voltage phase  $\theta$  are known. Therefore, for each Load Bus, the voltage magnitude and angle are unknown and must be solved for; for each Generator Bus, the voltage angle must be solved for; there are no variables that must be solved for the Slack Bus. In a system with  $N$  buses and  $R$  generators, there are then

$$2(N - 1) - (R - 1) \text{ unknowns}$$

In order to solve for the above equation, the possible equations to use are power balance equations, which can be written for real and reactive power for each bus.

$$0 = -P_i + \sum_{k=1}^N |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik})$$

The real power balance equation is

Where,  $P_i$  is the net power injected at bus  $i$ ,

$G_{ik}$  is the real part of the element in the bus admittance matrix,

$Y_{bus}$  corresponding to the  $i$ th row and  $k$ th column,

$B_{ik}$  is the imaginary part of the element.

$$0 = -Q_i + \sum_{k=1}^N |V_i| |V_k| (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik})$$

The reactive power balance equation is:

Where,  $Q_i$  is the net reactive power injected at bus  $i$ ,

$$\text{and } \theta_{ik} = \delta_i - \delta_k$$

Equations included are the real and reactive power balance equations for each Load Bus and the real power balance equation for each Generator Bus. Only the real power balance equation is written for a Generator Bus because the net reactive power injected is not assumed to be known and therefore including the reactive power balance equation would result in an additional unknown variable. For similar reasons, there are no equations written for the Slack Bus.

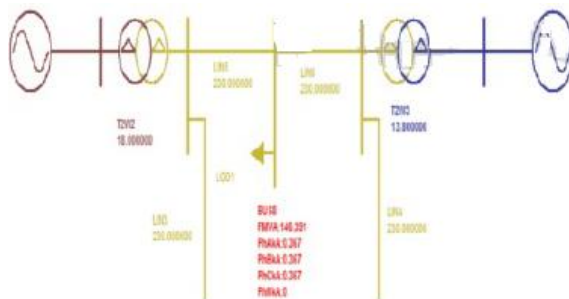


Fig1:power system analysis

### III. POWER FLOW EQUATION

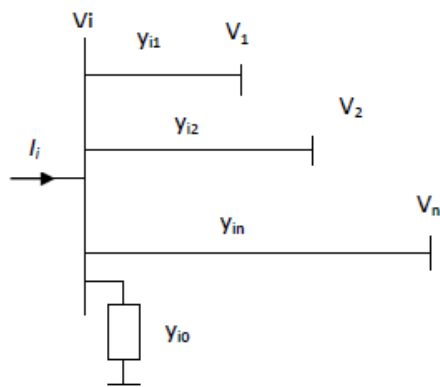


Fig 2- A typical bus of the power system

Applying KCL to this bus results in;

$$I_i = y_{io}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \dots + y_{in}(V_i - V_n) \\ = (y_{io} + y_{i1} + y_{i2} + \dots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \dots - y_{in}V_n \quad (1)$$

Representing eq. (1) in summation form;

$$I_i = V_i \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij}V_j \quad j \neq i \quad (2)$$

The complex power at its *i*th bus is;

$$P_i + jQ_i = V_i I_i^* \quad (3)$$

This is for reactive power.

$$I_i = \frac{P_i - jQ_i}{V_i^*} \quad (4)$$

This is for active power.

Substituting for  $I_i$  in (2) yields;

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij}V_j \quad j \neq i \quad (5)$$

Equation (5) is an algebraic non linear equation which must be solved by iterative techniques.

### IV. NEWTON RAPHSON METHOD

If you've ever tried to find a root of a complicated function algebraically, you may have had some difficulty. Using some basic concepts of calculus, we have ways of numerically evaluating roots of complicated functions. Commonly, we use the Newton-Raphson method. This iterative process follows a set guideline to approximate one root, considering the function, its derivative, and an initial *x*-value.

Power flow equations formulated in polar form. For the system in Fig 2, Eqn (2) can be written in terms of bus admittance matrix as;

$$I_i = \sum_{j=1}^n Y_{ij} V_j \quad (6)$$

Expressing in polar form;

$$I_i = \sum_{j=1}^n |V_{ij}| |V_j| \angle \theta_{ij} + \delta_j \quad (7)$$

Substituting for Ii from Eqn (15) in Eqn (4);

$$P_i - jQ_i = |V_i| \angle -\delta_i \sum_{j=1}^n |V_{ij}| |V_j| \angle \theta_{ij} + \delta_j \quad (8)$$

Separating the real and imaginary parts;

$$P_i = \sum_{j=1}^n |V_i| |V_j| |V_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (9)$$

$$Q_i = -\sum_{j=1}^n |V_i| |V_j| |V_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad (10)$$

$\delta_i$  is phase angle.

Expanding Eqn (9) & (10) in Taylor's series about the initial estimate neglecting high order terms we get;

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \left( \frac{\partial P_2^{(k)}}{\partial \delta_2^{(k)}} \quad \dots \quad \frac{\partial P_2^{(k)}}{\partial \delta_n^{(k)}} \right) & \left( \frac{\partial P_2^{(k)}}{\partial |V_2|} \quad \dots \quad \frac{\partial P_2^{(k)}}{\partial |V_n|} \right) \\ \vdots & \vdots \\ \left( \frac{\partial P_n^{(k)}}{\partial \delta_2^{(k)}} \quad \dots \quad \frac{\partial P_n^{(k)}}{\partial \delta_n^{(k)}} \right) & \left( \frac{\partial P_n^{(k)}}{\partial |V_2|} \quad \dots \quad \frac{\partial P_n^{(k)}}{\partial |V_n|} \right) \\ \left( \frac{\partial Q_2^{(k)}}{\partial \delta_2^{(k)}} \quad \dots \quad \frac{\partial Q_2^{(k)}}{\partial \delta_n^{(k)}} \right) & \left( \frac{\partial Q_2^{(k)}}{\partial |V_2|} \quad \dots \quad \frac{\partial Q_2^{(k)}}{\partial |V_n|} \right) \\ \vdots & \vdots \\ \left( \frac{\partial Q_n^{(k)}}{\partial \delta_2^{(k)}} \quad \dots \quad \frac{\partial Q_n^{(k)}}{\partial \delta_n^{(k)}} \right) & \left( \frac{\partial Q_n^{(k)}}{\partial |V_2|} \quad \dots \quad \frac{\partial Q_n^{(k)}}{\partial |V_n|} \right) \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}$$

The Jacobian matrix gives the linearized relationship between small changes in  $\Delta \delta_i^{(k)}$  and voltage magnitude  $\Delta |V_i|$  with the small changes in real and reactive power  $\Delta P_i^{(k)}$  and  $\Delta Q_i^{(k)}$ .

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad (11)$$

The diagonal and the off-diagonal elements of J1 are,

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (12)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad (13)$$

Similarly we can find the diagonal and off-diagonal elements of J2,J3 and J4;

The terms  $\Delta P_i^{(k)}$  and  $\Delta Q_i^{(k)}$  are the difference between the scheduled and calculated values, known as the power residuals.

$$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \quad (14)$$

$$\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)} \quad (15)$$

Continue until scheduled errors  $\Delta P_i^{(k)}$  and  $\Delta Q_i^{(k)}$  for all load buses are within a specified tolerance.

#### 4.1 Figures

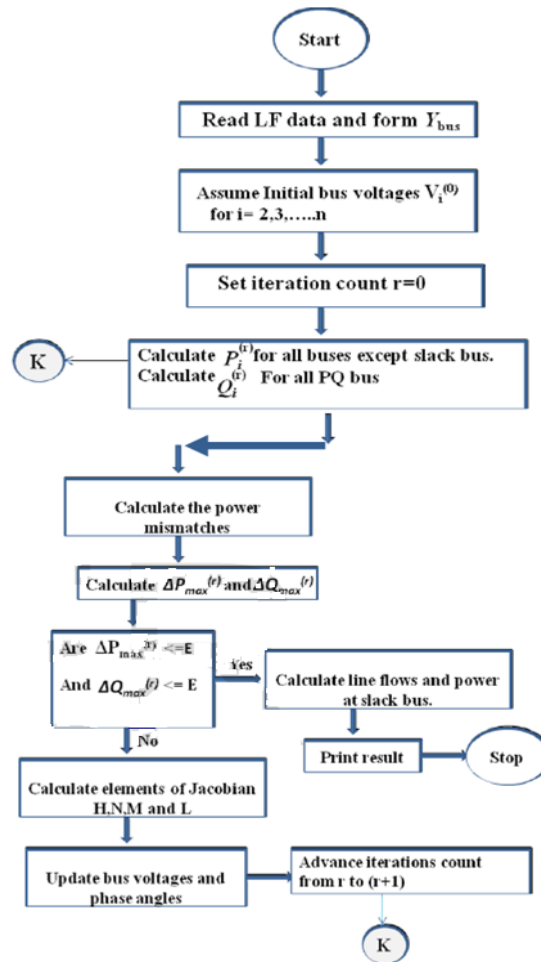


Fig 3- Flow chart of Newton Raphson Method

#### V. CONTINUOUS POWER FLOW

The Jacobian matrix of power flow equations becomes singular at the voltage stability limit. Continuous power flow overcomes this problem. Continuous power flow finds successive load flow solutions according to a load scenario.

It consists of prediction and correction steps. From a known base solution, a tangent predictor is used so as to estimate next solution for a specified pattern of load increase. The corrector step then determines the exact solution using Newton-Raphson technique employed by a conventional power flow. After that a new prediction is made for a specified increase in load based upon the new tangent vector. Then corrector step is applied. This process goes until critical point is reached. The critical point is the point where the tangent vector is zero. The illustration of predictor-corrector scheme is depicted in Figure 4.

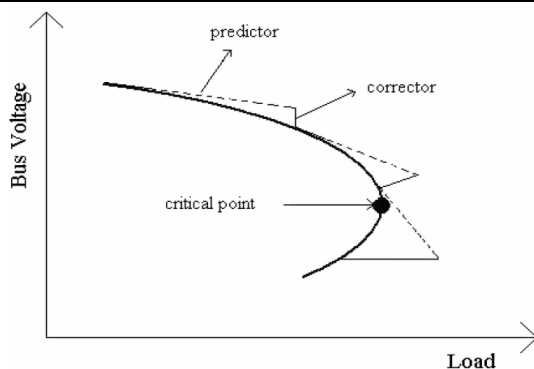


Fig 4: Illustration of prediction-correction steps

### 5.1 Mathematical Reformulation

Injected powers can be written for the  $i^{\text{th}}$  bus of an  $n$ -bus system as follows:

$$P_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \quad (16)$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik})$$

$$P_i = P_{Gi} - P_{Di}, \quad Q_i = Q_{Gi} - Q_{Di} \quad (17)$$

where the subscripts G and D denote generation and load demand respectively on the related bus.

In order to simulate a load change, a load parameter  $\lambda$  is inserted into demand powers  $P_{Di}$  and  $Q_{Di}$ .

$$\begin{aligned} P_{Di} &= P_{Dio} + \lambda(P_{\Delta base}) \\ Q_{Di} &= Q_{Dio} + \lambda(Q_{\Delta base}) \end{aligned} \quad (18)$$

$P_{Dio}$  and  $Q_{Dio}$  are original load demands on  $i^{\text{th}}$  bus whereas  $P_{\Delta base}$  and  $Q_{\Delta base}$  are given quantities of powers chosen to scale appropriately. After substituting new demand powers in Equation 3.4 to Equation 3.3, new set of equations can be represented as:

$$F(\theta, V, \lambda) = 0 \quad (19)$$

where  $\theta$  denotes the vector of bus voltage angles and  $V$  denotes the vector of bus voltage magnitudes.

### 5.2 Prediction Step

In this step, a linear approximation is used by taking an appropriately sized step in a direction tangent to the solution path. Therefore, the derivative of both sides of Equation 19 is taken.

$$\begin{aligned} F_{\theta} d\theta + F_V dV + F_{\lambda} d\lambda &= 0 \\ \begin{bmatrix} F_{\theta} & F_V & F_{\lambda} \end{bmatrix} \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} &= \underline{0} \end{aligned} \quad (20)$$

In order to solve Equation 20, one more equation is needed since an unknown variable W is added to load flow equations. This can be satisfied by setting one of the tangent vector components to +1 or -1 which is also called continuation parameter. Setting one of the tangent vector components +1 or -1 imposes a non-zero value on the tangent vector and makes Jacobian nonsingular at the critical point. As a result Equation 20 becomes:

$$\begin{bmatrix} F_{\theta} & F_V & F_{\lambda} \\ & e_k & \end{bmatrix} \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix}$$

(21)

where  $e_k$  is the appropriate row vector with all elements equal to zero except the  $k^{\text{th}}$  element equals 1. At first step is chosen as the continuation parameter. As the process continues, the state variable with the greatest rate of change is selected as continuation parameter due to nature of parameterization. By solving Equation 3.7, the tangent vector can be found. Then, the prediction can be made as follows:

$$\begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix}^{p+1} = \begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix}^p + \sigma \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix}$$

(22)

where the subscript “p+1” denotes the next predicted solution. The step size  $\sigma$  is chosen so that the predicted solution is within the radius of convergence of the corrector. If it is not satisfied, a smaller step size is chosen.

### 5.3 Correction Step

In correction step, the predicted solution is corrected by using local parameterization. The original set of equation is increased by one equation that specifies the value of state variable chosen and it results in:

$$\begin{bmatrix} F(\theta, V, \lambda) \\ x_k - \eta \end{bmatrix} = [0]$$

(23)

where  $x_k$  is the state variable chosen as continuation parameter and  $\eta$  is the predicted value of this state variable. Equation 23 can be solved by using a slightly modified Newton-Raphson power flow method.

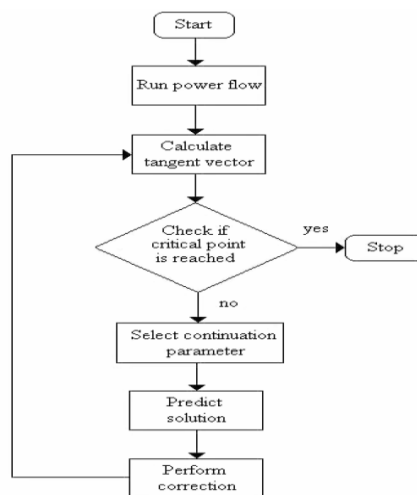


Fig 5: Flow chart for continuation power flow

## VI. RESULT

Comparison between Newton Raphson and continuous power flow

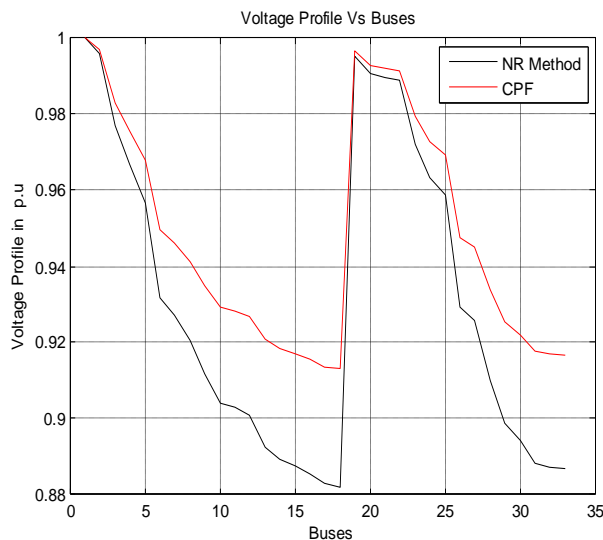


Fig 6:Graph between voltage Vs buses

## VII. CONCLUSIONS

The Newton-Raphson method fails to converge near critical operation point. At the loadability limit, Jacobian matrix of power flow equations become singular as the slope of the curve tends to infinite.

Continuous method overcomes the singularity problem by adopting a predictor-corrector scheme that finds successive load flow solutions.

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